(1)至(9)為選擇題，包含單選及複選型態．完全答對始給分，答錯倒扣該題之50%.

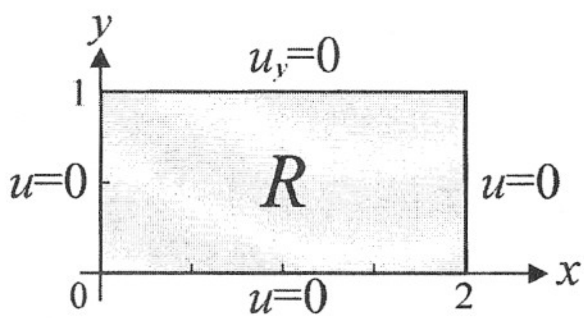
The temperature distribution $u(x,y,t)$ in a rectangular region R:$\{0\le x\le 2,\ 0\le y\le 1\}$ is governed by a partial differential equation (PDE): $u_t=\alpha^2(u_{xx}+u_{yy})$, where α is a real constant, and the subscripts t, xx, yy denote partial derivatives $\partial/\partial t$, $\partial^2/\partial x^2$, $\partial^2/\partial y^2$, respectively.

(1) (6%) If the three ordinary differential equations (ODEs) derived by separation of variables: $u(x,y,t)=X(x)\cdot Y(y)\cdot T(t)$ are: $X''+k^2X=0$, $Y''+h^2Y=0$, $\dot{T}+\frac{1}{\tau}T=0$, what is the relation among the three eigenvalues k, h, τ?

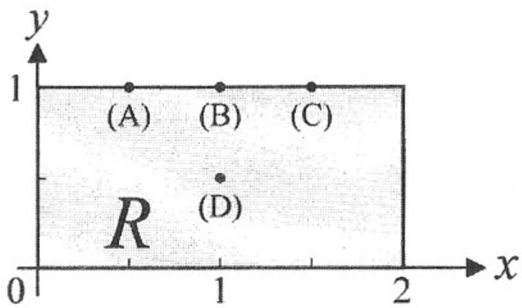
(A) $k^2-h^2=\frac{\alpha^2}{\tau}$; (B) $k^2+h^2=\frac{\tau}{\alpha^2}$; (C) $k^2+h^2=\frac{\alpha^2}{\tau}$; (D) $k^2-h^2=\frac{\alpha^2}{\tau^2}$;

(E) $k^2+h^2=\frac{1}{\tau\alpha^2}$.

(2) (6%) The boundary conditions (BCs) are specified as follows ($u_y\equiv\partial u/\partial y$):



Let the fundamental mode $u_{min}(x,y,t)$ be the solution to the PDE and BCs, which corresponds to the maximum of eigenvalue τ. What is the position $(x_0,\ y_0)\in R$ where the fundamental mode has peak magnitude, i.e. $|u_{min}(x_0, y_0, t)|$ is maximum at any time t?



(A) $\left(\frac{1}{2},1\right)$; (B) $(1,1)$; (C) $\left(\frac{3}{2},1\right)$; (D) $\left(1,\frac{1}{2}\right)$;

(E) none of the above.

The idea of d'Alembert's solution is to transform the wave equation into canonical form which can be solved more easily. Similar approaches can be applied to some other cases. Consider the linear first order PDE $u_x + 2u_y + u = 0$.

(3) (8%) which of the following transformations can reduce it to an ODE?

(A) $\Phi = x + y$, $\Psi = x - 2y$; (B) $\Phi = x + y$, $\Psi = 2x - y$;

(C) $\Phi = x + y$, $\Psi = x - y$; (D) $\Phi = x + 2y$, $\Psi = x - y$;

(E) none of the above.

(4) (8%) solve $u_x + 2u_y + u = 0$ with the condition $u = 1$ when $x + y = 1$. Calculate $u(x = 2, y = 2) = ?$

(A) e^{-1}; (B) e^{1} ; (C) $e^{-2/3}$; (D) $e^{2/3}$; (E) none of the above.

(5) (8%) Let A be an n-by-n matrix, and $x \in \Re^n$. Please find the correct statements from the following:

(A) $\|x\|_1 \le n\|x\|_\infty$; (B) $\|x\|_2 > \sqrt{n}\|x\|_\infty$; (C) $\|x\|_\infty \le \|x\|_2$; (D) $\|Ax\|_\infty > \sqrt{n}\|A\|_2\|x\|_\infty$;

(E) $\frac{1}{\sqrt{n}}\|A\|_2 \le \|A\|_\infty \le \sqrt{n}\|A\|_2$

(6) (8%) Evaluate the integral $\int_R e^{-z} dz$ along the path R that is the positive x axis (from the origin to the point $(\infty, 0)$, where ∞ represents some number approaching infinity).　(A) 0　(B) $2\pi j$　(C) 1　(D) $e^{-0.5\pi}$　(E) 0.5π

(7) (10%) Evaluate the integral $\oint_C \left[z^2 - \mathrm{Re}(z)\right] dz$ along the path C that is counterclockwise circle with $|z| = 2$.

(A) $-2\pi i$; (B) $-4\pi i$; (C) $2i - 4\pi$; (D) $2 - 4\pi i$; (E) $4 + 2\pi i$.

(8) (10%) $\oint_C \frac{e^{-z}}{\cos 4z} dz$ = (A) $2\pi j\, e^{-\frac{\pi}{8}}$; (B) 0; (C) $\pi j \sinh(\frac{\pi}{8})$; (D) $2\pi j\, e^{0.125\pi}$;

(E) $\pi j\, e^{\frac{\pi}{8}}$

(Here C is the counterclockwise path along the circle centered at the origin with a radius of unity.)

(9) (10%) Pick the correct statements regarding the matrix $\mathbf{A}=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$.

(A) There are three sets of linearly independent eigenvectors associating with three distinct eigenvalues. (B) The determinant of this matrix is 45. (C) $\begin{bmatrix}2\\1\\0\end{bmatrix}$ is one of the eigenvectors. (D) 5 is an eigenvalue. (E) The homogeneous linear system $\mathbf{Ax}$=0 has no non-trivial solution.

(10)至(13)爲計算題, 請在答案卷上寫出該題之推導過程.

For problems 10 and 11, please write down your work and pick the correct answer codes (ex. A1 x C5) from the list.

(10) (6%) Find the inverse Laplace transform $f(t)$ of $L^{-1}\left\{\dfrac{(s-1)^n}{s^{n+1}}\right\}$.

ANS： (10-1) × (10-2)

(11) (6%) Suppose： $F(w)=\begin{cases}1 & , |w|\le W\\ 0 & |w|\ge W\end{cases}$ Find the inverse Fourier transform $f(x)$

ANS： (11-1) × (11-2)

(A1) $\sqrt{\dfrac{\pi}{2}}$; (A2) $\sqrt{2\pi}$; (A3) $\sqrt{\dfrac{1}{2\pi}}$; (A4) $\sqrt{\dfrac{2}{\pi}}W$; (A5) $\dfrac{W}{\pi}$; (A6) $\dfrac{W}{2\pi}$; (A7) $2\dfrac{W}{\pi}$;

(A8) $\sqrt{\dfrac{1}{2\pi}}W$; (A9) $\sqrt{2\pi}W$; (B1) $x\sin(xW)$; (B2) $\operatorname{sinc}\left(\dfrac{W}{\pi}x\right)$; (B3) $\operatorname{sinc}(\pi Wx)$;

(B4) $\operatorname{sinc}(Wx)$; (B5) $\sin(xW)$; (C1) $\dfrac{e^{-2t}}{n!}$; (C2) $\dfrac{e^{2t}}{n!}$; (C3) $\dfrac{e^{t}}{n!}$; (C4) $\dfrac{e^{-t}}{2n!}$;

(C5) $(-1)^n\dfrac{d^n}{dt^n}\left(t^ne^{-t}\right)$; (C6) $\dfrac{d^n}{dt^n}\left(t^ne^{t}\right)$; (C7) $\dfrac{d^n}{dt^n}\left(t^ne^{-t}\right)$; (C8) $(-1)^n\dfrac{d^n}{dt^n}\left(t^ne^{t}\right)$;

(D1) none of above

(12) (7%) Solve the differential equation: $x^3 \dfrac{d^3y}{dx^3} - 5x^2 \dfrac{d^2y}{dx^2} + 18x\dfrac{dy}{dx} - 26y = 0$.

(13) (7%) Solve the differential equation: $x\dfrac{dy}{dx} + 2y = xy^3$.