國 立 清 華 大 學 命 題 紙

分生組丙

八十五學年度 生命科學 系（所）生醫組丙 組碩士班研究生入學考試

科目 應用數學 科號 1105 1405 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the following differential equations ($y' = dy/dx$):

(a) $yy' = x^3 + y^2/x$

(b) $y'' - 3y' + 2y = \cosh x \qquad y(0) = 1/12, \quad y'(0) = 5/12$ (14%)

2. (a) Determine whether the following linear systems have solutions. If they do, are the solutions unique or infinite in number?
(b) Find the solutions that exist.

(i) $\begin{aligned} x + y + z &= 2 \\ 2x - y + 2z &= 4 \\ -x + 4y + z &= 3 \end{aligned}$ (ii) $\begin{aligned} 2x + 4y + 6z &= 18 \\ 4x + 5y + 6z &= 24 \\ 2x + 7y + 12z &= 40 \end{aligned}$. (14%)

3. Are the following matrices diagonalizable? For those which are, find a diagonalizing matrix for each, and diagonalize them. For those which are not, explain why.

(a) $\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{pmatrix}$ (14%)

4. (a) What is the condition under which a vector field has a potential? Such a field is said to be conservative.
(b) Which of the following fields are conservative? Find a potential for each of the conservative fields. (14%)

(i) $\mathbf{F} = e^{xy}(1+xy)\mathbf{i} + x^2 e^{xy}\mathbf{j}$, (ii) $\mathbf{F} = 2x(\cos y)\mathbf{i} + x^2(\sin y)\mathbf{j}$.
(Note: $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the unit vectors in the x, y, z directions)

5. Given a vector field $\mathbf{F} = (y^3 - z)\mathbf{i} + x^2 e^z \mathbf{j} + \sin(xy)\mathbf{k}$, find its surface integral over the ellipsoid $(x^2/4) + (y^2/16) + (z^2/25) = 1$. (8%)

6. A periodic function with period 2π is given as

$$f(x) = x^2/4 \qquad -\pi < x < \pi.$$

(a) Find its Fourier series.
(b) Show, by using the result in (a), that

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}.$$ (10%)

7. A vibrating string of length 2 m, has both ends fixed. Assuming $c^2 = 1\ m^2/s^2$, the initial velocity equals 3 m/s, and the initial deflection is given by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } 1 \leq x \leq 2. \end{cases}$$

(a) Write the wave equation that governs the deflection $u(x,t)$ where t represents time. (b) Solve the equation for $u(x,t)$. (12%)

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8. Evaluate the following integrals:

(a) $\oint_C z^{-4} \sin z \, dz$ C: unit circle in the complex plane, in the counterclockwise sense

(b) $\int_0^{\infty} \frac{\cos 2x}{1 + x^{1}} dx$. (14%)