九十二學年度 電子工程研判所 系（所） 組碩士班研究生招生考試

科目 工程數學 科號 2601 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Find the general solution of $y' = 6\frac{y \ln y}{x}$. (5%)

2. Solve $x' - x = f(t)$, where $x(0) = 0$, and $f(t)$ is 20 on $0 < t < 1$, 10 on $1 < t < 2$, and 0 on $t > 2$. (10%)

3. Find the general solution of $y'' + 2y' + y = e^{-x} \cos x$. (10%)

4. Fourier series and Fourier transform:
   (a) Find the Fourier series representation of a periodic function f(t):
   $$f(t) = \begin{cases} 0, \text{if } -\pi < t < 0 \\ t^2, \text{if } 0 < t < \pi \end{cases}, f(t+2\pi) = f(t).$$
   Denote its Fourier coefficient as $a_n$ and $b_n$, where n = 0, 1, 2,… (10%)
   (b) What are the Fourier series representation for g(t) where $g(t) = 3f(-t) + 2f(t-\pi)$? (express the Fourier coefficients in terms of $a_n$ and $b_n$) (7%)
   (c) Find the solution y(t) in Fourier series form in terms of $a_n$ and $b_n$ if
   $$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = f(t), \quad -\infty < t < \infty.$$ (8%)

5. Partial differential equations (PDE's)
   (a) Write down the wave equation, diffusion equation and Laplace equation. (5%).
   (b) Write down the definition of a linear PDE.
   Is $u_{xx} + (2x-1)u_{xy} - 2xu_{yy} = 0$ a linear PDE? (5%)
   (c) Write down the definition of hyperbolic, elliptic and parabolic PDE's and classify the wave equation, diffusion equation and Laplace equation. (5%)
   (d) Similar to d'Alembert's solution to the wave equation, one can usually change the independent variables from $x$, $y$ to, say, $\xi = \xi(x, y)$ and $\eta = \eta(x, y)$ to simplify the second-order terms in the PDE
   $u_{xx} + (2x-1)u_{xy} - 2xu_{yy} = 0$. Explain how you will do to determine the functions $\xi(x, y)$ and $\eta(x, y)$. (10%)

# 國　立　清　華　大　學　命　題　紙

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6. Consider the real integral $\int_{-\infty}^{\infty}\frac{1}{x-i\delta}$, where δ is a positive number. A student is trying to evaluate this integral by contour integration in the complex plane. He chooses the contour C to enclose the **upper half** complex plane, that is, C = infinite semi-circle with diametrical side being the x-axis. He gets

$$\int_{-\infty}^{\infty}\frac{1}{x-i\delta}=\oint_C\frac{dz}{z-i\delta}=2\pi i, \qquad \textbf{(1)}$$

since there is exactly one pole inside C. On the other hand, if he lets C enclose the **lower half** plane, he would get

$$\int_{-\infty}^{\infty}\frac{1}{x-i\delta}=\oint_C\frac{dz}{z-i\delta}=0, \qquad \textbf{(2)}$$

since no poles sit inside C. The results **(1)** and **(2)** are obviously in conflict. Obviously something is wrong with his contour integration.

(a) Explain what went wrong in the contour integration? (13%)

(b) Correct it and re-evaluate the integral by **contour integration**. (You are not allowed to use other methods.) (12%)