

國立清華大學命題紙

98 學年度 統計學研究所 碩士班入學考試

科目 基礎數學 科目代碼 0101 共 3 頁第 1 頁 \*請在【答案卷卡】內作答

除了第 13 題以外，每一題都要有計算過程與說明。

1. (5%) Is the following statement correct? Prove or disprove it (by giving a counter example): If

the limits of the functions  $f$  and  $g$  exist as  $x$  approaches  $a$ , then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ .

2. (5%) Please find the root of  $x - \cos x = 0$ ,  $0 \leq x \leq \pi/2$  with an accuracy of three decimal places using Newton's method.

3. Suppose  $A$  is a  $2 \times 2$  matrix and  $A^2 = I$ .

(a) (5%) If  $A \neq I, -I$ , find the trace and determinant of  $A$ .

(b) (5%) If, in addition to (a), the first row of  $A$  is  $[3 \quad -1]$ , find the second row of  $A$ .

4. (5%) If  $W$  is the space spanned by the two vectors  $c_1 = (1, 0, 1)$  and  $c_2 = (0, 1, 1)$ , please find a matrix  $A$  such that  $Ay$  is the orthogonal projection of any vector  $y \in \mathbb{R}^3$  into  $W$ . That is, we have  $(y - Ay) \perp Ay$ .

5. (5%) Please solve the following set of differential equations:

$$\frac{du}{dt} = 3u - 2v, \frac{dv}{dt} = 5u - 4v \text{ with the initial value } u=13 \text{ and } v=22 \text{ for } t=0.$$

6. (5%) If  $v_1, v_2, v_3$  are three linearly independent vectors in  $\mathbb{R}^n$ , and we have

$w_1 = 3v_1, w_2 = 2v_1 - v_2, w_3 = v_1 + v_3$ , please check if  $w_1, w_2, w_3$  are also linearly independent.

7. (5%) Given  $A = \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$ , find  $A^k$ , for all  $k \geq 2$

8. Please calculate the following limit.

(a) (5%)  $\lim_{x \rightarrow 0} \left[ \frac{1}{\ln(x + \sqrt{1+x^2})} - \frac{1}{\ln(1+x)} \right]$

(b) (5%)  $\lim_{x \rightarrow 0} x \int_x^1 \frac{\cos t}{t^2} dt$

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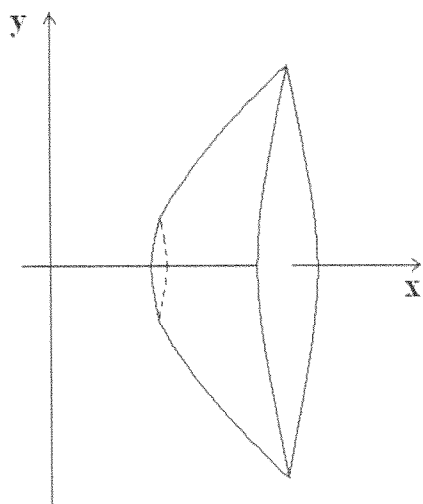
科目 基礎數學 科目代碼 0101 共 3 頁第 2 頁 \*請在【答案卷卡】內作答

9. Please determine if each of the following series is convergent. If so, please find the value it converges to.

(a) (5%)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}$

(b) (5%)  $\sum_{n=1}^{\infty} \frac{\cos(2n\pi/3)}{2^n}$

10. (8%) 把拋物線  $y = x \ln x$  從  $x=1$  到  $x=e$  的一段繞  $x$  軸旋轉 (如下圖所示), 求所得的旋轉體的體積。



11. (8%) 在橢圓  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  中嵌入有最大面積且四邊平行於橢圓軸的矩形, 請問此矩形的長寬為何?

12. (8%) Let  $F(x) = \sum_{j=n}^{\infty} e^{-x} \left(\frac{x^j}{j!}\right)$  and  $n$  is a positive integer. Please find  $\int_0^{\infty} xF'(x)dx$ .

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13. (16%) Mark each of the following statements true (T) or false(F).

- (a) If  $f(x) = x^3 \sin(1/x)$  for  $x \neq 0$  and  $f(0) = 0$ , then  $f'$  is a continuous function at  $x=0$ .
- (b) If  $f'(x) < 1$ , for all  $a \leq x \leq b$ , then there exists  $c$  and  $d$  such that  
 $f(d) - f(c) \leq d - c$  for  $a \leq c \leq d \leq b$ .
- (c) If  $f$  is a differentiable function, then  $f'$  is a continuous function
- (d) The sequence of functions  $f_n(x) = \frac{\sin nx}{n}$ ,  $n=1,2,\dots$ , are uniformly convergent in  $R$ , which is the 1-dimension real space.
- (e) The limit  $\lim_{x \rightarrow \pi/2} (\tan x)^{\cos x}$  does not exist
- (f) If  $v$  is an eigenvector of an invertible matrix  $A$ , then  $cv$  is an eigenvector of  $A^{-1}$  for all nonzero scalars  $c$ .
- (g) If  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  are continuous at  $(x_0, y_0)$ , then  $f$  is differentiable at  $(x_0, y_0)$ .
- (h) Suppose  $M_{22}$  stands for the vector space of all  $2 \times 2$  matrices. Then the set of all nonsingular matrices is a subspace of  $M_{22}$ . (The main point is, nonsingular matrices do not form a vector space. Nonsingular+nonsingular can be singular)