

參考用

科目：工程數學A(3003)

校系所組：中央大學電機工程學系(電子組)

交通大學電子研究所(甲組、乙A組、乙B組)

清華大學電機工程學系(甲組)

清華大學光電工程研究所

清華大學電子工程研究所

清華大學工程與系統科學系(丁組)

陽明大學生物醫學工程學系(醫學電子組)

1. (Total 15%)

Let $M_{n \times n}(C)$ be the vector space consisting of all $n \times n$ matrices with complex entries. Two matrices $A, B \in M_{n \times n}(C)$ are said to be unitarily equivalent if there exists a unitary matrix $P \in M_{n \times n}(C)$ such that $A = P^*BP$, where P^* is the conjugate transpose of P .

(a) (10%) Let $A, B \in M_{n \times n}(C)$ be unitarily equivalent. Show that $\text{tr}(A^*A) = \text{tr}(B^*B)$.

(b) (5%) Determine whether the matrices $A = \begin{pmatrix} 1 & 2 \\ 2+i & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1+2i & 1 \\ 4i & 2 \end{pmatrix}$ are unitarily equivalent (you need to justify your answer)?

2. (Total 10%)

Let $P_2(R)$ be a vector space that consists of all polynomials with real coefficients and with degree less or equal to 2. Let T be a linear operator on $P_2(R)$ defined by

$$Tf(x) = f(2x-1) - 2xf'(x),$$

for all $f(x) \in P_2(R)$.

(a) (6%) Suppose that $\beta = \{1+x^2, x+x^2, 1+x+x^2\}$ is an ordered basis for $P_2(R)$. Find the matrix representation of T in β , i.e., $[T]_\beta$.

(b) (4%) Let $A = [T]_\beta$ and let U be a linear operator on R^3 defined by

$$U(x) = Ax$$

for all $x \in R^3$. Find a basis for the range space of U .

3. (Total 20%)

(a) (6%) Solve $y' + \frac{1}{x} \cdot y = 3x^2$.

(b) (14%) Given the differential equation: $\ddot{x}(t) + a \cdot \dot{x}(t) + b \cdot x(t) = u(t)$ where $u(t)$ is a unit-step function, the response $x(t)$ is expressed as: $x(t) = 0.01 \cdot \left[1 - c \cdot e^{-5\sqrt{2}t} \cdot \sin(\sqrt{2}t + \theta) \right]$. All initial conditions are zero. Please calculate the constants a , b , c , and the angle θ (the unit is radian).

注意：背面有試題

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4. (Total 15%)

If a periodic function whose Laplace transform is shown as:

$$F(s) = \frac{s}{(s^2 + 1)(1 - e^{-\pi s})}$$

(a) (10%) Using power series to expand $\frac{1}{1 - e^{-\pi s}}$ and find the corresponding periodic function $f(t)$.(b) (5%) Please plot this function $f(t)$ in t -domain.

5. (Total 15%)

For a function: $f(x) = |\sin x|$, where $-\pi < x < \pi$. If we wish to use a function $g(x)$, which is in a finite-dimensional vector space V spanned by trigonometric functions: $\sin nx$ and $\cos nx$ for $n = 0$ to 5 , to approximate $f(x)$.

(a) (6%) Does the set $\{\sin x, \sin 2x, \sin 3x, \sin 4x, \sin 5x, \cos 0x, \cos x, \cos 2x, \cos 3x, \cos 4x, \cos 5x\}$ form an orthogonal basis in V for $-\pi < x < \pi$?(b) (9%) Please find $g(x)$ in V that is "closest" to $f(x)$, i.e., $g(x)$ is with minimum square error from $f(x)$ in V .

You may need the following formulas:

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)], \quad \cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)],$$

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)], \quad \sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

6. (Total 25%)

Evaluate the integrals (counterclockwise)

(a) (10%) $\oint_C \frac{e^z}{\cos z} dz$, $C: |z| = 4.5$ (b) (15%) $\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(z) dz$, where $F(z) = \frac{2kz}{(z^2 + k^2)^2}$ and k is a constant.