

國立清華大學命題紙

甲組(資訊系統管理組)

99 學年度 資訊系統與應用研究所丙組(生物及醫學資訊組) 碩士班入學考試

科目 機率論 科目代碼 2202 共 3 頁, 第 1 頁 \*請在【答案卷卡】作答

2002

I. (35%) Answer the following questions (7% for each question).

The probability that any particular hard disk will fail during its  $K$ th year of use is given by the probability mass function for  $K$ ,

$$P(K) = p \cdot (1-p)^{K-1}, K = 1, 2, 3, \dots$$

1. Four hard disks are tested simultaneously. Determine the probability that none of the four hard disks fails during its first year of use. (Express the answer in terms of  $p$ .)
2. Four hard disks are tested simultaneously. Determine the probability that exactly two hard disks have failed by the end of the third year. (Express the answer in terms of  $p$ .)
3. Four hard disks are tested simultaneously. Determine the probability that exactly one hard disks fails during each of the first three years. (Express the answer in terms of  $p$ .)
4. Four hard disks are tested simultaneously. Given that one hard disk has failed by the end of the first year, determine the probability that exactly two hard disks have failed by the end of the third year. (Express the answer in terms of  $p$ .)
5. Suppose that during the first three years the probability that a hard disk will fail is 0.488. What is the probability that a hard disk will fail during its first year of use? (That is, determine the value of  $p$ .)

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II. (35%) Answer the following questions.

1. (15%) Let the lifetime (in years) of a mobile phone of a particular type be modeled by an exponentially distributed random variable with a mean lifetime of 3 years.
- Write down the probability density function of this random variable. Justify that it satisfies the requirements of a probability density function for a continuous random variable.
  - Compute its cumulative distribution function.
  - What is the probability that a mobile phone of this type still functions after using 6 years?

2. (10%) Let  $X$  be a random variable with the following probability density function:

$$f(x) = \begin{cases} \frac{x+2}{8} & -2 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

- Find the probability density function of the random variable  $Y = X^2$ .
  - Compute the cumulative distribution function of the random variable  $Y$ .
3. (10%) The joint density function of random variables  $X$  and  $Y$  is given by

$$f(x, y) = \begin{cases} x+y & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Compute the probability density function for  $X$  and the expectation  $E(X)$ .
- Compute the probability  $P(X+Y < 1)$ .

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III. (30%) Answer the following questions (10% for each question).

1. The number of cells out of 100 that exhibit chromosome aberrations is a random variable  $X$  with a probability density function given by

$$P(X=i) = \beta[(i+1)^2/2^{i+1}], \quad i=0,1,2,3,4,5.$$

- (a) Determine the value of the constant  $\beta$ .  
(b) Compute the mean and variance of  $X$ .

2. Suppose that 2000 points are selected independently at random from the unit square  $S=\{(x,y)|0 \leq x \leq 1, 0 \leq y \leq 1\}$ . Let  $Y$  equal the number of points that fall in  $C=\{(x,y)|x^2+y^2 \leq 1\}$ .

- (a) How is  $Y$  distributed?  
(b) Compute the mean and variance of  $Y$ .  
(c) What is the expected value of  $Y/500$ ?

3. Let  $W$  have an exponential distribution with variance 4.

- (a) What is the probability density function of  $W$ ?  
(b) Compute the mean and median of  $W$ .