國立清華大學 102 學年度碩士班入學考試試題

系所班組別：生醫工程與環境科學系　丙組(醫學物理與工程組)

考試科目（代碼）：應用數學(2403)

　　*請在【答案卷、卡】作答

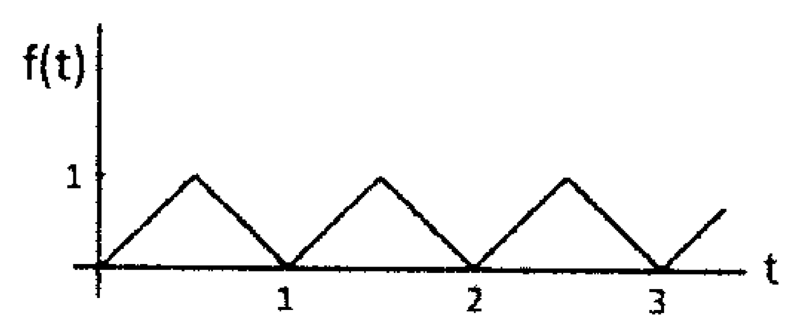
1. Solve the following ordinary differential equations.

   (a) $y'' - 10y' + 25y = 30x + 3$ (5 pts)

   (b) $x^3y''' - 6y = 0$ (5 pts)

2. Iodine-131 is a radioactive liquid used in the treatment of cancer of the thyroid. After one day in storage, analysis shows that initial amount of iodine-131 in a sample has decreased by 8.3%. Find the amount of iodine-131 remaining in the sample after 4 days. (10 pts)

3. (a) Find the Laplace transform of f(t). (5 pts)



   (b) Find the inverse Laplace transform of F(s). (5 pts)

$$F(s) = \frac{2s+5}{s^2+6s+34}$$

4. Find the eigenvalues and normalized eigenvectors of the given matrix.

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{pmatrix} \qquad \text{(10 pts)}$$

5. $f(x) = |x| - x \quad, -1 < x < 1$ Find the Fourier series of f(x). (10 pts)

6. Consider a second order linear differential equation

$$y'' + P(x)y' + Q(x)y = f(x).$$

Suppose $y_1(x)$ and $y_2(x)$ are the homogenous solutions of the above equation. Given $W(t) = y_1(t)y'_2(t) - y'_1(t)y_2(t)$ and $G(x,t) = \frac{y_1(t)y_2(x) - y_1(x)y_2(t)}{W(t)}$, show that $y_p = \int_{x_0}^{x} G(x,t)f(t)dt$ is the particular solution for the differential solution. (10pts)

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7. Find the integrating factor the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

and then solve the equation.

(10pts)

8. Find the inverse Laplace transform of

$$F(s) = \ln\left(1 + \frac{\omega^2}{s^2}\right)$$

Hint: If $\mathrm{L}\{g(t)\} = \mathrm{G}(s)$, $\mathrm{L}\{g(t)/t\} = \int_s^\infty \mathrm{G}(v)dv$. Also $\frac{\mathrm{d}}{\mathrm{ds}}\int_s^\infty \mathrm{G}(v)dv = -\mathrm{G}(s)$.

(10pts)

9. Consider the forced oscillation of a body on spring of modulus is governed by the equation

$$y'' + 0.02y' + 25y = r(t)$$

where $y(t)$ is the displacement from rest and $r(t)$ the external force depending on time t.

Let $r(t) = \begin{cases} t + \frac{\pi}{2} & -\pi < t < 0 \\ -t + \frac{\pi}{2} & 0 < t < \pi \end{cases}$, $r(t + 2\pi) = r(t)$.

(a) Find the Fourier series expansion of $r(t)$.

(b) Find the steady-state solution $y(t)$. $y_h(t) \approx Ae^{-0.01t}\cos 5t + Be^{-0.01t}\sin 5t$ and

$t \to \infty \Rightarrow y_h(t) \approx 0$.

(20pts)