Multi-plate crystal cavity with compound refractive lenses

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ABSTRACT

Multi-plate crystal cavities consisting of compound refractive lenses were prepared on silicon wafers by lithographic techniques. The crystallographic orientation of the crystal device is the same as that of the two-plate x-ray resonators reported (Phys. Rev. Lett. 94, 174801, 2005). X-ray (12 4 0) back diffraction from these monolithic silicon crystal devices showed interference fringes due to cavity resonance through the compound refractive lenses (CRL), but with less pronounced amplitudes. The transmitted beam size through this device is reduced by a factor of 5. Detailed analysis on cavity interference and beam suppression are discussed.

Keywords: Crystal cavity, compound refractive lens, x-ray back diffraction, focusing

1. INTRODUCTION

X-ray Fabry-Perot type resonators were proposed forty years ago [1, 2]. There seems a revival of interest in studying X-ray resonators and related optical components in recent years [3-6], mainly because of the availability of highly intense synchrotron X-ray sources, high-resolution monochromators, and new technology of tailoring materials. Very recently cavity resonant interference patterns have been observed for X-rays, using back diffraction in a monolithic multi-plate resonator [7]. In order to investigate focusing effects in this type of X-ray cavities, we adopt the structure of X-ray compound refractive lenses (CRL) [8] such that a multi-plate resonator consists of many concave lenses, while each lens acts still as reflection mirror for X-ray back diffraction. Diffraction experiments from this crystal device with successive forward transmission and backward reflection are then performed and the beam size of the transmitted beam through the device is measured to investigate possible focusing effects.

2. EXPERIMENTAL

Multi-plate crystal cavities consisting of compound refractive lenses were prepared on (001) silicon wafers by lithographic techniques. A series of cylindrical holes along [001] with circular cross section of radius R drilled on the wafers produced a series of concave lenses capable of focusing incident x-rays. The distance between two adjacent holes is d. The curved (concave) crystal plates are lined up in the [310] direction so that the (12 4 0) reflection can be used as the back reflection to generate consecutive mirror reflections among the crystal plates, thus leading to cavity resonance.

The focus length F of the CRL follows \( F=\frac{R}{(2N\delta)} \) [8] for non-absorbing materials, where R is the radius of the lens, N is the number of holes, and \( \delta \) is the correction of refractive index, i.e., \( \delta =2.33 \times 10^{-6} \) for the 14.4388 keV (0.8605 \( \mu \)m) x-ray photons, with which the back diffraction (12 4 0) of silicon takes place and leads to cavity resonance. The absorption of silicon at this photon energy is appreciable so the actual focus length would differ considerably from the value given by the above formula. Since the distance between the detector and the center of the diffractometer at which the crystal device is placed is about 90 cm, we choose \( F=1 \) m to facilitate the beam-size measurements of the transmitted beam through the crystal device.

To optimize the visibility of cavity resonance fringes, we choose the total crystal thickness of the device, excluding the diameters of the holes, to be about 140 \( \mu \)m, at which the reflectivity of the device is equal to the transmissivity. Also we
try to keep as high as possible the real gain $g$ of a CRL, defined as $g=aG\{\sigma_f/\sigma_1\}$, where the absorption $a=\exp[-\mu Nd]$, $G$ is the ideal gain ($=\{A/\sigma_f\}[1+\{r_f/r_o\}]$), $\sigma_f$ is the diffraction-limited resolution of the lens, $\sigma_1$ is the real focus size, and $A$ is the effective lens aperture defined as $A=2R[2/(\mu RN)]^{-1/2}$. The larger the real gain, the smaller the total crystal thickness $N \times d$. Under these considerations, we fabricate a CRL by etching 11 equally spaced cylindrical holes ($N=11$), each having a radius of $R=51 \mu m$ and the distance between the adjacent holes is $d=13 \mu m$ on silicon (001) wafers prepared by x-ray lithographic techniques. The real gain is $g=49.7$, corresponding to $\mu =25.18 \ (1/cm)$, $G=172$, $a=0.7$, $A=121 \mu m$, and $\sigma_f=0.7 \mu m$. The real focus size $\sigma_1$ is 2 $\mu m$, which is estimated according to $\sigma_1=\sigma_o(\sigma_f/r_o)$, where $\sigma_o$ is the source size equal to 25 $\mu m$, $r_f$ is the image distance about 1.02 m estimated via $r_f=F[r_o/(r_o-F)]$. $r_o$ is the source distance from the crystal which is 58.4 m for beam line BL12XU at SPring-8. The crystal orientation of the CRL is that the lenses are lined up in the [130] direction and the focal plane lies in the plane perpendicular to [001].

The experiment is performed at the Taiwan undulator beamline BL12XU at the SPring-8 synchrotron radiation facility in Japan. The storage ring is operating at 8 GeV and 100 mA. The experimental setup is the same as that reported in [7]. The incident radiation, monochromatized first by a Si (111) double-crystal and then by a four-crystal ultra-high resolution monochromator [9], gives the energy resolution $\Delta E=0.36 \text{ meV}$ at 14.4388 keV. The sample crystal, placed at the center of an 8-circle diffractometer, is aligned for the (1240) back diffraction. By tuning together the Bragg's angles of third and fourth crystals of the four-crystal monochromator, energy scan of the (000) transmitted beam can be performed.

3. RESULTS AND ANALYSIS

The energy scan of the (000) transmitted beam is shown in Fig. 1, where resonance fringes are clearly seen.

![Fig. 1 Energy scan of the transmitted (000) beam: Interference fringes due to cavity resonance are observed.](image-url)
The free spectral range $E_d$ measured from Fig. 1 is 5.01 meV in agreement with 5.4 meV estimated from $E_d = \frac{hc}{(2d_{eff})}$, where $h$, $c$, and $d_{eff}$ are the Planck constant, the speed of light, and the effective gap between the two adjacent lenses ($d_{eff} = d + 2R$) [10]. The measured distribution of the transmitted intensity as a function of photon energy covers approximately ± 60 meV from the exact resonance energy $E_o (=14.4388$ keV) at the minimum (Fig.1).

Under this cavity resonance condition, we then scan a knife-edge across the transmitted beam along [-1 3 0] in front of the detector to measure the size of the transmitted beam through the compound refractive lens. The beam sizes are 17, 17, 18, 13 $\mu$m, measured at the distances 61, 71, 81, 91 cm behind the CRL, respectively. Comparing with the beam sizes of the direct incident beam, which are 87, 87, 88, 86 $\mu$m measured at the distances 61, 71, 81, and 91 cm respectively from the diffractometer center without putting in the CRL crystal, the transmitted beam size is reduced by a factor of 5. This result is shown in Fig.2.

![Fig.2 Beam sizes of the transmitted (000) beam through the multiple-plate cavity measured at 61, 71, 81, 91 cm from the end of the cavity.](image)

The cavity resonance fringes for the CRL cavity shown in Fig.1 are not as distinct as for a plane-parallel two-plate cavity. This is because the entrance surface of the CRL plates is curved. On the other hand, considering only beam focusing, a CRL without satisfying a back diffraction, nor suffering from absorption, will focus an incident beam of 100 $\mu$m with a Gaussian distribution into 2 $\mu$m in the [-1 3 0] direction at focal length equal to 1 m. That means the transmitted beam through the CRL is no longer parallel. If crystal absorption ($\mu = 25.6 (1/cm)$ at $E = 14.4388$ keV) and the dynamical diffraction effect are considered [11-13], however, the incident beam size will be reduced to about 47 $\mu$m under focusing situation. When a CRL cavity fulfills both the focusing and back diffraction (12 4 0) for cavity resonance, the transmitted beam size through these 11 cylindrical holes will drop to 28 $\mu$m. This almost 3.6 times beam-size reduction is less than the measured one (5 times). The difference is probably due to anomalous absorption resulting from the coexisting 24-beam simultaneous diffraction at $E = 14.4388$ keV [12, 13] and refraction effect.

The experimental results imply that the smaller parallel transmitted beam size is mainly due to the successive back diffractions. Hence the back diffraction tries to produce more parallel beam, while the CRL focusing intends to generate focused (non-parallel) and yet small sized beam. There is indeed a competition between the cavity resonance due to back diffraction and beam focusing by the CRL, thus leading to a small sized parallel beam.
Fig. 3 Distributions of the transmitted beam: (a) calculated at the entrance; (b) calculated at the exit of the CRL; (c) measured intensity distribution at the entrance by a moving sharp knife edge; (d) measured intensity at the exit by the same knife edge; (e) the derivative of (c); and (f) the derivative of (d) with respect to the knife edge position. The small additional peak in (f) is probably due to scattering from a guard slit.
The analysis of beam size is carried out as follows: The incident beam of the beam size of 100 µm is assumed to have a Gaussian distribution (Fig. 3a). The integrated intensity is 86.17 in an arbitrary unit. The analysis of the diffraction from the CRL in terms of the dynamical theory of x-ray diffraction [11] is a formidable task, because the excitation of the dispersion surface for the CRL’s cylindrical surface is very complicated. The wave vectors of the excited waves inside the first crystal plate could change when the transmitted x-rays enter the second crystal plate and there are 10 plates involved in the diffraction process. As an alternative, we first deal with diffraction without considering focusing effect due to the cylindrical crystal boundaries and then compare the calculated intensity with the measured one. The focusing effect could become clear from this comparison, if the measured intensity per unit beam size is larger than the calculated one. In the calculation, because the CRL has a cylindrical surface, the incident beam is divided into many smaller beams of the beam size of 1 µm each and the curved crystal is assumed to consist of many rectangular slabs with the width of 1 µm (the cylindrical surface is now composed of many small rectangular steps). Each beam diffracts in a given crystal slab. And the contribution of each smaller beam in the diffraction process from a slab is summed up, which yields the integrated distribution. Since the finesse of the cavity, which is $F = E_d / \Gamma$ = 2.4 measured from Fig. 1, each incident beam of 1 µm size has been effectively reflected 2 times between two crystal plates while propagating through the 11 holes (10 crystal-plates). The dynamical theory of back diffraction [11, 13] is then used to calculate the diffraction intensities. Note that the Darwin width of the (12 4 0), proportional to $2 \sqrt{|\chi_G|}$, is equal to 0.071 deg., where $\chi_G = -3.927 \times 10^{-7} + i 1.8 \times 10^{-8}$ and $G = (12 4 0)$. The angle of refraction for one crystal plate is about 0.0002 deg. and is 0.0028 deg. for 10 crystal plates, which is yet much smaller than the Darwin width. Hence the effect of refraction due to curve crystals on the diffracted intensities is negligibly small. By considering all the facts, the calculated integrated intensity over beam size drops to 17.9 and the beam size becomes 29.4 µm (Fig. 3b).

Experimentally, the measured incident intensity versus the position of the knife edge during the translation of the knife edge across the beam at the position 90 cm behind the CRL along the incident-beam direction is shown in Fig.3c and the integrated intensity over the beam size is about 3030. Figure 3d is the intensity distribution of the transmitted beam through the CRL measured at the same position as Fig.3c. Figures 3e and 3f are the derivatives of Fig. 3c and 3d with respect to the position of the knife edge, respectively, of which the FWHM's (full width at half maxima) yield the beam sizes of 86.2 and 14.6 µm for Fig. 3c and 3d. The integrated intensity of Fig.3d is about 525. The ratio of the reduced intensity is 0.17 (~525/3030), which is in agreement with the theoretical value 0.21 (~17.9/86.17). Moreover, if we divide the two ratios by the corresponding beam sizes, 14.6 and 29.4 µm, then the ratio per beam size of the current experiment is 1.67 times larger than the theoretical value. This indicates that the focusing effect did play a role in affecting the measured beam size, while the calculation without considering the curved surface gives a wider beam.

In conclusion, we have observed optical effects in curved multi-plate x-ray crystal cavities consisting of compound refractive lenses, i.e., the expected focusing effect from the CRL was not very pronounced but rather beam compression was detected. Thus, a small sized x-ray beam was produced. This beam compression is believed to be attributed to the competition between the multiple back reflection in the crystal cavity and the focusing of the CRL, in addition to crystal absorption. This competing mechanism may find usage in producing small sized quasi-coherent beams in very low-emittance synchrotron facilities.

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