國立清華大學 102 學年度碩士班考試入學試題

系所班組別：聯合招生(工科丙組、先進光源工科組)

考試科目（代碼）：9801 工程數學

　　*請在【答案卷、卡】作答

1. (a) Consider $\underline{y}' = \underline{\underline{A}}\,\underline{y} + \underline{g}$ [Note: $\underline{y}, \underline{g}$ *are vectors*; $\underline{\underline{A}}$ *represents a* maxtrix]
$\underline{y}^{(h)}$ is the homogeneous solution for above system and $\underline{y}^{(1)}, \underline{y}^{(2)}$ are its basis of solution vectors. Then, $\underline{y}^{(h)} = \underline{\underline{Y}}(t)c$, and $\underline{\underline{Y}}(t) = [\underline{y}^{(1)}, \underline{y}^{(2)}]^T$ is the fundamental matrix. If a particular solution of this nonhomogeneous system is $\underline{y}^{(p)}$
Set $\underline{y}^{(p)} = \underline{\underline{Y}}(t)\underline{u}(t)$
Prove $\underline{u}' = \underline{\underline{Y}}^{-1}\underline{g}$ (so-called Method of Variation of Parameters)

(b) Solve the following system by the Method of Variation of Parameters

$$y_1' = y_2 + t$$
$$y_2' = y_1 - 3t$$

(25%)

2. Let $f=e^z+yz$. Compute the rate of change of f in the direction of unit vector $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ at the point (1, -1, 1). (13%)

3. Let **F**(x,y,z)=x**i**+y**j**+z**k**. Evaluate the integral of **F** along the followings path
c(t)=(cost, sint, t), $0 \leq t \leq 4\pi$. (12%)

4. Solve the diffusion equation

$$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad \text{for } 0 < x < L,\ t > 0$$

subject to the boundary conditions

$$\frac{\partial u(0,t)}{\partial x} = 0,\ u(L,t) = u_0 \quad \text{for } t > 0,$$

and the initial condition

$$u(x,0) = f(x) \quad \text{for } 0 < x < L. \qquad (13\%)$$

5. Use the residue theorem to evaluate the integral

$$\int_0^{\infty} \frac{x^2}{x^6+1}\,dx. \qquad (12\%)$$

6. Use an appropriate infinite series method about x=0 to find two solutions of the given differential equation.

$$xy'' - (x+2)y'(x) + 2y(x) = 0$$

(13%)

7. Use Laplace transform to solve the given equation.

$$f(t) = 3 + \int_0^t f(\tau)\cos 2(t-\tau)\,d\tau \qquad (12\%)$$

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參考資料:

Function f(t)	Laplace transform F(s)
e^{at}	$\frac{1}{s-a}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - s\,f(0) - f'(0)$
t^n	$\frac{n!}{s^{n+1}}$
$\int_0^t f(\tau)g(t-\tau)d\tau = [f(t)*g(t)]$	$F(s)\,G(s)$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
$f(t-a)\,U(t-a)$	$e^{-as}F(s)$
$e^{at}f(t)$	$F(s-a)$
$t\,f(t)$	$-\frac{dF(s)}{ds}$
$\frac{f(t)}{t}$	$\int_s^\infty F(s')ds'$