國立清華大學命題紙

九十一學年度 經濟 系（所） ____________ 組碩士班研究生招生考試

科目 微積分與統計 科號 5103 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

一、微積分（共五十分）

1. a. (5 points) (Inverse Function Theorem) Let $g(z)$ be the inverse function of $f(x)$. If $g(z)$ is continuously differentiable, show that

$$g'(z)=\frac{1}{f'(g(z))}.$$

b. (5 points) $y(x)=x^2-2x+4$ for $x\geq 1$. Apply the Inverse Function Theorem to evaluate $x'(y)$ at $x=2$.

2. a. (5 points) $\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=4e^{-t}$. Solve $\frac{dy}{dt}$.

b. (5 points) Write down the Taylor's series of $\ln x$ about the point 1 and make use of the series to approximate $\ln 0.99$ up to 3 decimal places.

3. $f(x,y)=\begin{cases} 0 & \text{if } (x,y)=(0,0) \\ \frac{x^3y-xy^3}{x^2+y^2} & \text{Otherwise} \end{cases}$

a. (5 points) Show that $(\partial f/\partial x)(0,0)$ and $(\partial f/\partial y)(0,0)$ are both 0.

b. (5 points) Show that $\frac{\partial^2 f}{\partial x\partial y}(0,0)\neq\frac{\partial^2 f}{\partial y\partial x}(0,0)$.

4. a. (5 points) Evaluate the indefinite integral $\int\frac{e^x-1}{e^x+1}dx$.

b. (5 points) Evaluate the indefinite integral $\int e^x\sin x\,dx$.

5. (10 points) $f(x,y)=4xy-x^4-y^4$. Find and classify the critical points of $f(x,y)$ as yielding relative maxima, relative minima, saddle point, or none of these.

二. 統計 (共五十分)

Instructions: Answer all questions and show all calculations. Point for each question is given in the margin.

1. (10 points) Consider two discrete random variables X and Y with joint probability distribution:

		X = 0	X = 1	X = 2
	0	0	1/4	0
Y	1	1/4	0	1/4
	2	0	1/4	0

(a) Compute the covariance $Cov(X,Y)$.

(b) Are X and Y independent? Show your assertion.

2. (10 points) Let X have a Poisson distribution with mean $E(X)=100$.
The Chebyshev's inequality states: let $U(X)$ be a non-negative function of random variable X, then $\Pr\{U(X)\geq C\}\leq \dfrac{E[U(X)]}{C}$, where C is a positive constant.
Use this to determine a lower bound for the probability $\Pr(75<X<125)$.

3. (10 points) Suppose the random variable X have the following probability density function:

$$
\begin{aligned}
f(x;\theta) &= \theta^2 && \text{if } x=1,\\
&= 2\theta(1-\theta) && \text{if } x=2,\\
&= (1-\theta)^2 && \text{if } x=3,
\end{aligned}
$$

and zero elsewhere, where $0<\theta<1$ is the unknown parameter. If we observe three $i.i.d.$ samples with $x_1=1, x_2=2$ and $x_3=1$.

(a) Derive the likelihood function for the parameter θ.

(b) Find the maximum likelihood estimator of θ.

4. (10 points) Suppose the random variable X have the gamma distribution $G(1,\beta)$, i.e.

$$f(x)=\frac{1}{\beta}\exp\{-\frac{x}{\beta}\},\qquad x>0,\beta>0,$$

and zero elsewhere. Show that X has "*no memory*". i.e.,

$\Pr\{X>r+s\mid X>s\}=\Pr\{X>r\}$ for any positive constants r and s.

5. (10 points) An investigator estimates the simple normal linear model

$$y_i=\beta_1+\beta_2 x_i+\varepsilon_i \qquad (i=1,\ldots,12)$$

by least squares, and reports the conventional 95% confidence interval (.1772, .6228) for $\beta_1+\beta_2$, and (.0860, 2.3140) for $\beta_1-\beta_2$.

(a) What are the least squares estimates, b_1 and b_2, respectively? Show your work.

(b) What is the standard error of b_1+b_2? Note: $t_{.025,10}=2.228$