Lepton flavor violation in $\tau$ decays

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We study the lepton flavor violation (LFV) in $\tau$ decays in the framework of the supersymmetric seesaw mechanism with nonholomorphic terms for the lepton sector at a large $\tan\beta$. In particular, we analyze two new decay modes $\tau \to \ell \ell_0(980)$ and $\tau \to \ell K^+ K^-$ arising from the scalar boson exchanges contrast to $\tau \to \ell \eta^0$ from the pseudoscalar ones. We find that the decay branching ratios of the two new modes could be not only as large as the current upper limits of $O(10^{-7})$, but also larger than those of $\tau \to \ell \eta^0$. Experimental searches for the two modes are important for the LFV induced by the scalar-mediated mechanism. In addition, we show that the decay branching ratios of $\tau \to \ell \mu^+ \mu^-$ are related to those of $\tau \to \ell \eta$ and $\tau \to \ell f_0(980)$.

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In the standard model (SM), since the neutrinos are regarded as massless particles, the processes associated with lepton flavors are always conserved. Inspired by the discoveries of nonzero neutrino masses [1,2], it has been studied enormously how to generate the neutrino masses which are less than a few eV. By supplementing with singlet right-handed Majorana neutrinos with masses $M_R$ required to be around the scale of unified theory, it is found that the seesaw mechanism is one of the natural ways [3] to obtain the small neutrino masses. Accordingly, in non-SUSY models, it is easy to understand that the effects of the lepton flavor violation (LFV) are suppressed by $1/M_R$. However, in models with SUSY, due to the non-diagonal neutrino mass matrix, the flavor conservation in the lepton sector at the unified scale will be violated at the $M_R$ scale via renormalization [4–6]. The flavor violating effects could propagate to the electroweak scale so that instead of $1/M_R$, the suppression of the LFV could be $1/M_{\text{SUSY}}$ with $M_{\text{SUSY}} \sim O(\text{TeV})$ being the typical mass of the SUSY particle. Consequently, the lepton flavor violating processes, such as $\ell_i^- \to \ell_j \gamma$ and $\ell_i^- \to \ell_j^+ \ell_i^- \ell_i^-$ become detectable at the low energy scale. The LFV has been extensively studied in the literature. For example, $\tau \to \mu \eta$, $B \to (e, \mu)\tau$, and the $\mu \to (e, \tau)$ conversions can be found in Refs. [7–12], while that to the detection of the LFV in colliders is given in Ref. [13].

In the large $\tan\beta$ region, it has been pointed out that the nonholomorphic Yukawa interactions [14–17] play very important roles for flavor changing neutral currents (FCNCs) in the quark sector. In the SUSY-seesaw model, the nonholomorphic terms [6] in the lepton sector naturally induce the LFV due to the Higgs couplings. It has been shown that the contribution to the decay of $\tau \to 3\mu$ from the Higgs-mediated LFV at large $\tan\beta$ could be much larger than that from $\tau \to \ell \gamma \mu \mu^+$ [6,18]. Recently, the experimental limits on the radiative decays of $\tau \to \ell \gamma$ $(\ell = e, \mu)$ have been improved from $O(10^{-6})$ [19] to $O(10^{-7})$ [20,21]. Moreover, the sensitivity of probing the LFV in $\tau$ decays with single pseudoscalar ($P$) or vector ($V$) and double mesons in the final states, i.e., $\tau \to \ell (P, V)$ and $\tau \to \ell PP$, have also reached $O(10^{-7})$ [22]. In this paper, we will simultaneously analyze $\tau \to \ell \gamma$ and $\tau \to \ell X$, where $X$ are $\mu^+ \mu^-$, $\eta^0$, $\phi$, $f_0(980)$, $\sigma(600)$, and $K^+ K^-$, respectively, in the Higgs-mediated mechanism. In particular, we would like to check whether it is possible to have large rates for the processes beside the mode of $\tau \to 3\mu$. Note that the decays of $\tau \to \ell S$ with $S = f_0(980)$ and $\sigma(600)$ and $\tau \to \ell (\phi, K^+ K^-)$ have not been explored previously based on the Higgs-mediated mechanism in the literature, while $\tau \to \ell P$ have been studied in Refs. [7,9,23].

We start with the Higgs-mediated mechanism. It is known that, by the induced slepton flavor mixing, the effective Lagrangian with induced nonholomorphic terms for the Higgs bosons coupling to leptons is given by [6]

$$-\mathcal{L}_{\text{eff}} = \tilde{E}_R i Y_{ij} [\delta_{ij} H_0^0 + (\epsilon_1 \delta_{ij} + \epsilon_2 I_{ij}) H_0^{0*}] E_{Lj} + \text{H.c.}$$

$$= \tilde{E}_R M^0_{ij} E_L + \text{H.c.},$$

where $Y$ denotes the diagonalized Yukawa matrix of leptons, $I_{ij} = (\Delta m^2_{ij})/m^2_0$ and $\epsilon_{(2)}$ is related to the induced lepton flavor conserving (violating) effect, expressed by [6]
where $M_{1,2}$ are the masses of gauginos from the soft SUSY-breaking terms, $\mu$ stands for the mixing of $H_u$ and $H_d$, 
\[
M_1 = \frac{\alpha_1}{8\pi} \mu M_4 \left[ 2 f_1(M_{1}^2, m_{\ell_1}^2, m_{\ell_2}^2) - f_1(M_{2}^2, m_{\ell_1}^2, m_{\ell_2}^2) + 2 f_2(M_{1}^2, m_{\ell_1}^2, m_{\ell_2}^2) \right] + \frac{\alpha_2}{8\pi} \mu M_4 \left[ 2 f_2(M_{1}^2, m_{\ell_1}^2, m_{\ell_2}^2) - f_2(M_{2}^2, m_{\ell_1}^2, m_{\ell_2}^2) \right] + 2 f_2(M_{2}^2, m_{\ell_1}^2, m_{\ell_2}^2) \right],
\]
\[
M_2 = \frac{\alpha_1}{8\pi} \mu M_4 \left[ 2 f_2(M_{1}^2, m_{\ell_1}^2, m_{\ell_2}^2) - f_2(M_{2}^2, m_{\ell_1}^2, m_{\ell_2}^2) \right] + \frac{\alpha_2}{8\pi} \mu M_4 \left[ 2 f_2(M_{1}^2, m_{\ell_1}^2, m_{\ell_2}^2) - f_2(M_{2}^2, m_{\ell_1}^2, m_{\ell_2}^2) \right],
\]
and $\alpha_{1(2)} = g_{1(2)}^2/4\pi$ with $g_{1(2)}$ corresponding to the gauge coupling of the $U(1)(SU(2))$ symmetry. Because of the nonholomorphic term $\varepsilon_i E_{ij}$, the lepton mass matrix is not diagonal anymore. Consequently, after rediagonalizing the lepton mass matrix, the lepton flavor changing neutral interactions through the Higgs bosons appear. Since the nonholomorphic terms are expected to be much less than unity, to obtain the LFV, we take the unitary matrices used for diagonalizing lepton mass matrix to be $U_{\ell\ell' \ell''} = 1 + \Delta_{\ell\ell'}$ as a leading expansion of $\varepsilon_i E_{ij}$, where $\Delta_{\ell\ell'}$ are 3 × 3 matrices. From Eq. (15) and $\Delta M_{\ell\ell'}^{2ij} = (\Delta M_{\ell \ell'})_{ij}$, we may set $\Delta_{\ell \ell'} = \Delta_{\ell \ell'} = \Delta$. Hence, the diagonal mass matrix in Eq. (1) could be obtained by
\[
UM_{\ell\ell'}^0 U^\dagger = (1 + \Delta)M_{\ell\ell'}^0(1 - \Delta) = M_{\ell\ell'}^{\text{dia}},
\]
where $M_{\ell\ell'}^{\text{dia}}$ is the physical mass matrix of the lepton with the diagonal elements being $(M_{\ell\ell'}^{\text{dia}})_{ij} = (m_e, m_\mu, m_\tau)$. At the leading order, we get
\[
(M_{\ell\ell'}^{\text{dia}})_{ij} = (M_{\ell\ell'}^{\text{dia}})_{ii}, \quad \Delta_{ij} \approx \frac{(M_{\ell\ell'}^{\text{dia}})_{ij}}{(M_{\ell\ell'}^{\text{dia}})_{ii} - (M_{\ell\ell'}^{\text{dia}})_{jj}} \quad (i \neq j).
\]
In terms of the physical mass eigenstates of the Higgs bosons, represented by [24]
\[
\text{Re} H_d^0 = v_d + \frac{1}{\sqrt{2}} \left[ \cos \alpha H^0 - \sin \alpha h^0 \right],
\]
\[
\text{Re} H_u^0 = v_u + \frac{1}{\sqrt{2}} \left[ \sin \alpha H^0 + \cos \alpha h^0 \right],
\]
\[
\text{Im} H_d^0 = \frac{1}{\sqrt{2}} \left[ \cos \beta G^0 - \sin \beta A^0 \right],
\]
\[
\text{Im} H_u^0 = \frac{1}{\sqrt{2}} \left[ \sin \beta G^0 + \cos \beta A^0 \right],
\]
where $\alpha$ is the mixing angle of the two CP-even neutral scalars, the interactions for the LFV via the Higgs-mediated mechanism are expressed by
\[
\mathcal{H}_{\ell\ell'}^{i\neq j} = \left( \sqrt{2} G_F \right)^{1/2} \frac{m_{\ell_i} C_{ij}}{\cos^2 \beta} \bar{E}_{iR} \ell_{jL} [\sin(\alpha - \beta)H^0 + \cos(\alpha - \beta)h^0 - ia^0] + \text{H.c.},
\]
where $m_{\ell_i}$ is the mass of the $i$th flavor lepton and $C_{ij} = \varepsilon_i I_{ij}/(1 + (\varepsilon_i + \varepsilon_i I_{ij}) \tan \beta)^2$.

From Eq. (3), we see that the decays of $\tau \rightarrow \ell P$ only pick up the contributions from the pseudoscalar boson $A^0$, while $\tau \rightarrow \ell S$ and $\tau \rightarrow \ell PP$ are governed by both scalar bosons $H^0$ and $h^0$ due to the parity properties. In our following analysis, we only concentrate on the processes associated with the productions of $ss$ and $\mu^+ \mu^-$ pairs to avoid small Higgs couplings. We choose the decays of $\tau \rightarrow X$ with $X = \mu^+ \mu^-$, $\eta^0$, $f_0(980)(\sigma(600))$, and $K^+ K^-$ as the representative modes. For $\tau \rightarrow \ell \mu^+ \mu^-$, the formalisms for the decay rates dictated by scalar and pseudoscalar bosons are given by
\[
\Gamma(\tau \rightarrow \ell \mu^+ \mu^-) \approx c_e G_F^2 m_{\mu_i}^2 C_{ij}^{1/2} \left[ \frac{c}{m_h^2} - \frac{s}{m_H^2} \right]^2 \left[ \sin^2 \beta \right]^2.
\]
where $c_e = 3/2$ and 1 with $\ell = \mu$ and $e$, $c_s = \cos(\alpha - \beta)\sin \alpha$, and $s_c = \sin(\alpha - \beta)\cos \alpha$, respectively. To study the production of $\eta^0$, we adopt the quark-flavor scheme, defined by [25]
\[
\left( \begin{array}{c}
\eta_1 \\
\sin \phi \\
\cos \phi
\end{array} \right),
\]
where $\eta_q = (u + d \bar{d})/\sqrt{2}$ and $\eta_s = s \bar{s}$. From $\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \eta_q(p) \rangle = f_{\eta_q} \gamma_\mu p_\mu$, the mass of $\eta_q$ can be expressed by $m_{\eta_q}^2 = f_{\eta_q} (0)_s \bar{s} \bar{d} s u + m_d \bar{d} \bar{s} \bar{d} s \eta_q (m_{\eta_q}^2 = \frac{1}{2} (0)_s \bar{s} \bar{d} s \eta_q)$. If we neglect the $\eta_q$ contribution due to small $m_{u,d}$, the decay rates for $\tau \rightarrow \ell \eta$ can be written as
\[
\Gamma(\tau \rightarrow \ell \eta) \approx G_F^2 m_{\eta_q}^2 C_{\ell\ell'} |\sin \beta| \left[ \frac{\sin^2 f_{\eta_q}}{m_{\eta_q}^2} \frac{m_{\ell_1}^2}{m_{\ell_2}^2} \right]^2 \left[ \frac{1}{m_{\eta_q}^2} \right]^2.
\]
Similarly, the rate for $\tau \rightarrow \ell \eta'$ is given by
\[
\Gamma(\tau \rightarrow \ell \eta') = \cot^2 \phi \frac{1}{1 - m_{\eta_q}^2/m_{\eta'}^2} \left[ \frac{1}{1 - m_{\eta_q}^2/m_{\eta'}^2} \right]^2.
\]
For $\tau \rightarrow f_0(980)(\sigma(600))$ decays, although the quark contents of $f_0(980)$ and $\sigma(600)$ are still uncertain, we
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adopt two quark contents to describe the states. In terms of the notations in Refs. [26,27], the isoscalar states $f_0(980)$ and $\sigma(600)$ are described by $f_0(980) = \cos \theta [s\bar{s}] + \sin \theta [n\bar{n}]$ and $\sigma(600) = -\sin \theta [s\bar{s}] + \cos \theta [n\bar{n}]$ with $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\theta$ being the mixing angle. The decay constants are defined as

$$\langle f_0^*|s\bar{s}|0\rangle = m_{f_0} f_{s\bar{s}}, \quad \langle \sigma|s\bar{s}|0\rangle = m_{\sigma} f_{s\bar{s}}, \quad (8)$$

where $f_{s\bar{s}}$ and $f_{\sigma}$ represent the $s\bar{s}$ component in $f_0(980)$ and $\sigma(600)$, respectively. As a result, the decay rates of $\tau \to f_0(980)$ are given by

$$\Gamma(\tau \to f_0(980)) \approx \frac{G_F^2 m_{\tau}^3 |C_{\tau\tau}|^2}{16\pi \cos^3 \beta} (m_{m_{f_0}^*} f_{s\bar{s}} \cos \theta)^2 \times \left( \frac{m_s}{m_h} - \frac{m_c}{m_{\tau}} \right)^2 \left( 1 - \frac{m_{\mu}^2}{m_{\tau}^2} \right)^2. \quad (9)$$

On the other hand, the rates for $\tau \to \sigma(600)$ can be obtained by

$$\Gamma(\tau \to \sigma(600)) \approx \frac{\langle m_{\sigma} f_{s\bar{s}} \tan \theta \rangle^2}{m_{f_0} f_{s\bar{s}}^2} \left( 1 - \frac{m_{\mu}^2}{m_{\tau}^2} \right)^2. \quad (10)$$

For the three-body decays of $\tau \to \ell K^+ K^-$, the associated hadronic effects are much more complicated and unclear. Nevertheless, the uncertainties could be fixed by the $B$ decays, such as $B \to KKK$. The related form factor including resonant and nonresonant effects is defined by [28]

$$\langle K^+(p_1)K^-(p_2)|s\bar{s}|0\rangle = f_s^+ s^- K^-(Q^2) = \sum_S \frac{m_S f_{s\bar{s}}^S g^{S-KK}}{m_S - Q^2 - i\delta m_S} + f_{sNR}^S, \quad (11)$$

where $S$ stands for the possible scalar meson state, $m_S f_{s\bar{s}}^S = \langle S|s\bar{s}|0\rangle$, $g^{S-KK}$ denotes the strong coupling for $S \to KK$, and

$$f_{sNR} = \frac{\nu}{\sqrt{2}} \left( 3F_{NR}^1 + 2F_{NR}^2 \right) + \nu \frac{\kappa}{Q^2} \left( \ln^2 \frac{Q^2}{\Lambda^2} \right)^{-1}, \quad (12)$$

$$F_{NR}^1(2) = \left( x_1(2)^2 + x_2(2)^2 \right) \left( \ln^2 \frac{Q^2}{\Lambda^2} \right)^{-1},$$

with $\nu = (m_S^2 - m_{\ell}^2)/(m_{\tau} - m_{S})$, $x_1 = -3.26$ GeV$^2$, $x_2 = 5.02$ GeV$^2$, $x_1 = 0.47$ GeV$^2$, and $x_2 = 0$. It is found that only $f_0(980)$ and $f_0(1530)$ have the largest couplings to the $KK$ pair [29]. Note that in calculating $B \to KKK$ [28], the factorization approach in Ref. [30] has been used. In our numerical estimations, we will only consider these two scalar contributions. The differential decay rates as a function of the invariant mass in the $KK$ system are given by

$$d\Gamma(\tau \to \ell K^+ K^-)/dQ^2 \approx G_F^2 m_{\tau}^3 |C_{\tau\tau}|^2 \left( m_{f_0}^* f_{s\bar{s}} \right)^2 \left( \frac{c_s}{m_h} - \frac{s_c}{m_{\tau}} \right)^2 \times \left( 1 - \frac{Q^2}{m_{\tau}^2} \right)^2 \left( 1 - 4\frac{m_{\tau}^2}{Q^2} \right)^{1/2}. \quad (13)$$

From Eqs. (4), (6), and (9), it is interesting to see that the various decay rates mediated by the Higgs bosons have the relationship

$$\Gamma(\tau \to \ell \mu^+ \mu^-) = -\frac{c_{\tau\ell}}{3 \times 2^5 \pi^3} \left( \frac{m_{\tau}^2 m_{\mu}^2 m_{\nu}}{C_{\mu}} - \frac{m_{\mu}^2 m_{\tau}^2 m_{\nu}}{C_{\nu}} \right). \quad (14)$$

where $C_{\eta} = (\sin \beta \sin \phi f_s m_{\mu}^2/m_{\tau}^2)^2 (1 - m_{\mu}^2/m_{\tau}^2)^2$ and $C_{f_0} = (m_{m_{f_0}^*} f_{s\bar{s}} \sigma(600)^2 (1 - m_{\mu}^2/m_{\tau}^2)^2)$.

We now consider the radiative modes of $\tau \to \ell \gamma$. At the large tan$\beta$ scenario, the dominant contributions to the decays are illustrated in Fig. 1. To simplify the estimations, we use the mass insertion method to formulate the decay amplitudes. The induced LFVs in the slepton mass matrix can be approximately written as [4,5,31]

$$\langle \Delta m_{L}\rangle_{ij} \approx -\frac{1}{(4\pi^2)} (6m_{0}^2 Y_{\nu} Y_{\nu} + 2A_{ij} A_{ij} \ln \frac{M_{U}}{M_{R}}). \quad (15)$$

where $m_{0}, Y_{\nu}$, and $A_{ij}$ denote the typical initial soft SUSY-breaking mass of the slepton, the neutrino Yukawa couplings, and the trilinear soft SUSY-breaking effects, respectively, at the unified scale of $M_{U}$. From Fig. 1 and Eq. (15), the effective interactions for $\tau \to \ell \gamma$ are given by

$$T = G_F \epsilon_{\mu
u} e^{\mu
u}(k) \epsilon(\ell(p - k)) i\sigma_{\mu\rho}k^\rho A_{R}(1 + \gamma_5)\ell(p), \quad (16)$$

where

$$A_{R} = \frac{M_{S} \mu}{4(4\pi^2)^2 M_{2}} \tan \beta (\Delta m_{L})_{\tau} \sum_{S \to \ell, \bar{\ell}} G_{S}. \quad (17)$$

$$G_{\ell} = \frac{1}{m_{\ell}^2 - m_{\ell}^2 - m_{\ell}^2} \left[ f_{\ell}(x_{\ell}) - f_{\ell}(x_{\bar{\ell}}) \right],$$

$$G_{\bar{\ell}} = \frac{4}{m_{\ell}^2 - m_{\ell}^2} \left[ f_{\ell}(x_{\bar{\ell}}) - f_{\ell}(x_{\ell}) \right], \quad (18)$$

with $x_{\ell} = M_{L}^2/m_{\ell}^2 [5]$. Here, we have set the masses of Higgsinos and gauginos to be the same, denoted as $M_{2}$. Subsequently, the decay rates of $\tau \to \ell \gamma$ are given by

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cal values, we set $h$ and $M$ the form factors given in Refs. [28,32], respectively. It is easy to estimate the ratios of branching ratios (BRs) to be [5]

$$R^L_\ell = \frac{\text{BR}(\tau \to \ell X) \gamma}{\text{BR}(\tau \to \ell \gamma)} = O\left(\frac{\alpha_{em}}{\pi}\right) \sim 10^{-3},$$

(20)

$$(X_\gamma = \mu^+ \mu^-, \phi, K^+ K^-).$$

Note that it is impossible to produce modes with $X_\gamma$ being a single pseudoscalar or scalar by the dipole operators in Eq. (16). In our estimations for the modes with $X_\gamma = \phi$ and $K^+ K^-$, we have used the hadronic matrix elements defined by $\langle 0|\bar{q}\gamma^\mu q|\phi\rangle = im_\phi \epsilon_\phi^\mu(k)$ and $\langle 0|\bar{q}\gamma^\mu q|K^+(p_1)K^-(p_2)\rangle = (p_1^\mu - p_2^\mu) F^a_{\gamma} K^-(Q^2)$, with the form factors given in Refs. [28,32], respectively. It is clear that from the current limits on $\text{BR}(\tau \to \ell \gamma)$, $\text{BR}(\tau \to \ell X_\gamma)$, are too small to be observed. We remark that other loop contributions to the decays, such as those from box diagrams, are expected to be small due to the light fermion final states.

For the numerical estimations on $\tau \to \ell \gamma$ and $\tau \to \ell X$, we assume that $M_1 \sim M_2 \sim m_0 \sim \mu \sim m_\tilde{\tau} \sim m_\tilde{\tau}$ to simplify our discussions. Consequently, Eqs. (2) and (17) become

$$e_1 = \frac{3\alpha_{em}}{4\pi \sin^2(2\theta_W)}, \quad e_2 = \frac{\alpha_{em}}{16\pi} \left(\frac{1}{3 \cos^2\theta_W} + \frac{1}{3 \sin^2\theta_W}\right),$$

$$A_R = \frac{1}{64\pi^2} \frac{m_0^2}{m_\tilde{\tau}^2} \tan\beta(1 + \tan^2\theta_W),$$

(21)

respectively. If we regard $A^A_{1\ell}$ in Eq. (15) as $(A^A_{1\ell})_{\tau\ell} \sim m_0^A (Y^A_\tau Y^A_\ell)_{\tau\ell} = m_0^A O(1)$, we get $(\Delta m^2_{\ell\tau}/m_0^A) \sim -8/(4\pi^2) \ln(M_\mu/M_\tau)$. Thus, we find that $C_{\tau\ell}$ are insensitive to the SUSY-breaking scale and the decays of $\tau \to \ell \gamma$ and $\tau \to \ell X$ are only sensitive to the masses of the slepton and Higgs bosons, respectively. In calculating the numerical values, we set $G_{U(8)} = 10^{10}(10^{10})$ GeV and $\tan\beta = 60$. Other parameters in various modes are taken to be as follows: $\phi = 39^\circ$, $f_\tau = 0.17$ GeV, and $m_{\tilde{s}} = 0.69$ GeV for $\tau \to \ell \eta^{(0)}$ [25]; $\theta = 30^\circ$, $m_s = 0.15$ GeV, and $f_\tau \sim \frac{f_{f_0}}{3} = 0.33$ GeV [27] for $\tau \to \ell (f_0(980), \sigma(600))$; $\nu = 2.87$ GeV, $\kappa = -10.4$ GeV$^4$, $f_{f_0(1530)} \sim f_{f_0(980)} = 0.33$ GeV, $g_{f_0(980) \to KK} = 1.50$ GeV, $g_{f_0(1530) \to KK} = 3.18$ GeV [28], $\Gamma_{f_0(980)} = 80$ MeV, and $\Gamma_{f_0(1530)} = 1.16$ GeV [29] for $\tau \to \ell K^+ K^-$. For simplicity, we do not distinguish the difference between $(Y^A_\tau Y^A_\ell)_{\tau\ell}$ and $(Y^A_\tau Y^A_\ell)_{\tau\mu}$, i.e., $(\Delta m^2_{\ell\tau})_{\tau\ell} = (\Delta m^2_{\ell\tau})_{\tau\mu}$.

In Fig. 2, we present the BRs for $\tau \to \ell \gamma$ as a function of the slepton mass. In comparison with the BELLE and BABAR results of $\text{BR}(\tau \to \ell \gamma) < 3.1 \times 10^{-7}$ [20] and $0.68 \times 10^{-7}$ [21], we see clearly that $m_\tau > 1$ TeV is favorable. The BRs of $\tau \to \ell \gamma$ as a function of the pseudoscalar mass are displayed in Fig. 3(a). From Eq. (7), we have $\text{BR}(\tau \to \ell \eta') = 0.93\text{BR}(\tau \to \ell \eta)$. The BRs of $\tau \to f_{0}(980)$ and $\tau \to (K^+ K^-)$ as a function of $M_H = (c s / m_{\tilde{\tau}} - s c / m_{H})^{-1/2}$ are shown in Figs. 3(b) and 3(c), respectively. In terms of Eq. (10), we get $\text{BR}(\tau \to \ell (\sigma(600))) = 0.28\text{BR}(\tau \to f_{0}(980))$. In addition, from Eq. (14), we obtain $\text{BR}(\tau \to \ell (\mu^+ \mu^-)) = 0.33[\text{BR}(\tau \to \ell \eta) + 1.6\text{BR}(\tau \to f_{0}(980))]$. Clearly, all $\tau \to \ell X$ modes except $\tau \to \ell \sigma(600)$ are suitable to search for the LFV. Finally, it is worth mentioning that if we take the decoupling limit, i.e., $m_H = m_\lambda$ and $\alpha = \beta - \pi/2$ [24], leading to $M_H = m_\lambda$, we get $\Gamma(\tau \to f_{0}(980)) : \Gamma(\tau \to \ell \mu^+ \mu^-) : \Gamma(\tau \to \ell \eta) = 1.3 : 0.36 c_\ell : 1$. 

FIG. 1. The Feynman diagrams for $\tau \to \ell \gamma$ with a large $\tan\beta$. The crosses represent the various mixing effects.

$$\Gamma(\tau \to \ell \gamma) = \frac{\alpha_{em}}{2} G^2_{\tau} m_\tau^3 |A_R|^2.$$

FIG. 2. Branching ratios (in units of $10^{-7}$) for $\tau \to \ell \gamma$ as a function of the stau mass.
In summary, we have studied the lepton flavor violating \( \tau \) decays through the Higgs-mediated mechanism with the nonholomorphic terms from the couplings between the Higgs bosons and leptons at the large tan\( \beta \). By assuming that all masses associated with SUSY breaking are the same, we have demonstrated that BRs of \( \tau \rightarrow \ell \eta \) only depend on the stau mass. In the Higgs-mediated mechanism, we have shown that the BRs of the new proposed decays of \( \tau \rightarrow \ell f_0(980) \) and \( \tau \rightarrow \ell K^+ K^- \) arising from the scalar exchanges can be as large as the upper limits \( O(10^{-7}) \) of the current data and, moreover, they can be larger than those of \( \tau \rightarrow \ell \eta \) from pseudoscalar exchanges. We have also pointed out that \( \tau \rightarrow \ell \mu^+ \mu^- \) are related with \( \tau \rightarrow \ell \eta \) and \( \tau \rightarrow \ell f_0(980) \). It is clear that future experimental searches for the LFV in the leptonic and semileptonic tau flavor violating decays are important for us to identify the Higgs-mediated mechanism.

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\[ \begin{align*}
\text{FIG. 3. Branching ratios (in units of } 10^{-7} \text{) for (a) } & \tau \rightarrow \ell \eta \text{ and (b) } \tau \rightarrow \ell f_0(980)[K^+ K^-] \text{ as functions of the pseudoscalar and scalar Higgs masses, respectively.} \\
\end{align*} \]