

國 立 清 華 大 學 命 題 紙

98 學年度工程與系統科學系乙組、工程與系統科學系丙組、核子工程與科學研究所甲組、先進光源科技碩士學位學程工程與系統科學組碩士班入學考試

科目 工程數學 科目代碼 2701、2801、2901、3001 共 2 頁第 1 頁 *請在【答案卷卡】內作答

In all the problems below, show the details of your work.

1. (a) [5%] Determine the rank, nullity, number of linear independent (LI) rows and number of LI

columns for the matrix
$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}.$$

- (b) [5%] Determine the eigenvalues of the matrix
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$
 Obtain an orthogonal basis for \mathbb{R}^4 from the eigenvector of this matrix.

2. (a) [5%] Identify and classify any local extrema and saddles of the function $f(x, y) = \ln[2x(y-1)+1]$.

(b) [5%] Evaluate the line integral $I = \int_C \vec{v} \cdot d\vec{R}$ where $\vec{v} = 3x^2y^2\hat{i} + (2x^3y - e^z)\hat{j} + (2z - ye^z)\hat{k}$, and C is any path connecting from $(1, -2, -1)$ to $(-2, 3, 1)$.

- (c) [5%] Determine the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad (0 < x < L), \quad y(0) = 0, \quad y(L) = 0.$$

3. [12%] Use residues to evaluate the integral

$$\oint_C \frac{dz}{z^2 \sinh z},$$

where C is the positively oriented unit circle $|z|=1$.

4. [13%] Solve the wave equation

$$u_{tt} = c^2 u_{xx} \quad \text{for } 0 < x < 1, \quad t > 0$$

subject to the boundary conditions $u(0, t) = 0, \quad u_x(1, t) = 0$ for $t > 0$,

and the initial conditions $u(x, 0) = x, \quad u_t(x, 0) = 0$ for $0 < x < 1$.

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5. [10%] Solve the set of differential equations

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\frac{dN_3}{dt} = \lambda_2 N_2$$

for $N_1(t)$, $N_2(t)$ and $N_3(t)$, given the initial conditions :

$$N_1(0) = N_{10},$$

$$N_2(0) = N_3(0) = 0.$$

Note that λ_1, λ_2 , and N_{10} are positive constants.

6. [15%] Solve the mixed data problem by Laplace transform method:

$$y'' - 4y' + 4y(t) = t, \quad y(1) = 0, \quad y'(0) = -2.$$

7. [10%] Using power series method, find at least the first three nonzero terms of each of the linearly independent solutions for the following equation:

$$y'' + 2xy' - y(x) = 0.$$

8. [15%] Find the general solution of the following equation.

$$x^2 y'' + xy' - y(x) = \frac{1}{x^2}.$$