

九十三年學年度 經濟系(所) 組碩士班入學考試

科目 微積分與統計 科號 5603 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

一、微積分 (共五十分)

Instructions: Answer all questions and show all calculations.

1. a. (5 points) Evaluate the limit: $\lim_{x \rightarrow \infty} \frac{\log(1 + xe^{2x})}{x^2}$

b. (5 points) Evaluate the limit: $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{x^2}$

2. a. (5 points) If the function f is defined by: $f(x) = x$ if x is rational and $f(x) = 1 - x$ if x is irrational. If any, find the point(s) at which f is continuous. Show your assertion.

b. (5 points) Define $f(x) = e^{-1/x^2}$ (if $x \neq 0$),
 $= 0$ (if $x = 0$)

Find the second derivative $f''(0)$ if it exists otherwise show your assertion.

3. a. (5 points) Find the area bounded by the curves $y = \sqrt{x+14}$, $x = \sqrt{y}$, and $y = 0$.

b. (5 points) Evaluate the indefinite integral $\int \frac{x^{1/3}}{1+x^{2/3}} dx$.

4. Suppose a and c are real numbers, $c > 0$, and f is defined on interval $[-1, 1]$ by

$$f(x) = x^a \sin(x^{-c}) \quad (\text{if } x \neq 0),$$

$$= 0 \quad (\text{if } x = 0)$$

a. (5 points) Obtain the condition for parameters a and c such that $f'(0)$ exists.

b. (5 points) Obtain the condition for parameters a and c such that $f''(0)$ exists.

5. a. (5 points) Consider the function $f(x, y) = x^2 e^y$, what is the slope of the level set at $x = 2, y = 0$? (i.e. $dy/dx = ?$)

b. (5 points) Let $f(x, y) = x^4 + x^2 - 6xy + 3y^2$, find and classify the critical points of $f(x, y)$ as yielding relative maxima, relative minima, saddle point, or none of these.

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二、統計 (共五十分) (Please note that you can get points by laying down correct equations, even if you do not have the time to crank out exact numbers.)

1. (10 points) Calculate the mean, variance, and standard deviation using the following distribution of the random variable X .

$X(i)$	162	167	172	177	182	187	192	197	202	207
$f(i)$	2	10	16	19	14	8	5	3	2	1

2. (5 points) Experience has shown that the average number of phone calls arriving to a switchboard is 4 per minute. If the switchboard can handle a maximum of 8 calls per minute, what is the probability that it is unable to handle all the calls that come during a period of 1 minute?

3. (5 points) Show that if two random variables X and Y are independent, then the $COV(X,Y)$ is also zero.

4. (10 points) Suppose a machine drills holes into plates with a diameter of 1.02 inches and standard deviation of 0.01 inches, where diameters are normally distributed. What percentage of the plates containing two holes will be rejected because at least one of the holes will be greater than 1.05 inches or smaller than 1 inch?

5. (10 points) 200 people choose from a set of 4 different designs, A, B, C, and D, with the following results:

Design	A	B	C	D
Chosen times	33	42	67	58

Test the hypothesis that there is equal preference among all 4 designs. Note: with 3 degrees of freedom, the 5% right tail of the CHI-SQUARE distribution is 7.8.

6. (10 points) Maximum likelihood: Suppose you make 10 independent observations from a Poisson distribution: 0, 0, 1, 1, 1, 2, 3, 3, 4, 5. Calculate the unknown parameter θ that is most likely to have generated this data set.