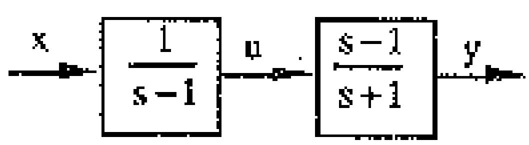
1. Answer **TRUE** or **FALSE** for the following questions (1% each). Briefly explain your answers (2% each).

   (a) To control the steady-state error constant K_v, using the root-locus method is easier than using the frequency response method of Bode.
   (b) The speed of a system response can be determined by the bandwidth.
   (c) In general, adding a zero to a system will increase the overshoot of the system response.
   (d) Consider the system shown below



   This system is stable with perfect pole-zero cancellation.
   (e) When use state-variable method to design a control system, all closed-loop poles can be assigned to any desired locations by using state feedback. (15%)

2. A mechanical system is shown in Figure 2. The motion of mass M is small and is constrained by a spring with constant k and a damper with coefficient c. A rigid bar is used to connect the mass and the disk. The motion of the disk is constrained by a torsional spring with spring constant G. The inertia of the disk is J and the friction of the disk is negligible.
   (a) Derive the equation of motion in which y is the output and f(t) is the input. (10%)
   (b) Find the natural frequency and the damping ratio of the system in terms of the system parameters. (5%)

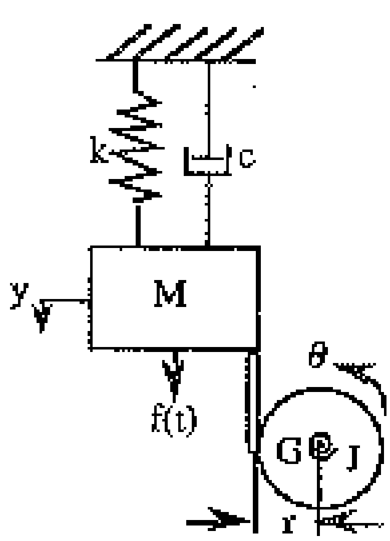


Figure 2

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3. A feedback control system is shown in Figure 3. Design a controller D(s) such that the closed-loop system satisfies the following three conditions:
   (i) The steady-state error is less than 8% to a step input of r(t) assuming W(s) = 0.
   (ii) The overshoot $M_p \leq 10\%$.
   (iii) When r(t) = 0, the steady-state value of y due to the disturbance, w(t) = 4 sin (5t), has magnitude smaller than 0.04. (10%)

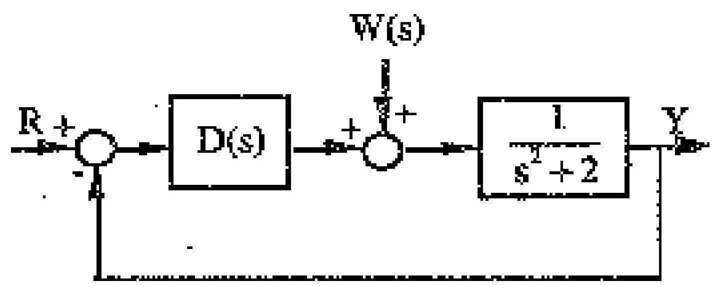

Figure 3

4. Give an open-loop transfer function

$$G(s)=\frac{K(1+\tau_1 s)^2}{s^n(1+\tau_2 s)(1+\tau_3 s)(1+\tau_4 s)^m}$$

where τ_1, τ_2, τ_3, and τ_4 are greater than zero. The polar plot of G(s) is shown in Figure 4.
   (a) Determine the value of C in the polar plot, and n and m in G(s). (5%)
   (b) Based on the results of part (a) and the given plot, determine the order of the magnitude of τ_1, τ_2, τ_3, and τ_4; that is, which one is the greatest? which one is the smallest? and which one is in the middle? Justify your answer. (10%)

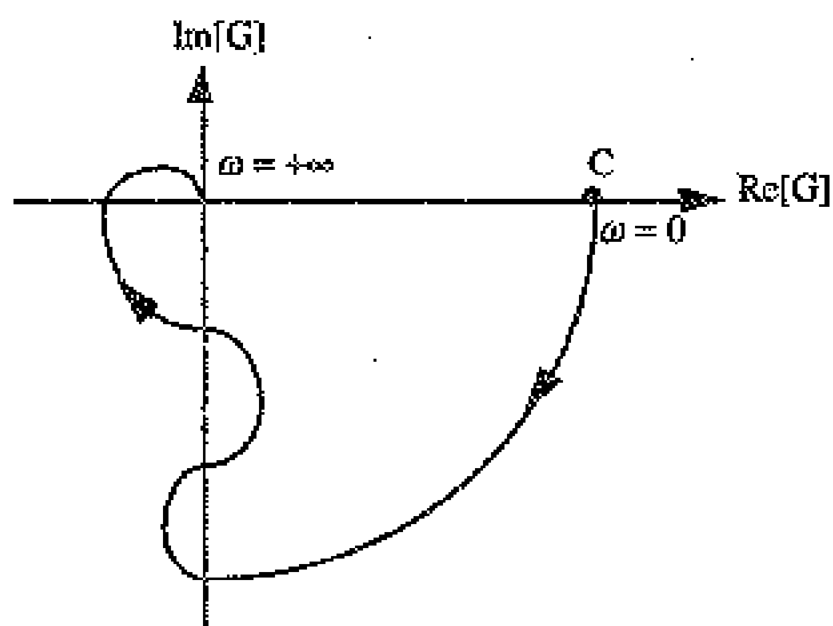

Figure 4

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5. For the system shown in Figure 5, sketch the root locus of closed loop with respect to K from 0 to infinite. Assume that $\beta > \alpha > 0$. Note that it is not necessary to calculate the break-in and breakaway points carefully. Just show these points roughly. (10%)

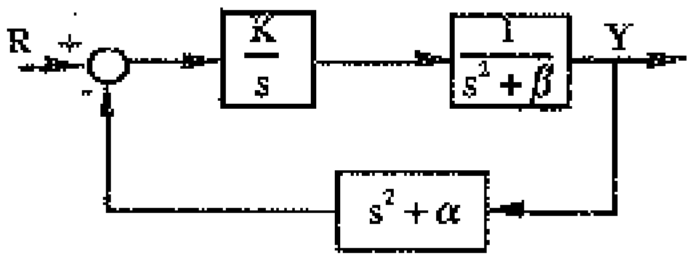

Figure 5

6. Consider the discrete-time system represented by the state equation

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = -2x_1(k) + 3x_2(k) + u(k)$$

and the output equation

$$y(k) = x_1(k)$$

(a) Determine the stability of the system. (5%)

(b) Given: y(k) = 0 for $k \le 0$; u(0) = 1; u(k) = 0 for k < 0, k > 0. Find Y(z), which is the z-transform of y(k), and the value of y (3). (5%)

7. Consider the continuous-time system represented by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

and y(t) = [3 1 0] x(t) where

$$\mathbf{x}(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(a) Determine the gain matrix $\mathbf{K} = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$ in the state-feedback controller u = r - **K**x such that the closed-loop transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + 3s + 2}$$

where Y(s) and R(s) are the Laplace transform of y(t) and r(t), respectively. (15%)

(b) Discuss the controllability and observability of the closed-loop system. (5%)

8. For the nonlinear system represented by the differential equation

$$\ddot{x} + 3\dot{x} - \dot{x}^2 + 2x - x^3 = 0$$

Sketch a few state trajectories on the phase plane $(x, \dot{x})$ in the neighborhood of the equilibrium point (0, 0) and give your reasoning. (5%)