科目：工程數學A(3003)

校系所組：中央大學電機工程學系(電子組)
交通大學電子研究所(甲組、乙A組、乙B組)
清華大學電機工程學系(甲組)
清華大學光電工程研究所
清華大學電子工程研究所
清華大學工程與系統科學系（丁組）
陽明大學生物醫學工程學系（醫學電子組）

1. (Total 15%)
Let $M_{n\times n}(C)$ be the vector space consisting of all $n\times n$ matrices with complex entries. Two matrices $A,B\in M_{n\times n}(C)$ are said to be unitarily equivalent if there exists a unitary matrix $P\in M_{n\times n}(C)$ such that $A=P^*BP$, where P^* is the conjugate transpose of P.

(a) (10%) Let $A,B\in M_{n\times n}(C)$ be unitarily equivalent. Show that $tr(A^*A)=tr(B^*B)$.

(b) (5%) Determine whether the matrices $A=\begin{pmatrix}1 & 2\\ 2+i & 3\end{pmatrix}$ and $B=\begin{pmatrix}1+2i & 1\\ 4i & 2\end{pmatrix}$ are unitarily equivalent (you need to justify your answer)?

2. (Total 10%)
Let $P_2(R)$ be a vector space that consists of all polynomials with real coefficients and with degree less or equal to *2*. Let T be a linear operator on $P_2(R)$ defined by

$$T f(x)=f(2x-1)-2xf'(x),$$

for all $f(x)\in P_2(R)$.

(a) (6%) Suppose that $\beta=\{1+x^2,x+x^2,1+x+x^2\}$ is an ordered basis for $P_2(R)$. Find the matrix representation of T in β, i.e., $[T]_\beta$.

(b) (4%) Let $A\overset{\Delta}{=}[T]_\beta$ and let U be a linear operator on R^3 defined by

$$U(x)=Ax$$

for all $x\in R^3$. Find a basis for the range space of U.

3. (Total 20%)

(a) (6%) Solve $y'+\frac{1}{x}\cdot y=3x^2$.

(b) (14%) Given the differential equation: $\ddot{x}(t)+a\cdot\dot{x}(t)+b\cdot x(t)=u(t)$ where u(t) is a unit-step function, the response x(t) is expressed as: $x(t)=0.01\cdot\left[1-c\cdot e^{-5\sqrt{2}\cdot t}\cdot sin\left(5\sqrt{2}\,t+\theta\right)\right]$. All initial conditions are zero. Please calculate the constants a, b, c, and the angle θ (the unit is radian).

注意：背面有試題

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4. (Total 15%)
If a periodic function whose Laplace transform is shown as:

$$F(s)=\frac{s}{(s^2+1)(1-e^{-\pi s})}$$

(a) (10%) Using power series to expand $\frac{1}{1-e^{-\pi s}}$ and find the corresponding periodic function $f(t)$.

(b) (5%) Please plot this function $f(t)$ in t-domain.

5. (Total 15%)

For a function: $f(x)=|\sin x|$, where $-\pi < x < \pi$. If we wish to use a function $g(x)$, which is in a finite-dimensional vector space V spanned by trigonometric functions: $\sin nx$ and $\cos nx$ for $n = 0$ to 5, to approximate $f(x)$.

(a) (6%) Does the set $\{\sin x, \sin 2x, \sin 3x, \sin 4x, \sin 5x, \cos 0x, \cos x, \cos 2x, \cos 3x, \cos 4x, \cos 5x\}$ form an orthogonal basis in V for $-\pi < x < \pi$?

(b) (9%) Please find $g(x)$ in V that is "closest" to $f(x)$, i.e., $g(x)$ is with minimum square error from $f(x)$ in V.

You may need the following formulas:

$$\sin x\cos y=\tfrac{1}{2}[\sin(x+y)+\sin(x-y)],\quad \cos x\sin y=\tfrac{1}{2}[\sin(x+y)-\sin(x-y)],$$
$$\cos x\cos y=\tfrac{1}{2}[\cos(x+y)+\cos(x-y)],\quad \sin x\sin y=\tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$

6. (Total 25%)
Evaluate the integrals (counterclockwise)

(a) (10%) $\oint_C \frac{e^z}{\cos z}dz,\quad C:|z|=4.5$

(b) (15%) $\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(z)dz$, where $F(z)=\frac{2kz}{(z^2+k^2)^2}$ and k is a constant.