Part I: 微積分 (50 points)

1. (10 points)

Prove that if $f(x,y)$ is homogeneous of degree k, then

a. $\dfrac{\partial f(x,y)}{\partial x}$ and $\dfrac{\partial f(x,y)}{\partial y}$ is homogeneous of degree k-1.

b. $x\dfrac{\partial f(x,y)}{\partial x}+y\dfrac{\partial f(x,y)}{\partial y}=kf(x,y)$

2. (10 points)

a. Suppose m is a positive integer, find the Taylor's expansion for $(1+x)^m$

b. Show that

$$(a+b)^m=a^m+\binom{m}{1}a^{m-1}b+\binom{m}{2}a^{m-2}b^2+\cdots+\binom{m}{m}b^m$$

3. (10 points)

Evaluate the following limits.

a. $\lim\limits_{x\to 1}\dfrac{x^2+3x-4}{x^2-x}$

b. $\lim\limits_{x\to\infty}x^{1/x}$

4. (10 points)

a. Show that the function $f(x)=x^7+5x^5+2x-2$ has an inverse function g.

b. Compute $g'(-2)$

5. (10 points)

Evaluate the following integrals and derivative.

a. $\int\dfrac{1}{x}\ln x\,dx$

b. $\dfrac{d}{dt}\int^{et}\dfrac{e^{tx}}{x}dx$

Part II: 統計 (50 points)

1. An accident has occurred on a busy highway between city A, of 100,000 people, and city B, of 200,000 people. It is known only that the victim is from one of the two cities and his name is Smith. A check of the records reveals that 10% of city A's population is named Smith and 5% of city B's population is named Smith. The police want to know where to start looking for relatives of the victim. What could you tell them about the probability that the victim is from city A?

2. Let Y denote a random variable with a uniform distribution over the interval (a, b).

   Show that

   $$E(Y) = (1/2)(a + b) \text{ and } V(Y) = (1/12)(b - a)^2$$

3. Suppose two electronic components in the guidance system for a missile operate independently, but each has a length of life governed by the exponential distribution with a mean of 1 (with measurements in hundreds of hours).
   (a) Find the probability density function for the average length of life of the two components.
   (b) Find the mean and variance of this average, using the answer in (a).

4. A scientific laboratory is sent five specimens of tobacco to test for nicotine content, and finds that they contain 24, 27, 26, 21 and 22 milligrams of nicotine respectively. Test the hypotheses that these specimens come from a population (assumed to be Normal) in which the mean nicotine content is 22 milligrams. Use a 10% test.

5. A market research economist randomly samples 400 people to estimate the average amount spent last month on fast-food meals. Assuming the standard deviation of the population is \$20, what is the probability that the sample mean will be within \$2 of the true mean? Justify your procedure.

Note: Tables of the Normal distribution and the t distribution are on the other sheet.

TABLE III

The Normal Distribution

$$\Pr(X \le x) = N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[N(-x) = 1 - N(x)]$$

x	N(x)	x	N(x)	x	N(x)
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.988
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.960	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999

TABLE IV

The t Distribution*

$$\Pr(T \le t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1 + w^2/r)^{(r+1)/2}}\, dw$$

$$[\Pr(T \le -t) = 1 - \Pr(T \le t)]$$

	Pr (T ≤ t)				
r	0.90	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

* This table is abridged from Table III of Fisher and Yates: *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Oliver and Boyd, Ltd., Edinburgh, by permission of the authors and publishers.