國立清華大學100學年度碩士班入學考試試題

系所班組別：聯合招生(工科丙組、先進光源工科組)

考試科目（代碼）：工程數學(9801)

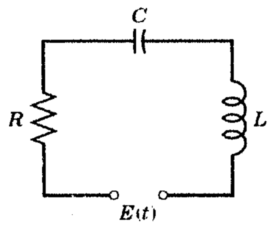
　　　　*請在【答案卷、卡】作答

1. Find the transient current if

   $R = 6\,\Omega,\ L = 1\ H,\ C = 0.04\ F,\ E = 600\,(\cos t + 4\sin t)\ V;$

   (L, R, C, E, are measured in henrys, ohms, farads, volts, respectively.)

   Initial current and charge are assumed to be zero. (10%)



2. Consider a general nonhomogeneous linear ODE

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \ldots + p_1(x)y' + p_0(x)y = r(x)$$

The particular solution $y_p(x)$ can be solved by the method of variation of parameters. That is,

$$y_p(x) = \sum_{k=1}^{n} y_k(x)\int \frac{W_k(x)}{W(x)} r(x)dx$$

(Where the y_k' s are n linearly independent homogeneous solutions. W is the Wronskian of $y_1, \ldots, y_n$, and W_k is identical to W, but with the kth column replaced by a column of zeros-except for the bottom element, which is 1.)

Try to solve the following equation using the method of variation of parameters.

$$y''' + \frac{3}{4}x^{-2}y' - \frac{3}{4}x^{-3}y = 9x^{5/2}$$ (10%)

3. Solve the following equation by Laplace Transform method.

$$y'+y=f(t),\quad y(0)=3,$$
$$where\quad f(t)=\begin{cases}0 & 0\le t<\pi\\ 2\cos t & t\ge\pi\end{cases}. \qquad (10\%)$$

4. Find the basis of solutions y(x) of the following differential equation. Show the details of your work.

$$xy''+(2x+1)y'+(x+1)y=0. \qquad (10\%)$$

5. Find a unit vector normal to surface S given by $\cos(xy)=e^z-1$ at the point $(1, \pi, 0)$. (10%)

6. Let **F**=(x-y)**i**+ (y-z)**j**+ (z-x)**k**. Evaluate the surface integral of **F** over the unit sphere defined by $x^2+y^2+z^2=1$. (10%)

7. Define the Fourier transform of $f(x)$ to be $\hat{f}(w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iwx}dx$.

(a) [5%] Calculate the Fourier transform of $f(x)=\begin{cases}a-|x| & ,|x|<a\\ 0 & ,\text{ otherwise}\end{cases}$

(b) [5%] Consider the one-dimensional diffusion equation:

$$\frac{\partial}{\partial t}u(x,t)=D\frac{\partial^2}{\partial x^2}u(x,t)\quad\text{for}\quad -\infty<x<\infty$$

with the initial condition $u(x,0)=f(x)$. Use the Fourier transform to show that the solution of the diffusion equation takes the form $u(x,t)=\int_{-\infty}^{\infty}K(x-\xi,t)f(\xi)d\xi$.

Find $K(x-\xi,t)$, which is called the kernel.

[Hint: Gaussian integral $\int_{-\infty}^{\infty}\exp\left(\frac{-x^2}{2\sigma^2}\right)dx=\sqrt{2\pi\sigma^2}$]

8. (a) [5%] Find and classify all local maxima, local minima and saddles for

$$f(x,y,z) = \exp(2x^2 + xz - 5z^2).$$

(b) [5%] Consider a forced vibration system which is described by the equations

$$\frac{d^2x_1}{dt^2} + 2x_1 - x_2 = A\sin(\omega t)$$
$$\frac{d^2x_2}{dt^2} - x_1 + 2x_2 = B\sin(\omega t)$$

, where A, B, and ω are constant.

To seek a particular solution, we assume $x_1(t) = q_1 \sin(\omega t)$ and $x_2(t) = q_2 \sin(\omega t)$. Find q_1 and q_2.

9. Along the circumference of the circle $r = b$ a solution $T(r,\theta)$ of Laplace's equation is required to take on the value T_0 when $0 < \theta < \pi$ and the value $-T_0$ when $\pi < \theta < 2\pi$. Determine an expression for T valid when $r > b$. (Show the details of your work.)

$$[\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}] \qquad (12\%)$$

10. (complex analysis) Prove the following identity

$$\sin^{-1} z + \cos^{-1} z = \frac{1}{2}(4n+1)\pi, \quad n = 0, \pm 1, \pm 2, --- \qquad (8\%)$$