

國立清華大學命題紙

97 學年度 奈米工程與微系統研究所 (所) \_\_\_\_\_ 組碩士班入學考試

科目 工程數學 科目代碼 1803 共 2 頁第 1 頁 \*請在【答案卷卡】內作答

1. Laplace Transform can be used to solve differential equations.

The model of the system in the figure 1 is:

$$m_1 y_1'' = -k_1 y_1 + k_2 (y_2 - y_1)$$

$$m_2 y_2'' = -k_2 (y_2 - y_1) - k_3 y_2$$

while

$$m_1 = m_2 = 10 \text{ kg,}$$

$$k_1 = k_3 = 20 \text{ kg/sec}^2$$

$$k_2 = 40 \text{ kg/sec}^2$$

(a) Please find the solutions  $[y_1(t) = ?, y_2(t) = ?]$  which satisfying the initial conditions:

$$y_1(0) = y_2(0) = 0$$

$$y_1'(0) = 1 \text{ m/sec}$$

$$y_2'(0) = -1 \text{ m/sec}$$

through Laplace Transform (15 points)

(b) When

$$y_1(0) = y_2(0) = 1 \text{ meter}$$

$$y_1'(0) = y_2'(0) = 0$$

Please find the solutions  $[y_1(t) = ?, y_2(t) = ?]$  through Laplace Transform and compare the solutions in (a) and (b) (frequency, type of motion...etc) (12 points)

(c) These differential equations are also a typical eigenvalue problem. Please solve the (a) by the method of eigenvalue problem. (15 points)

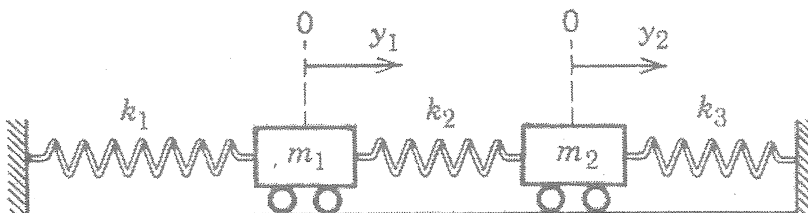


Figure 1. Mass-Spring System

2. Partial Differential Equations

Find the solution  $u(x,y)$  of following equations using separation variables:

- (a)  $u_x = y u_y$  (7 points)
- (b)  $a y u_x = b x u_y$  (8 points)
- (c)  $x^2 u_{xy} + 3y^2 u = 0$  (8 points)

3. Find the solution of the following Bernoulli equation. (15 points)

$$y' - 4y = 4y^2$$

4. (Vector transformation in 3D space) A vector in 3D space can be expressed by different coordination, for example, in both rectangular and cylindrical systems,

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} = A_\rho \hat{\rho} + A_\phi \hat{\phi} + A_z \hat{z}, \text{ where } [A_\rho, A_\phi, A_z]^T = Q_{rc} [A_x, A_y, A_z]^T.$$

- (a) Find the coordinate transformation matrix  $Q_{rc}$  (15 points.)
- (b) Similarly,  $[A_x, A_y, A_z]^T = Q_{cr} [A_\rho, A_\phi, A_z]^T$ . Find  $Q_{cr}$  (5 points)

