

科目：工程數學 A(5003)

校系所組：中央大學電機工程學系(電子組、系統與生醫組)

交通大學電子研究所(甲組、乙組)

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1. (13%) For a 3x3 matrix  $A$ , where  $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$

(a) Please find its eigenvalues and corresponding eigenvectors. (6 %)

(b) Assume the 3 eigenvalues are in the order of  $\lambda_1 \leq \lambda_2 \leq \lambda_3$ . Starting from the eigenvector corresponding to  $\lambda_1$ , please find the corresponding orthonormal basis. (7 %)

2. (12%) Define the space  $P_n$  as the set of all polynomials of degree less than  $n$ . Let  $L$  be the operator on  $P_3$  and

$$L(p(x)) = \alpha + x \frac{d}{dx} p(x) + \frac{d^2}{dx^2} p(x)$$

(a) (3%) Find the matrix  $A$  representing  $L$  with respect to  $[1, x, x^2]$ .

(b) (3%) Find the matrix  $B$  representing  $L$  with respect to  $[1, x, 1+x^2]$ .

(c) (3%) Find the condition of  $\alpha$  such that  $A$  and  $B$  are similar matrices.

(d) (3%) If  $p(x) = a_0 + a_1x + a_2x^2$ , calculate  $L^n(p(x))$  given the condition of  $\alpha$  in (c).

3. (7%) Develop  $f(z) = \frac{1}{1-z^4}$  in a Maclaurin series and find the radius of convergence.

4. (8%) Evaluate  $\int_C \bar{z} dz$ , where  $C$  is from 0 along the parabola  $y = x^2$  to  $3 + 9i$ .

5. (10 %) Calculate  $\int_{-\infty}^{\infty} \frac{e^{ax}}{1+x} dx$ , where  $0 < a < 1$ .

6. (2%) (a) Solve  $y' = y^2, y(0) = 2$ . Call the solution  $y_c$ .

(2%) (b) Solve  $y' = y^2 - 1, y(0) = 1$ . Call the solution  $y_p$ .

(1%) (c) Does  $y_c + y_p$  solve  $y' = y^2 - 1, y(0) = 3$ ? Explain.

7. (8%) One solution of the equation  $y'' + p(t)y' + q(t)y = 0$  is  $(1+t)^2$ , and the Wronskian of any two solutions is constant. Find the general solution of  $y'' + p(t)y' + q(t)y = 1+t$ .

8. (5%) Three solutions of a 2nd-order linear equation  $L(y) = g(t)$  are

$$\psi_1 = 2e^{t^2} + e', \psi_2 = te^{t^2} + e' \text{ and } \psi_3 = (1+t)e^{t^2} + e'$$

Find the solution of the initial problem  $L(y) = g(t); y(0) = 3, y'(0) = 0$

注意：背面有試題

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9. (8%) Let  $y$  be a real function of  $x$ . Find two linearly independent Frobenius solutions of the following differential equation at  $x = 0$ :

$$2x^2y'' + x(x-3)y' + 3y = 0$$

10. (8%) Let  $x_1$  and  $x_2$  be two real functions of  $t$ . Solve  $x_1$  and  $x_2$  for the following system of differential equations

$$\begin{cases} x_1' = 4x_1 - x_2 \\ x_2' = x_1 + 2x_2 \end{cases}, x_1(1) = 5, x_2(1) = 3$$

11. (7%) Given the initial value problem,  $x'' + 4x' + 13x = f(t)$ ;  $x(0) = x'(0) = 0$

(a) (3%) Express  $x(t)$  in terms of  $f(t)$  and convolution.

(b) (4%) Solve  $x(t)$  for  $f(t) = u(t) - u(t-1)$ , where  $u(t)$  denotes the unit step (or Heaviside Step) function.

12. (9%)  $f(t) = \begin{cases} 1, & 0 < t < 5 \\ 0, & 5 < t < 10 \end{cases}$  with  $f(t+10) = f(t)$  is a piecewise continuous and periodic function that

satisfies  $f(t) = \frac{[f(t^+) + f(t^-)]}{2}$ , where  $f(t^+)$  and  $f(t^-)$  are the right-hand and left-hand limits of  $f(t)$  at each discontinuity.

(c) (3%) Find the Fourier series of  $f(t)$ .

(d) (3%) Let  $f(t)$  be defined for  $t \geq 0$ ; find its Laplace transform  $F(s)$  for  $s > 0$ .

(e) (3%) Find a particular solution for  $x'' + 16x = f(t)$ .