1. Find the transient current if
\[ R = 6 \, \Omega, \, L = 1 \, H, \, C = 0.04 \, F, \, E = 600 \, (\cos t + 4 \sin t) \, V; \]
(L, R, C, E, are measured in henrys, ohms, farads, volts, respectively.)
Initial current and charge are assumed to be zero. 
\[ (10\%) \]

2. Consider a general nonhomogeneous linear ODE
\[ y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = r(x) \]
The particular solution \( y_p(x) \) can be solved by the method of variation of parameters. That is,
\[ y_p(x) = \sum_{k=1}^{n} y_k(x) \int \frac{W_k(x)}{W(x)} r(x) \, dx \]
(Where the \( y_k \)'s are \( n \) linearly independent homogeneous solutions. \( W \) is the Wronskian of \( y_1, \ldots, y_n \), and \( W_k \) is identical to \( W \), but with the \( k \)th column replaced by a column of zeros except for the bottom element, which is 1.)
Try to solve the following equation using the method of variation of parameters.
\[ y'' + \frac{3}{4} x^{-2} y' - \frac{3}{4} x^{-3} y = 9x^{5/2} \] 
\[ (10\%) \]
3. Solve the following equation by Laplace Transform method.

\[ y' + y = f(t), \quad y(0) = 3, \]

where

\[ f(t) = \begin{cases} 
0 & 0 \leq t < \pi \\
2\cos t & t \geq \pi 
\end{cases} \]  

(10%)  

4. Find the basis of solutions \( y(x) \) of the following differential equation. Show the details of your work.

\[ xy'' + (2x + 1)y' + (x + 1)y = 0. \]  

(10%)  

5. Find a unit vector normal to surface \( S \) given by \( \cos(xy) = e^x - 1 \) at the point \( (1, \pi, 0) \).

(10%)  

6. Let \( \mathbf{F} = (x-y)i + (y-z)j + (z-x)k \). Evaluate the surface integral of \( \mathbf{F} \) over the unit sphere defined by \( x^2 + y^2 + z^2 = 1 \).

(10%)  

7. Define the Fourier transform of \( f(x) \) to be

\[ \hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} \, dx. \]

(a) [5%] Calculate the Fourier transform of

\[ f(x) = \begin{cases} 
a - |x| & |x| < a \\
0 & \text{otherwise}
\end{cases} \]

(b) [5%] Consider the one-dimensional diffusion equation:

\[ \frac{\partial}{\partial t} u(x,t) = D \frac{\partial^2}{\partial x^2} u(x,t) \quad \text{for} \quad -\infty < x < \infty \]

with the initial condition \( u(x,0) = f(x) \). Use the Fourier transform to show that the solution of the diffusion equation takes the form

\[ u(x,t) = \int_{-\infty}^{\infty} K(x - \xi,t) f(\xi) \, d\xi. \]

Find \( K(x - \xi,t) \), which is called the kernel.

[Hint: Gaussian integral \( \int_{-\infty}^{\infty} \exp \left(-\frac{x^2}{2\sigma^2}\right) \, dx = \sqrt{2\pi\sigma^2} \) ]
8. (a) [5%] Find and classify all local maxima, local minima and saddles for

\[ f(x, y, z) = \exp(2x^2 + xz - 5z^2). \]

(b) [5%] Consider a forced vibration system which is described by the equations

\[ \frac{d^2 x_1}{dt^2} + 2x_1 - x_2 = A \sin(\omega t) \]
\[ \frac{d^2 x_2}{dt^2} - x_1 + 2x_2 = B \sin(\omega t) \]

To seek a particular solution, we assume \( x_1(t) = q_1 \sin(\omega t) \) and \( x_2(t) = q_2 \sin(\omega t) \). Find \( q_1 \) and \( q_2 \).

9. Along the circumference of the circle \( r = b \) a solution \( T(r, \theta) \) of Laplace's equation is required to take on the value \( T_0 \) when \( 0 < \theta < \pi \) and the value \( -T_0 \) when \( \pi < \theta < 2\pi \). Determine an expression for \( T \) valid when \( r > b \). (Show the details of your work.)

\[
[\nabla^2 T = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}] \quad (12\%) \]

10. (complex analysis) Prove the following identity

\[ \sin^{-1} z + \cos^{-1} z = \frac{1}{2} (4n+1)\pi, \quad n = 0, \pm 1, \pm 2, \ldots \quad (8\%) \]