Lepton flavor violation as a probe of quark-lepton unification

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(Received 14 March 2005; published 11 August 2005)

The recent measurements of the solar neutrino mixing angle $\theta_{\text{sol}}$ and the Cabibbo mixing angle $\theta_C$ reveal a surprising relation, $\theta_{\text{sol}} + \theta_C \approx \frac{\pi}{4}$, which has been interpreted as an evidence for quark-lepton unification. We show in realizations of quark-lepton unification that the PMNS mixing matrix can be decomposed into a CKM-like matrix and maximal mixing matrices. We explore a possibility to probe such implications by considering the relative sizes of branching ratios for the lepton flavor violating radiative decay processes, $l_i \rightarrow l_j \gamma$, in the context of the supersymmetric standard model with heavy right-handed Majorana neutrinos.

DOI: 10.1103/PhysRevD.72.036003

PACS numbers: 11.30.Hv, 12.15.Ff, 13.35.−r, 14.60.Pq

Neutrino studies will enter a new era when the MINOS experiment starts firing a neutrino beam toward the Soudan mine in March 2005. Until now, while the atmospheric neutrino deficit still points toward a maximal mixing between the tau and muon neutrinos, however the solar neutrino problem favors a not-so-maximal mixing between the electron and muon neutrinos. Surprisingly, it has recently been noted that the solar neutrino mixing angle $\theta_{\text{sol}}$ required for a solution of the solar neutrino problem and the Cabibbo angle $\theta_C$ reveal a striking relation [1]

$$\theta_{\text{sol}} + \theta_C \approx \frac{\pi}{4}, \tag{1}$$

which is satisfied by the experimental results within a few percent accuracy $\theta_{\text{sol}} + \theta_C = 45.4^\circ \pm 1.7^\circ$ [2–4]. This quark-lepton complementarity (QLC) relation (1) has been simply interpreted as an evidence for certain quark-lepton symmetry or quark-lepton unification as shown in Refs. [1,5–7].

To effectively describe the deviation from maximal mixing of solar neutrino as well as a small mixing element $U_{e3}$ and possible deviation from maximal mixing of atmospheric neutrino, three possible combinations of maximal mixing and a certain mixing matrix $U(\lambda)$ parameterized in terms of a small parameter $\lambda \sim \sin \theta_C$ have been proposed as parametrization of $U_{\text{PMNS}}$ [8–11]:

(a) $U^\dagger(\lambda)U_{\text{bimax}}$,

(b) $U_{\text{bimax}}U^\dagger(\lambda)$,

(c) $U_{23}^m U_{12}^m U^\dagger(\lambda)$.

Here $U_{\text{bimax}}$ corresponds to the bi-maximal lepton mixing matrix [12], and $U_{23}^m$, $U_{12}^m$ denote the rotation matrices with (2,3) and (1,2) maximal mixing, respectively. Unfortunately, the present oscillation data are not sufficient to determine which combination can give correct flavor structure in $U_{\text{PMNS}}$. It is, however, very important to distinguish these various decompositions of $U_{\text{PMNS}}$ because they are realized in different quark-lepton unification scenarios. As extensively studied in [5], the QLC relation can be derived from the parametrization given above but up to some corrections. These corrections to the QLC relation can be compensated with renormalization effects [7].

In this paper, we will show that among possible forms of $U(\lambda)$ which are consistent with the neutrino experimental results, the “CKM-like” form of $U(\lambda)$ implied by quark-lepton unification would give rise to interesting lepton flavor violating processes. Motivated by this observation, we will study the implication of the parametrization composed of bi-maximal mixing and $U_{\text{CKM}}$ reflecting quark-lepton unification by considering the lepton flavor violating (LFV) decays particularly in the context of supersymmetric standard model (SSM). We also examine a possibility to differentiate the above combinations by considering the relative size of branching ratios of the radiative LFV decays, $Br(l_i \rightarrow l_j \gamma)(i,j = e, \mu, \tau)$. While the LFV processes have tiny rates in the minimal extensions of the standard model (SM) with heavy right-handed Majorana neutrinos, the supersymmetric extensions of the SM can lead to sizable effects on the LFV processes due to new sources of lepton flavor violation. As is well known, the LFV decays in SSM can be caused by the misalignment of lepton and slepton mass matrices [13] and the branching ratios of the LFV decays depend on the specific structure of the neutrino Dirac Yukawa matrix $Y_\nu$. Therefore, we expect that a specific structure of $Y_\nu$ reflecting quark-lepton unification can lead to distinctive predictions for the branching ratios of the LFV decays. However, the branching ratios of the LFV decays in SSM strongly depend on several parameters which make it difficult to probe the
structure of $Y_p$. Instead of considering the branching ratios of each LFV process, we can rely on the relative size of $Br(l_i \to l_j \gamma)$ among the three different flavors, because the relative size is almost free from arbitrary supersymmetric parameters. These ratios of $Br(l_i \to l_j \gamma)$ can be useful to probe the structure of $Y_p$ with the help of the parametrization of $U_{PMNS}$ given in Eq. (2). In particular, we expect that a hierarchical structure of $Y_p$ predicted by quark-lepton unification may be responsible for the hierarchy of $Br(l_i \to l_j \gamma)$ if they are observed in the future experiments. In such a way, the quark-lepton unification could be tested from the determination of the relative size of the branching ratios in future experiments.

Let us begin by considering how the parametrization given by the forms of Eq. (2) can be realized in the framework of the quark-lepton unification. For our purpose, it is useful to work in a basis where the quark and lepton Yukawa matrices are related to each other by a certain symmetry. In general, the quark Yukawa matrices $Y_u$, $Y_d$ are given by

$$Y_u = U_u V_{PMNS} Y_{u,\text{diag}} V_u^\dagger, Y_d = U_d V_{PMNS} Y_{d,\text{diag}} V_d^\dagger,$$

from which the observable CKM quark mixing matrix is described as $U_{CKM} = U_u^T U_d$. For the lepton sector, we consider the following leptonic superpotential, which implements the seesaw mechanism:

$$W_{\text{lepton}} = Y_l \tilde{L}_L \tilde{H}_d + Y_e \tilde{L}_L \tilde{N}_L \tilde{H}_u - \frac{1}{2} \tilde{N}_L^c M_R \tilde{N}_L^c,$$

where the family indices have been suppressed and $\tilde{L}_j$, $j = e, \mu, \tau = 1, 2, 3$, represent the chiral supermultiplets of the $SU(2)_L$ doublet lepton fields, $\tilde{N}_L^c$, $\tilde{N}_L^c$ are the supermultiplet of the $SU(2)_L$ singlet neutrino and charged lepton field, respectively. In the superpotential $W_{\text{lepton}}$, $M_R$ is the heavy Majorana neutrino mass matrix. $Y_l$ and $Y_e$ are the $3 \times 3$ charged lepton and neutrino Dirac Yukawa matrices, respectively, and can be parameterized as

$$Y_l = U_l Y_{l,\text{diag}} V_l^\dagger, Y_e = U_e Y_{e,\text{diag}} V_e^\dagger.$$

We note that in the framework of the minimal unification and the symmetric basis where the quarks and leptons are interrelated, $M_R$ is generally not diagonal. The light neutrino mass matrix can be generated through the seesaw mechanism after the breaking of the electroweak symmetry as

$$M_\nu = (U_0 M_{\text{Dirac}}^{\text{diag}} V_0^\dagger) M_R^{-1} (V_0^T M_{\text{Dirac}}^{\text{diag}} U_0^T),$$

where $M_{\text{Dirac}} = Y_e u_e / \sqrt{2}$ with $u_e = v \sin \beta$. We can then rewrite $M_\nu$ as follows

$$M_\nu = U_0 V_M M_{\text{Dirac}}^{\text{diag}} V_M^T U_0^T,$$

where $V_M$ represents the diagonalizing matrix of

$$M_{\text{Dirac}}^{\text{diag}} V_0^T M_R^{-1} V_0^T M_{\text{Dirac}}^{\text{diag}}.$$

Then, the observable PMNS mixing matrix can be written as

$$U_{PMNS} = U_1^T U_\nu = U_1^T U_0 V_M.$$

Now, let us consider how $U_{PMNS}$ given by Eq. (7) can be related with $U_{CKM}$ in the context of quark-lepton unification. The quark-lepton unification based on the minimal $SU(5)$ leads to the following simple relations, $Y_\ell$

$$Y_\ell = Y_d^T, Y_u = Y_u^T.$$

Then, we deduce that $U_\ell = V_d^*$ from which

$$U_{PMNS} = V_d^T U_0 V_M.$$

As one can easily see, the contribution of $U_{CKM}$ may appear in $U_{PMNS}$ if we further assume that the Yukawa matrix of the up-type quark sector is related with that of Dirac-type neutrinos such as $Y_e = Y_u$ which can be realized in some larger unified gauge group such as $SO(10)$. Then, the lepton flavor mixing matrix can be written as

$$U_{PMNS} = V_d^T U_{\text{Dirac}} V_M.$$

In addition, requiring symmetric form of the down-type quark Yukawa matrix, we obtain

$$U_{PMNS} = U_{\text{Dirac}} V_M.$$

where the mixing matrix $V_M$ should have two almost maximal mixings so as to account for the solar and atmospheric neutrino oscillations. This expression for $U_{PMNS}$ corresponds to the parametrization given by Eq. (2)-a. On the other hand, in order to achieve the parametrization given by Eq. (2)-b, one should take $V_M$ to be identity matrix and the product of two matrix $V_d^T U_d$ in Eq. (10) should give bi-maximal mixing pattern. Since the left-handed rotation matrix $U_d$ for down-type quark can be almost diagonal to leading order, $V_d^T$ should have almost bi-maximal mixing form, which can be achieved in the so-called lopsided form of Yukawa matrix. The case given by Eq. (2)-c can also be achieved by taking $V_d^T \approx U^T_{33}$ and $V_M \approx U^T_{12}$ in Eq. (10). In such ways, $U_{PMNS}$ can be connected with $U_{CKM}$ in the framework of the quark-lepton unification.

Although the minimal quark-lepton unification can lead to an elegant relation between $U_{PMNS}$ and $U_{CKM}$ as shown above, it indicates undesirable mass relations between quarks and leptons at the GUT scale such as $m^{\text{diag}} = m_1^{\text{diag}}$. Thus, we need to modify the simple relations between quark and lepton Yukawa matrices so as to achieve desirable mass relations. From the well known empirical relation

$$|V_{us}| \approx \frac{m_d}{m_s} \approx 3 \frac{m_e}{m_\mu},$$

it has been shown that the $U(\lambda)$ in Eq. (2) should have the CKM-like form but with the replacement of $\lambda$ with $\lambda/3$ as
shown in Refs. [7,14], which can be obtained by introducing the Higgs sector transforming under the representation 45 of SU(5) or 126 of SO(10) [15].

Now, let us consider how the relative ratio of \( Br(l_i \rightarrow l_j \gamma) \) can be connected with the structure of the neutrino Dirac Yukawa matrix, which is constructed from the grand unification scenario above. It is well known that the RG running induces off-diagonal terms in the slepton mass matrix even for the case of universal slepton masses at GUT scale [16]:

\[
m_{ij}^2 = -\frac{1}{8\pi^2} (3m_0^2 + A_0) (Y_d^\dagger Y_u^\dagger)_{ij} \log \frac{M_G}{M_X}, \quad (13)
\]

where \( m_0, A_0 \) are universal soft scalar mass and soft trilinear \( A \) parameter, and \( Y_d^\dagger \) is defined in the basis where the charged lepton Yukawa matrix and the heavy Majorana mass matrix are real and diagonal. Here \( M_G \) and \( M_X \) denote the GUT scale and the characteristic scale of the right-handed neutrinos at which off-diagonal contributions are decoupled [16], respectively. Thus, one can expect that some specific form of \( Y_d^\dagger \) is crucial to estimate the sizes of LFV processes which are caused by nondiagonal slepton mass matrix. First of all, let us consider the parametrization (a). It follows from Eqs. (2)-a, (11) that

\[
Y_d^\dagger = U_{CKM}^T Y_d V_0^T V_R,
\]

where \( Y_d^\dagger \) is diagonal neutrino Dirac Yukawa matrix and

\[
Y_d^\dagger Y_u^\dagger \sim \left( \frac{m_d}{v_u} \right)^2 \left( \frac{\lambda^{2n_1 + 1} + \lambda^{2n_2 + 2} + \lambda^6}{\lambda^{2n_1 + 1} - \lambda^{2n_2 + 1} - \lambda^5} \right)^2 \lambda^3:1.
\]

Inserting this into Eq. (16), we can estimate how large \( Br(\mu \rightarrow e \gamma):Br(\tau \rightarrow e \gamma):Br(\tau \rightarrow \mu \gamma) \) could be by fixing the parameters \( m_S \) and \( M_X \). Instead of considering the values of \( Br(l_i \rightarrow l_j \gamma) \), we focus on the ratio of \( Br(l_i \rightarrow l_j \gamma) \). The ratio of \( Br(l_i \rightarrow l_j \gamma) \) only depends on \( Y_d^\dagger Y_u^\dagger \), and thus from Eqs. (13), (16), and (18), we can simply obtain the ratio:

\[
Br(\mu \rightarrow e \gamma):Br(\tau \rightarrow e \gamma):Br(\tau \rightarrow \mu \gamma) \approx (\lambda^{2n_1 - 1} + \lambda^{2n_2 - 1} + \lambda^3)^2:1.
\]

When \( Y_d^\dagger \) is the same as \( Y_u^\dagger \) due to the quark-lepton unification, we predict that the ratio given in Eq. (19) should be \( (\lambda^8; \lambda^2)^2:1 \). But, we note that this ratio may not necessarily indicate quark-lepton unification just considered because we can obtain the same ratio in the limit of large values of \( n_1, n_2 \). However, if the ratio of \( Br(l_i \rightarrow l_j \gamma) \) is measured to be inconsistent with the prediction of the ratio given above, it may indicate \( Y_d^\dagger \neq Y_u^\dagger \).

For the case of realistic quark-lepton unification satisfying Eq. (12), the terms \( (Y_d^\dagger Y_u^\dagger)_{i,j} \) for \( (i,j) = (2,1), (3,1), (3,2) \) are roughly given as

\[
(\lambda^{2n_1 - 1} + \lambda^{2n_2 - 1} + \lambda^3)^2:1.
\]

\[
(\lambda^{2n_1 + 1} + \lambda^{2n_2 + 2} + \lambda^6)^2:1.
\]

\[
(\lambda^{2n_1 + 1} - \lambda^{2n_2 + 1} - \lambda^5)^2:1.
\]

Then, the ratio of \( Br(l_i \rightarrow l_j \gamma) \) among the three different flavors is

\[
Br(\mu \rightarrow e \gamma):Br(\tau \rightarrow e \gamma):Br(\tau \rightarrow \mu \gamma) \approx (\lambda^{2n_1} - \lambda^{2n_2} + \lambda^4)^2:1,
\]

in order of magnitude estimation. For \( n_1 = 8, n_2 = 4 \), the ratio becomes \( \lambda^8; \lambda^4)^2:1 \). Therefore, we may confirm the validity or breaking of the quark-lepton unification through the measurements of the ratios of \( Br(l_i \rightarrow l_j \gamma) \). As can be seen from Eq. (18), the important elements in \( U_{CKM} \) which actually determine the hierarchy among \( Br(l_i \rightarrow l_j \gamma) \) are \( (U_{CKM})_{13} \) and \( (U_{CKM})_{23} \). Note that the lepton mixing matrix given by Eq. (11) leads to a new QLC relation,
\[
(U_{\text{PMNS}})_{e3} = \left[ -\lambda + (U_{\text{CKM}})_{31} \right] / \sqrt{2}.
\]  

(22)

Therefore, we are led to further confirm or discard quark-lepton unification through the measurement of \((U_{\text{CKM}})_{31}\) and \((U_{\text{PMNS}})_{e3}\). Note that similar to the QLC relation between \(\theta_{\text{sol}}\) and \(\theta_{c}\), we can get another QLC relation between the mixing angle \(\theta_{\text{atm}}\) and \((\theta_{23})_{\text{CKM}}\) [1],

\[
\theta_{\text{atm}} + (\theta_{23})_{\text{CKM}} \approx \pi / 4.
\]

(23)

Similar to the parametrization (2)-a, we can easily estimate the relative ratios of \(Br(l_i \to l_j \gamma)\) for the parameterizations in (2)-b and (2)-c. In these cases, the term \(Y^{\dagger} Y\)

\[
Y^{\dagger} Y = \begin{cases}
U_{\text{bimax}} (Y^D)^2 U_{\text{CKM}} U_{\text{bimax}}^{\dagger} & \text{(2)-b}, \\
U_{\text{23}} U_{\text{CKM}} (Y^D)^2 U_{\text{CKM}} U_{\text{23}}^{\dagger} & \text{(2)-c}.
\end{cases}
\]

(24)

Imposing the hierarchy of \(Y^D\) given by Eq. (17), the relative ratios of \(Br(l_i \to l_j \gamma)\) becomes

\[
Br(\mu \to e \gamma) : Br(\tau \to e \gamma) : Br(\tau \to \mu \gamma) \\
\approx \lambda^4 : \lambda^3 : 1(2)-b, \lambda^6 : \lambda^5 : 1(2)-c.
\]

(25)

From the predictions (19) and (25), one can see that experimental determination of the relative ratios of \(Br(l_i \to l_j \gamma)\) can differentiate the parameterizations of the quark-lepton unification if the empirical QLC relations indeed indicate the quark-lepton unification.

The current limits on radiative decays are \(Br(\mu \to e \gamma) < 1.2 \times 10^{-11}\), \(Br(\tau \to e \gamma) < 2.7 \times 10^{-6}\), and \(Br(\tau \to \mu \gamma) < 1.1 \times 10^{-6}\) [17]. Given \(\mu \to e \gamma\) would be observed soon, i.e., \(Br(\mu \to e \gamma) \approx 10^{-12} - 10^{-11}\), Eq. (25) predicted that \(Br(\tau \to e \gamma) \approx 10^{-10} - 10^{-11}\) and \(Br(\tau \to \mu \gamma) \approx 4 \times 10^{-10} - 9 \times 10^{-8}\), the largest of which is still 3 orders of magnitude below the current limit. However, Eq. (21) predicts that \(Br(\tau \to e \gamma) \approx 4 \times 10^{-10} - 4 \times 10^{-9}\) and \(Br(\tau \to \mu \gamma) \approx 2 \times 10^{-7} - 2 \times 10^{-6}\). Especially, \(Br(\tau \to \mu \gamma)\) is very close to the current limit. Therefore, if \(\tau \to \mu \gamma\) is observed in the near future together with \(\mu \to e \gamma\), the quark-lepton unification scenarios that lead to Eq. (25) can be ruled out. On the other hand, if \(\mu \to e \gamma\) is observed but not \(\tau \to \mu \gamma\), then the scenario that leads to Eq. (21) may be ruled out.

We note that the RG-induced off-diagonal terms in the slepton mass matrix is more precisely given by [18]

\[
m_{l_{ij}}^2 \approx - \frac{1}{8 \pi^2} (3m_0^2 + A_0^2) \left( y_{\nu k}^i M_M^C \right) y_{\nu k j}^{\dagger}.
\]

(26)

In this expression, we see that the prediction of \(Br(l_i \to l_j \gamma)\) depends on the hierarchy of the heavy Majorana neutrino mass eigenvalues \(M_R\). However, numerically, the dependence is only a mild logarithm such that the hierarchy pattern obtained in Eq. (19), (21), and (25) stays about the same.

In summary, interpreting the surprising empirical relation, \(\theta_{\text{sol}} + \theta_{c} \approx \pi / 4\), as a support of the quark-lepton unification, we find that the PMNS mixing matrix can be parameterized by a CKM-like matrix and maximal mixing matrices in various ways. Each parametrization may imply very different fundamental flavor structure. We have shown that the various parameterizations of \(U_{\text{PMNS}}\) with regard to quark-lepton unification would give very different and profound implication to the radiative leptonic decays, \(l_i \to l_j \gamma\), in the context of SSM. Therefore, by measuring the relative size of the radiative decay branching ratios, we will be able to pin down the \(U_{\text{PMNS}}\) parametrization, assuming the quark-lepton unification. There have been proposed experiments [19] to measure these radiative decays. The proposal in this paper can soon be tested for the quark-lepton unification.

S. K. and J. L. are supported by BK21 program of the Ministry of Education in Korea. C. S. K. is supported by the KRF Grant funded by the Korean Government No. R02-2003-000-10050-0. K. C. is supported by the NSC of Taiwan.


