

八十六學年度 動力機械 系(所)甲乙丙丁 組碩士班研究生入學考試

科目 工程數學

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1. Consider an ordinary differential equation in the form

$$y'' + a(x)y' + b(x)y = 0$$

- (a) Under what conditions will  $x = 0$  be an ordinary point? Write a possible power series form for the solution  $y(x)$ .
- (b) Under what conditions will  $x = 0$  be a regular singular point? Write a possible power series form for the solution  $y(x)$ .
- (c) Under what conditions will  $x = 0$  be an irregular singular point? Write a possible power series form for the solution  $y(x)$ .

(15%)

2. (a) Solve the Laplace equation by the method of separation of variables.

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

$$\theta(0, y) = 1 \quad \theta(x, 0) = 1$$

$$\theta(1, y) = 0 \quad \theta(x, 1) = 0$$

(10%)

- (b) Explain why the solution  $\theta(x, y)$  from part (a) cannot be expressed as a single Fourier series. Two of them are needed. (5%)

3. Given  $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ , find  $A^n$ . (15%)

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4. Find the Fourier series representation of the square wave which is given by

$$f(x) = \begin{cases} 3 & -\pi \leq x < 0 \\ 5 & 0 \leq x < \pi \end{cases}$$

and  $f(x+2\pi) = f(x)$  (15%)

5. Please give the mathematical expressions and physical meanings of
- Gradient of a velocity potential,  $\text{Grad } \phi$ ,
  - Divergent of a velocity field,  $\text{Div } \mathbf{V}$ , and
  - Curl of a velocity field,  $\text{Curl } \mathbf{V}$ ,
- please also prove that
- $\text{Curl}(\text{grad } \phi) = 0$ , and give the physical explanation of the equation. (5% each)
6. If  $f(z) = u(x, y) + iv(x, y)$ , where  $z = x + iy$  is the complex number, please prove the **Cauchy-Riemann Equations**

$$u_x = v_y \quad \text{and} \quad u_y = -v_x \quad (8\%)$$

and prove the **Laplace equation**

$$u_{xx} + u_{yy} = 0 \quad \text{and} \quad v_{xx} + v_{yy} = 0 \quad (4\%)$$

If one define further  $z^* = x - iy$  is the complex conjugate of  $z$ , then  $u_{zz^*} = 0$  and  $v_{zz^*} = 0$ .

(8%)