科目：工程數學 A(3004)　　校系所組：中央大學電機工程學系(電子組)
交通大學電子研究所(甲組、乙 A 組、乙 B 組)
交通大學電控工程研究所
交通大學生醫工程研究所(乙組)
清華大學電機工程學系(甲組)
清華大學光電工程研究所
清華大學電子工程研究所
清華大學工程與系統科學系（丁組）
陽明大學生物醫學工程學系（醫學電子組）

1~5 題為複選題：(答案可能有一個或是多個。請標明題號，並將答案寫在答案卷上)

1. (5%) Let **x** and **y** be non-zero vectors in $\mathbf{R}^m$ and $\mathbf{R}^n$, respectively. Assume A is a matrix that represents the linear transformation $\mathbf{L_A}$: $\mathbf{R}^n \to \mathbf{R}^m$, then
   (a) A is an m x n matrix.
   (b) A and A^T have the same rank, where A^T denotes the transpose matrix of A.
   (c) A and A^T have the same nullity.
   (d) A has an inverse matrix.
   (e) If the rank of A is r, then A can be transformed to a matrix D via Gaussian elimination process such that $D = \begin{pmatrix} I_r & O_1 \\ O_2 & O_3 \end{pmatrix}$, where I_r is an r x r identity matrix and O_1, O_2 and O_3 are zero matrices.

2. (5%) What are the following statements are true?
   (a) If U, V, and W are subspaces of $\mathbf{R}^3$ and $U \perp V$ and $V \perp W$, then $U \perp W$.
   (b) Let $\{v_1, v_2, \ldots, v_n\}$ be an orthogonal basis in **V**. If a vector y is in **V**, we can express y as $y = \sum_{i=1}^{n} a_i v_i$, where $a_1, a_2, \ldots, a_n$ are scalars. Therefore the norm of y is $\|y\| = \left(\sum_{i=1}^{n} |a_i|^2\right)^{1/2}$.
   (c) If a vector y is in an inner product space **V** and **W** is a subspace in **V**, then the shortest vector from y to **W** lies in the orthogonal subspace of **W** in **V**.
   (d) The Legendre polynomials from an inner product space.
   (e) The Gram-Schmidt process can construct an orthogonal set with n orthogonal vectors from any arbitrary set of n vectors.

3. (5%) Let **T** be a linear operator on a finite-dimensional vector space. We define the determinant of **T**, denoted det(**T**), as follows: Choose any basis β for **V**, and define det(**T**) = det($[\mathbf{T}]_\beta$).
   (a) **T** is invertible if and only if det(**T**) $\neq 0$.
   (b) If **T** is invertible, then $\det(\mathbf{T}^{-1}) = [\det(\mathbf{T})]^{-1}$.
   (c) If **U**: $\mathbf{V} \to \mathbf{V}$ is linear, then det(**TU**) = det(**T**)·det(**U**).
   (d) If λ is any scalar and β is any basis for **V**, then $\det(\mathbf{T} - \lambda \mathbf{I_V}) = \det(A - \lambda I)$, where $\mathbf{I_V}$ is the identity transformation in **V**, I is the identity matrix and $A = [\mathbf{T}]_\beta$.
   (e) None of the above statements is true.

注意：背面有試題

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4. (5%) For a 3 x 3 matrix: $A=\begin{bmatrix}1&1&2\\0&1&1\\1&3&4\end{bmatrix}$,

(a) det(A) = 1.
(b) One of the eigenvalue is 0.
(c) One of the eigenvector is $(1, 1, 3)^T$.
(d) The basis of the null space of A is $\{(-1, -1, 1)^T\}$.
(e) rank(A) = 3.

5. (5%) For a linear equation system:
(a) Any linear equation system has at least one solution.
(b) Any linear equation system has at most one solution.
(c) Any system of n linear equations in n unknowns has at least one solution.
(d) Any system of n linear equations in n unknowns has at most one solution.
(e) None of the above statements is true.

6. (15%) Solve the following ordinary differential equations:
(a) (7%) Solve the differential equation: $x^3\frac{d^3y}{dx^3}-3x^2\frac{d^2y}{dx^2}+7x\frac{dy}{dx}-8y=0$.
(b) (8%) $\begin{cases}y_1'-y_1-y_2=3x\\y_1'+y_2'-5y_1-2y_2=5\end{cases}$. Please solve $y_1(x)$ and $y_2(x)$.

7. (10%) Solve $\cos(y)\cdot\frac{dy}{dx}-\frac{2}{x}\cdot\sin(y)=-\frac{1}{x^2}$.

8. (10%) Evaluate the integrals (counterclockwise) of $\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}F(z)dz$, where $F(z)=\frac{2kz}{(z^2+k^2)^2}$ and k is a constant.

9. (15%) Consider the complex function: $w=\frac{z-z_0}{\bar{z}_0 z-1}$, where z_0 is a constant with $|z_0|<1$ and $\overline{z_0}$ is the complex conjugate of z_0. Show that this function maps the unit disk $|z|<1$ in the complex z-plane onto the unit disk in the complex w-plan.

10. (25%) Suppose $F(s)$ is the Laplace transform of the function $f(t)$ and given by the equation: $F(s)=\frac{1}{s}\exp[\exp(-s)]$

(a) (15%) Please plot $f(t)$ for $3\geq t\geq 0$ and indicate the scale.
(b) (10%) Find $f(3/2)$ and $f(7/2)$