

國立清華大學命題紙

甲組(熱流組)、丁組(設計、製造組)

99 學年度 動力機械工程學系丙組(固體與奈微米力學組) 碩士班入學考試

科目 工程數學 科目代碼 1003 共 3 頁, 第 1 頁 \*請在【答案卷卡】作答

0803, 1103

1. Solve the following ordinary differential equations

(1a)  $y' = 2 + \sqrt{y - 2x + 3}$  (7%)

(1b)  $4y'' - 4y' + y = e^{x/2} \sqrt{1 - x^2}$  (8%)

2. Solve the following pair of simultaneous differential equations by Laplace transform

$$\frac{d^2 x}{dt^2} - x = y$$

$$\frac{d^2 y}{dt^2} + y = -x$$

given that at  $t = 0$ ,  $x = 2$ ,  $y = -1$ ,  $\frac{dx}{dt} = 0$ , and  $\frac{dy}{dt} = 0$  (15%)

3.

(3a) Consider a linear system  $Ax = b$ , where  $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$  and  $b = \begin{bmatrix} 3 \\ 2 \\ 6 \\ -1 \end{bmatrix}$

Find the QR decomposition of the matrix A and the least squares solution of the above system. (5%)

(3b) Let the eigenvectors be  $v_1, v_2$ , and  $v_3$ , describing the general solution to the differential equation

$$\frac{du}{dt} = Ku, \text{ where } K = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}; \text{ Find the eigenvalues of the matrix } K. (5\%)$$

(3c) At what time T is the solution  $u(T)$  guaranteed to equal its initial value  $u(0)$ ? (5%)

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0803、1103

4. Given a vector function  $\vec{E} = \vec{i}xy + \vec{j}(3x - y^2)$ , and the coordinates of three points  $P_1(5, 6)$ ,  $P_2(5, 3)$  and  $P_3(3, 3)$ , where  $\vec{i}$  and  $\vec{j}$  are unit vectors along x- and y- axes respectively.
- (4a) Evaluate the integral  $\int \vec{E} \cdot d\vec{l}$  from  $P_1$  straight to  $P_3$ . (5%)
- (4b) Evaluate the integral  $\int \vec{E} \cdot d\vec{l}$  from  $P_1$  to  $P_3$  along the piecewise straight path  $P_1 P_2 P_3$ , i.e. integrate from  $P_1$  along straight-line segment to  $P_2$  and then along another straight-line from  $P_2$  to  $P_3$ . (5%)
- (4c) Is this  $\vec{E}$  a conservative field? And why? (5%)

5. Consider the following partial differential equation for the function  $\varphi(\xi, t)$

$$\omega_0^2 \varphi_{\xi\xi} - \varphi_{tt} = 0$$

The boundary conditions are:

$$\omega_0^2 \varphi_{\xi}(0, t) - \varepsilon \varphi_{tt}(0, t) = 0$$

$$\omega_0^2 \varphi_{\xi}(1, t) + \varepsilon \varphi_{tt}(1, t) = 0$$

In the above equations,  $\omega_0$  and  $\varepsilon$  are constants.

- (5a) Assume that  $\varphi(\xi, t) = \Phi(\xi)T(t)$ , find the time free behavior equation  $\Phi(\xi)$  and the boundary conditions by the method of separating variables or the product method. (3%)
- (5b) Write the eigenvalue equation of the system from the time free behavior equation and boundary conditions. (3%)
- (5c) Find the complete eigenvalues, when  $\varepsilon = 0$ . (3%)
- (5d) From the first nonzero eigenfunction  $\Phi_1(\xi)$ , find the behavior equation with time  $T_1(t)$  and the corresponding solution  $\varphi_1(\xi, t)$  (3%)

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0803、1103

6. Consider the following ordinary differential equation

$$\ddot{y} + 2D\dot{y} = x(t)$$

where  $D$  is a constant. The right hand side is given by

$$x(t) = \begin{cases} 0 & t < 0 \\ \exp(-2Dt) & t \geq 0 \end{cases}$$

Solve the equation by the method of Fourier transformation:

$$Y(i\omega) = F(i\omega) \cdot X(i\omega)$$

(6a) Determine the complex input spectrum  $X(i\omega)$ . (2 %)

(6b) Find the relationship in the complex form of the system  $F(i\omega)$  and  $X(i\omega)$  (3 %)

(6c) Find the complex output spectrum  $Y(i\omega)$  (3 %)

(6d) Calculate in the spectrum density  $S(\omega) = |Y(i\omega)|$  and the value  $S(0)$  and  $S(\infty)$ . (2%)

(6e) Calculate the solution in the time domain  $y(t)$  for  $t > 0$  through the inverse Fourier transform. (3%)

**Remark:**

$$\int_{-\infty}^{\infty} \frac{e^{iat}}{i\omega(a+i\omega)^2} d\omega = \begin{cases} 0 & , t < 0 \\ \frac{2\pi \cdot \gamma(2, at)}{a^2 \cdot \Gamma(2)} & , t > 0 \end{cases}$$

$$\text{with } \Gamma(2) = \int_0^{\infty} te^{-t} dt, \quad \gamma(2, at) = \int_0^{at} te^{-t} dt,$$

$$\int xe^{-x} dx = \frac{-(x+1)}{e^x}$$

7 Find all Laurent series of

$$\frac{1}{z^2 + 2z}$$

with center  $z = 0$ , and indicate their respective regions of convergence. (9%)

8 Evaluate the following integral

$$\oint_C \text{Im}(z) dz$$

where  $C$  is the unit circle around  $z = 0$  in the complex plane. (6%)