

科目：工程數學 A(5003)

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1. The field \mathbb{Z}_2 consists of two elements 0 and 1 with the operations of addition (+) and multiplication (\cdot) defined by $0+0=0, 0+1=1, 1+0=1, 1+1=0, 0\cdot 0=0, 0\cdot 1=0, 1\cdot 0=0, \text{ and } 1\cdot 1=1$.

$$\text{Let } A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad \text{and } b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \text{where all entries are in } \mathbb{Z}_2.$$

- (a) (3%) For the matrix A , compute the rank and the inverse if it exists.
- (b) (4%) Determine whether the system $Ax = b$ is consistent. If the system is consistent, find all solutions.
- (c) (3%) Find a basis for the solution set of the corresponding homogeneous system.
2. Let $S = \{(1, 0, 1), (2, 2, 2)\}$ in \mathbb{R}^3 , and $W = \text{span}(S)$.
- (a) (6%) Find an orthonormal basis of W and W^\perp .
- (b) (3%) If $x = (2, 0, 0)$, find the closet vector u on W to x .
- (c) (3%) What is the closest distant from W to x ? Please also specify the corresponding vector z .
- (d) (3%) Please plot a schematic diagram that specifies the relation of x, u, z, W and W^\perp .
3. The differential equation: $\ddot{y}(t) + a\dot{y}(t) + by(t) = u(t)$, where a and b are constants and $u(t)$ is the unit step function. All initial conditions are zero.
- (a) (5%) Solve $y(t)$ when $a = 2$ and $b = 4$.
- (b) (5%) Solve $y(t)$ when $a = 4$ and $b = 4$.
4. (5%) EM wave propagates inside an absorptive material. The absorbed intensity amount per penetration depth is proportional to the intensity at that position. Write down a mathematic model to describe the phenomenon above and obtain the general solution.
5. Solve the initial value problems,
- (a) (5%) $xy' = y + \sqrt{x^2 + y^2}, y(2) = 0$.
- (b) (5%) Solve the initial value problem, $y_1' - y_1 - y_2 = 3x$
 $y_1' + y_2' - 5y_1 - 2y_2 = 5$, with $y_1(0) = 3, y_2(0) = 4$.
6. Evaluate the following integrals.
- (a) (6%) $\int_{-\infty}^{\infty} \frac{x^3 + 1}{x^4 + 1} dx$;
- (b) (6%) $\int_{-\infty}^{\infty} \frac{\sin(kx)}{x - a} dx$, where $ka = \pi$.

注意：背面有試題

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7. (5%) The following Legendre Equation

$$(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$$

with $\alpha = 3$ has two linearly independent solutions. Calculate to find the solution that is a polynomial.

8. Let
- $f(t)$
- be a periodic function with period
- p
- , and its corresponding Laplace Transform
- $F(s) = \mathcal{L}\{f(t)\}$
- exists.

(a) (4%) Derive the general form of $F(s)$ in terms of a single integral within a finite range. "Finite range" means that neither the upper or lower bound of the integral is infinity.(b) (4%) Calculate $X(s)$, the Laplace Transform of $x(t)$, from the following initial value problem

$$x'' + 6x' + 10x = 5f(t); \quad x(0) = x'(0) = 0,$$

where $f(t)$ is periodic with period 2, and is defined as $f(t) = \delta(t-1)$, for $0 \leq t < 2$, where $\delta(t)$ is the delta function that describes an infinite-sharp impulse.You do not need to perform inverse Laplace Transform to further calculate $x(t)$, so your answer for $X(s)$ should contain a single term but not an infinite series.

9. Consider all piecewise continuous periodic functions
- $f(t)$
- with a period
- $2L$
- that satisfy
- $f(t) = f(t^-) + f(t^+)$
- , where
- $f(t^\pm)$
- are the function's right (left) limits at
- t
- . We can define the inner product between any two such functions
- $f_1(t)$
- and
- $f_2(t)$
- as

$$f_1 \cdot f_2 = \int_0^{2L} f_1(t)f_2(t)dt.$$

Moreover, there is a theorem telling that the corresponding Fourier basis functions 1 , $\cos \frac{m\pi t}{L}$, and $\sin \frac{m\pi t}{L}$ with $m=1, 2, 3, \dots$ form a complete basis set for all such functions, i.e., any such $f(t)$ can be expressed as

$$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} \left(a_m \cos \frac{m\pi t}{L} + b_m \sin \frac{m\pi t}{L} \right)$$

(a) (4%) Using the fact that all the Fourier basis functions are orthogonal to each other, calculate all a_m and b_m .(b) (8%) Calculate the exact value of Leibniz's series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$.

10. Consider the complex function
- $f(z) = x^2 - iy^2$
- .

(a) (3%) Where is $f(z)$ differentiable?(b) (3%) Where is $f(z)$ analytic?

11. (7%) Consider a branch
- $f(z)$
- of
- $(z^2 - 1)^{1/2}$
- that is analytic in the exterior of the unit circle,
- $|z| > 1$
- . If
- $f(\sqrt{2}) = -1$
- , find
- $f(-i\sqrt{2})$
- .