

國立清華大學命題紙

九十五學年度 微機電系統工程研究 (系)所 _____ 組碩士班入學考試
 科目 工程數學 科號 2403 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. Consider the following equation for the temperature $u(x)$ in a chemical reacting slab of material:

$$\frac{d^2u}{dx^2} + \lambda [e^u - 1] = 0, \quad 0 < x < 1 \quad \text{and} \quad u(0) = u(1) = 0.$$

Find $u_0(x)$ and λ_0 for a small amplitude positive solution of the form

$$u(x) = \varepsilon u_0(x) + \varepsilon^2 u_1(x) + (\text{higher order terms of } \varepsilon) \quad \text{and} \quad \varepsilon = \lambda - \lambda_0 \quad (20\%)$$

2. Use Laplace transform to solve the $y(t)$:

$$\frac{d^2y}{dt^2} + 9y = f(t), \quad y(0) = \frac{dy}{dt}(0) = 0,$$

$f(t) = t$ if $0 < t < a$ and $f(t) = 0$ else. Here a is a positive number (15%)

3. Solve the equation

$$\frac{d^2y}{dt^2} - y = e^t, \quad y(0) = \frac{dy}{dt}(0) = 0 \quad (10\%)$$

4. $AX = Y + \text{noise}$, where $A = \begin{bmatrix} 1 & -2 & -1 & 2 & 1 \\ 1 & -2 & 0 & 3 & 0 \\ -1 & 2 & 2 & -1 & -2 \\ 2 & -4 & -3 & 3 & 3 \\ -1 & 2 & -1 & -4 & 1 \end{bmatrix}$, $Y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(a) Find the Rank of A (10%)

(b) Find the complete base for the row space of A (10%)

(c) Find a general solution for vector X based on the row space that you got (10%)

5. $\vec{z}_k = A\vec{z}_{k-1}$ where $\vec{z}_k = \begin{bmatrix} x_{k+1} \\ x_k \end{bmatrix}$, $A = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix}$ and $k = 1, 2, 3, \dots$

(a) Derive an expression for \vec{z}_k in terms of \vec{z}_0 (10%)

(b) The ratio $\frac{x_{k+1}}{x_k}$ approach a constant when k becomes very large. For the case in which

$a = -1$ and $b = 1$, determine the ration $\frac{x_{k+1}}{x_k}$. Hint: It is possible to answer this question without a lot of computation. (15%)