

國立清華大學命題紙

98 學年度 統計學研究所 碩士班入學考試

科目 基礎數學 科目代碼 0101 共 3 頁第 1 頁 *請在【答案卷卡】內作答

除了第 13 題以外，每一題都要有計算過程與說明。

1. (5%) Is the following statement correct? Prove or disprove it (by giving a counter example): If

the limits of the functions f and g exist as x approaches a , then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$.

2. (5%) Please find the root of $x - \cos x = 0$, $0 \leq x \leq \pi/2$ with an accuracy of three decimal places using Newton's method.

3. Suppose A is a 2×2 matrix and $A^2 = I$.

(a) (5%) If $A \neq I, -I$, find the trace and determinant of A .

(b) (5%) If, in addition to (a), the first row of A is $[3 \quad -1]$, find the second row of A .

4. (5%) If W is the space spanned by the two vectors $c_1 = (1, 0, 1)$ and $c_2 = (0, 1, 1)$, please find a matrix A such that Ay is the orthogonal projection of any vector $y \in \mathbb{R}^3$ into W . That is, we have $(y - Ay) \perp Ay$.

5. (5%) Please solve the following set of differential equations:

$$\frac{du}{dt} = 3u - 2v, \frac{dv}{dt} = 5u - 4v \text{ with the initial value } u=13 \text{ and } v=22 \text{ for } t=0.$$

6. (5%) If v_1, v_2, v_3 are three linearly independent vectors in \mathbb{R}^n , and we have

$w_1 = 3v_1, w_2 = 2v_1 - v_2, w_3 = v_1 + v_3$, please check if w_1, w_2, w_3 are also linearly independent.

7. (5%) Given $A = \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$, find A^k , for all $k \geq 2$

8. Please calculate the following limit.

(a) (5%) $\lim_{x \rightarrow 0} \left[\frac{1}{\ln(x + \sqrt{1+x^2})} - \frac{1}{\ln(1+x)} \right]$

(b) (5%) $\lim_{x \rightarrow 0} x \int_x^1 \frac{\cos t}{t^2} dt$

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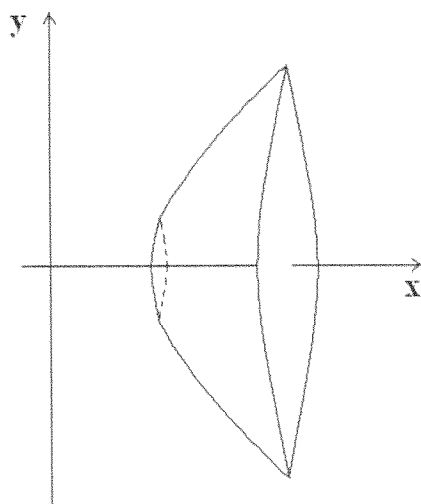
科目 基礎數學 科目代碼 0101 共 3 頁第 2 頁 *請在【答案卷卡】內作答

9. Please determine if each of the following series is convergent. If so, please find the value it converges to.

(a) (5%) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}$

(b) (5%) $\sum_{n=1}^{\infty} \frac{\cos(2n\pi/3)}{2^n}$

10. (8%) 把拋物線 $y = x \ln x$ 從 $x=1$ 到 $x=e$ 的一段繞 x 軸旋轉 (如下圖所示), 求所得的旋轉體的體積。



11. (8%) 在橢圓 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 中嵌入有最大面積且四邊平行於橢圓軸的矩形, 請問此矩形的長寬為何?

12. (8%) Let $F(x) = \sum_{j=n}^{\infty} e^{-x} \left(\frac{x^j}{j!}\right)$ and n is a positive integer. Please find $\int_0^{\infty} xF'(x)dx$.

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13. (16%) Mark each of the following statements true (T) or false(F).

- (a) If $f(x) = x^3 \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$, then f' is a continuous function at $x=0$.
- (b) If $f'(x) < 1$, for all $a \leq x \leq b$, then there exists c and d such that
 $f(d) - f(c) \leq d - c$ for $a \leq c \leq d \leq b$.
- (c) If f is a differentiable function, then f' is a continuous function
- (d) The sequence of functions $f_n(x) = \frac{\sin nx}{n}$, $n=1,2,\dots$, are uniformly convergent in R , which is the 1-dimension real space.
- (e) The limit $\lim_{x \rightarrow \pi/2} (\tan x)^{\cos x}$ does not exist
- (f) If v is an eigenvector of an invertible matrix A , then cv is an eigenvector of A^{-1} for all nonzero scalars c .
- (g) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are continuous at (x_0, y_0) , then f is differentiable at (x_0, y_0) .
- (h) Suppose M_{22} stands for the vector space of all 2×2 matrices. Then the set of all nonsingular matrices is a subspace of M_{22} . (The main point is, nonsingular matrices do not form a vector space. Nonsingular+nonsingular can be singular)