Light pseudoscalar $\eta$ and $H \rightarrow \eta\eta$ decay in the simplest little Higgs model

Kingman Cheung*

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China
The National Center for Theoretical Sciences, Hsinchu, Taiwan

Jeonghyeon Song†

Department of Physics, Konkuk University, Seoul 143-701, Korea

(Received 12 March 2007; published 30 August 2007)

DOI: 10.1103/PhysRevD.76.035007 PACS numbers: 11.30.Pb, 12.60.Jv

I. INTRODUCTION

The Higgs boson is the last ingredient of the standard model (SM) to be probed at experiments. Precision measurements of the electroweak parameters with logarithmic dependence on the Higgs boson mass $m_H$ give an indirect but tantalizing limit on $m_H$ to be less than 186 GeV at the 95% confidence level (C.L.) [1]. A direct search by the four LEP collaborations, ALEPH, DELPHI, L3, and OPAL, resulted in no significant data. A lower bound on $m_H$ is established to be 114.4 GeV at the 95% C.L. [2], which is applicable to the SM and its extensions that preserve the nature of the SM Higgs boson, e.g., the minimal supersymmetric SM (MSSM) in most parameter space. In some other extensions, however, the nature of the light Higgs boson can be drastically modified. The limit from a direct search at LEP becomes weaker. Phenomenologically, evading the LEP data is possible when the Higgs boson coupling with the $Z$ boson is reduced and/or the Higgs boson decays into non-SM light particles. In the $CP$-conserving MSSM, for example, the lower bound on $m_H$ can be in the vicinity of 93 GeV at the 95% C.L. [3]. If we further allow $CP$ violation the result becomes more dramatic so that no absolute limits on $m_H$ can be set [4]. Since the Higgs mass bound has far-reaching implications on the Higgs search at the LHC, the examination of the LEP bound on $m_H$ in other new models is of great significance.

Recently, little Higgs models have drawn a lot of interest as they can solve the little hierarchy problem between the electroweak scale and the 10 TeV cutoff scale $\Lambda$ [5]. A relatively light Higgs boson mass compared to $\Lambda \sim 10$ TeV can be explained if the Higgs boson is a pseudo-Nambu-Goldstone boson (pNGB) of an enlarged global symmetry. The quadratically divergent Higgs boson mass at the one-loop level, through the gauge, Yukawa, and self-couplings of the Higgs boson, is prohibited by the collective symmetry breaking mechanism. According to the global symmetry breaking pattern, there are various models with the little Higgs mechanism [6]. Detailed studies have been also made, such as their implications on electroweak precision data (EWPD) [7] and phenomenologies at high energy colliders [8].

Considering the possibility of evading the LEP data on the Higgs mass, the simplest little Higgs model [9] is attractive as it accommodates a light pseudoscalar boson $\eta$, which the Higgs boson can dominantly decay into. The model is based on $[SU(3) \times U(1)_Y]_3$ global symmetry with its diagonal subgroup $SU(3) \times U(1)_Y$ gauged. The vacuum expectation values (VEV) of two SU(3)-triplet scalar fields, $\langle \Phi_{1,2} \rangle = (0, 0, f_{1,2})^T$, spontaneously break both the global symmetry and the gauge symmetry. Here $f_{1,2}$ are at the TeV scale. Uneaten pNGB’s consist of an SU(2)$_L$ doublet $h$ and a pseudoscalar $\eta$. Loops of gauge bosons and fermions generate the Coleman-Weinberg (CW) potential $V_{CW}$ which contains the terms such as $h^2 h$ and $(h^2 \eta)^2$. The Higgs boson mass and its self-coupling are radiatively generated. However the CW potential with nontrivial operators of $[\Phi_{1,2}]^n$ does not have the dependence of $\eta$ which is only a phase of sigma fields $\Phi_{1,2}$ [10,11]. This $\eta$ becomes massless, and thus problematic for $\eta$ production in rare $K$ and $B$ decays, $B$-$\bar{B}$ mixing, and $Y \rightarrow \gamma \gamma$, as well as for the cosmological axion limit.

*cheung@phys.nthu.edu.tw
†jhsong@konkuk.ac.kr
One of the simplest remedies was suggested by introducing the $-\mu^2 (\Phi_i^2 + \Phi_2^2 + \text{H.c.})$ term into the scalar potential by hand. Even though this breaks the global SU(3) symmetry and thus damages the little Higgs mechanism, its contribution to the Higgs boson mass is numerically insignificant. Now $\eta$ acquires nonzero mass with a scale of $\mu$. By requiring a negative Higgs mass-squared parameter for electroweak symmetry breaking (EWSB), we show that the $\mu$ (and thus $m_\eta$) is of the order of 10 GeV. Thus, we have light pseudoscalar particles. In addition, the $\mu$ term also generates the $\lambda' h^2 / m_\eta^2$ term in the CW potential [12]. As the $h$ field develops the VEV $v$, the $H-\eta-\eta$ coupling emerges with the strength proportional to $v^{2/3}$ at the TeV scale. The Higgs boson can then decay into two $\eta$ bosons. Furthermore, this light $\eta$ opens a new decay channel of $H \rightarrow Z \eta$. Indeed, these two new decay channels can be dominant, as shall be shown later.

Another issue for which we make a thorough investigation into is the condition for successful EWSB. The model with the $\mu$ term is determined by four parameters: $f_1$, $\tan \beta (\equiv f_2/f_1)$, $x_t$, and $\mu$. Here $x_t$ is the ratio of two Yukawa couplings in the third generation quark sector. The radiatively generated Higgs VEV $v$ is also determined by these four parameters: The SM EWSB condition $v = 246$ GeV fixes one parameter, e.g., $\tan \beta$. For $x_t \in [1, 15]$, $\mu \sim O(10)$ GeV, and $f = 2-4$ TeV, the $v = 246$ GeV condition limits $\tan \beta$ around 10. This large $\tan \beta$ reduces the effective $g_{ZHH}$ coupling in this model. With smaller $g_{ZHH}$ and $B(H \rightarrow b\bar{b})$ than in the SM, the LEP Higgs boson mass bound based on the limit $(g_{ZHH}/g_{ZSM})^2 B(H \rightarrow b\bar{b})$ can be reduced [2]. Yet there was a general search by the four LEP collaborations [13] in the channel $e^+e^- \rightarrow ZH \rightarrow Z(AA) \rightarrow Z + 4b$. The $\eta$ boson in the present model is similar to the $A$ boson. We shall apply their limit to the present model, which shall be shown entirely unconstrained.

The organization of the paper is as follows. In the next section, we highlight the essence of the original SU(3) simplest little Higgs model, in particular, the Higgs sector. We will show that the original model can accommodate proper EWSB as well as the Higgs mass $\sim 100$ GeV. After explicit demonstration of no $\eta$ dependence on the scalar potential, we will discuss the problem of the massless pseudoscalar $\eta$. In Sec. III, we introduce the $\mu$ term and discuss the EWSB implication as well as the mass spectra of the Higgs boson and $\eta$. In Sec. IV, we calculate the branching ratio $H \rightarrow \eta \eta$ and discuss its impact on the Higgs boson mass bound. We discuss further possibilities to investigate this scenario and then conclude in Sec. V.

II. SU(3) SIMPLEST GROUP MODEL WITHOUT THE $\mu$ TERM

The SU(3) simplest little Higgs model is based on $[SU(3) \times U(1)_X]^2$ global symmetry with its diagonal sub-

\begin{align}
\Phi_1 &= e^{i\mu \theta} \Phi_1^{(0)}, \\
\Phi_2 &= e^{-i\mu \theta} \Phi_2^{(0)}, 
\end{align}

(1)

where $t_\mu = \tan \beta$ and

\begin{align}
\Theta &= \frac{1}{f} \left[ \begin{array}{ccc}
0 & 0 & h \\
0 & 0 & h \\
h & h & 0
\end{array} \right] + \frac{\eta}{\sqrt{2}} \left[ \begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array} \right] \\
&= \frac{1}{f} \mathbb{1} + \frac{\eta}{\sqrt{2} f} \mathbb{1},
\end{align}

(2)

The kinetic term for $\Phi_{1,2}$ is

\begin{align}
\mathcal{L}_F &= \sum_{i=1,2} \left( \partial_{\mu}^2 + i g A_\mu^a T^a - i g_s \frac{3}{2} B_\mu \right)\Phi_i \right|^2, 
\end{align}

(3)

where $T^a$ are the SU(3) generators while $A_\mu^a$ and $B_\mu$ are the SU(3) and U(1) gauge fields, respectively. Two gauge couplings of $g$ and $g_s$ are fixed by the SM gauge couplings such that SU(3) gauge coupling $g$ is just the SM SU(2)_L gauge coupling and $g_s = g'/\sqrt{1 - t_\mu^2/3}$.

Each of the SM fermion doublets is promoted to an SU(3) triplet. Focusing on the third generation quarks, we introduce a 3 representation of SU(3), $\chi_L = (t_L, b_L, iU_L)^T$, as well as two weak-singlet quarks, $U_{R1}$ and $U_{R2}$. The Yukawa interaction is

\begin{align}
\mathcal{L} &= i\lambda_1 U_{R1}^\dagger \Phi_1^0 \chi_L + i\lambda_2 U_{R2}^\dagger \Phi_2^0 \chi_L + \text{H.c.},
\end{align}

(4)

where the complex number $i$'s guarantee positive masses for fermions. According to the SU(3) representation of the first two generation quarks and all generation leptons, there are two versions for fermion embedding. This variation in model building is possible since light quarks and leptons make very little contributions to the radiative Higgs mass. The first fermion embedding is called “universal” embedding [11], where all three generations have identical quantum numbers. The other is the “anomaly-free” embedding where anomaly-cancellation is required for easier UV completion [14]: The third generation quarks and all leptons are put into 3 representations of SU(3), while the first two generation quarks into 3. Yukawa couplings for light quarks and leptons in both embedding cases are referred to in Ref. [11].

When $\Phi_1$ and $\Phi_2$ develop the aligned VEV of

\begin{align}
\langle \Phi_1 \rangle &= \Phi_1^{(0)} = (0, 0, f \cos \beta)^T, \\
\langle \Phi_2 \rangle &= \Phi_2^{(0)} = (0, 0, f \sin \beta)^T, 
\end{align}

(5)

two kinds of symmetry breaking occur. First, the global symmetry is spontaneously broken into its subgroup of $[SU(2) \times U(1)]_X^2$, giving rise to ten Nambu-Goldstone bosons. Second, the gauge symmetry SU(3) $\times U(1)_X$ is broken into the SM SU(2)_L $\times U(1)_Y$, as five Nambu-
Goldstone bosons are eaten. Five new gauge bosons and one heavy toplike quark \( T \) appear with heavy mass of order \( f \sim \text{TeV} \). The heavy gauge bosons include a \( Z' \) gauge boson (a linear combination of \( A_\mu^\nu \) and \( B_\mu^\nu \)) and a complex SU(2) doublet \((Y^0, X^-)\) with masses of 

\[
M_{Z'} = \frac{2\sqrt{3 - r_W^2}}{g_f}, \quad \lambda x_\perp = M_Y = \frac{g_f}{\sqrt{2}}, \quad \lambda x_z = \frac{m_f}{v}.
\]

The new heavy \( T \) quark mass is 

\[
M_T = \sqrt{2} \frac{r_\perp^2 + x_\perp^2}{(1 + r_\perp^2)x_\perp} m_f, v,
\]

where \( x_\perp = \lambda_1/\lambda_2 \).

Brief comments on the EWPD constraint on \( f \) are in order here. According to Ref. [9], the anomaly-free model is less constrained. The strongest bound comes from atomic parity violation with \( f > 1.7 \text{ TeV} \) at the 95\% C.L. A more recent analysis in Ref. [15] gives a stronger bound of \( f > 4.5 \text{ TeV} \) at 99\% C.L. The main contribution comes from an oblique parameter \( \hat{S} \) due to the \( Z' \) gauge boson. They applied the approximation for \( \hat{Z'} \) that is eliminated by solving its equation of motion. Considering both analyses, we take \( f = 2-4 \text{ TeV} \) as reasonable choices.

The gauge and Yukawa interactions of the Higgs boson explicitly break the SU(3) global symmetry, generating the Higgs mass at loop level. In the CW potential up to dimension four operators, only the \( |\Phi_1\Phi_2|^2 \) term leads to a nontrivial result for the pNGB’s. A remarkable observation is that this \( |\Phi_1\Phi_2|^2 \) term does not have any dependence on \( \eta \) [16]. This can be easily seen by the expansion of, e.g., \( \Phi_1 \) as 

\[
\Phi_1 = \exp(i \frac{t_\beta \eta}{\sqrt{2} f}) \exp(i \frac{t_\beta \eta}{f}) \Phi_1^{(0)},
\]

which we have used the Baker-Hausdorff formula with \([\hat{H}, \hat{l}_i] = 0\). This compact form is very useful when calculating the \( \Phi_1\Phi_2 \):

\[
\Phi_1 \Phi_2 = f^2 \hat{s}_\beta \hat{c}_\beta \hat{e}^{-i(t_\beta + 1/\lambda_\perp)(\eta/\sqrt{2} f)} \cos(h_0) \frac{1}{\hat{f} \hat{c}_\beta \hat{s}_\beta}.
\]

The \( |\Phi_1\Phi_2|^2 \) term or the CW potential has no dependence on \( \eta \). The pseudoscalar \( \eta \) remains massless in the original model.

On the contrary, the Higgs boson mass is radiatively generated with one-loop logarithmic divergence and two-loop quadratic divergence. The troublesome one-loop quadratic divergence is eliminated by the little Higgs mechanism. The CW potential is

\[
V_{\text{CW}} = -m_0^2 h^4 h + \lambda_0(h^4 h^2),
\]

where

\[
m_0^2 = \frac{3}{8\pi^2} \left[ \frac{\lambda_0^2 M_T^2}{\lambda x_\perp^2} - \frac{g^2}{4} M_X^2 - \frac{\lambda_0 x_\perp^2}{\lambda x_\perp^2} \right],
\]

\[
\lambda_0 = \frac{1}{3s_\beta c_\beta} \frac{m_0^2}{f^2} + \frac{3}{16\pi^2} \left[ \frac{\lambda_0^2 m_T^2}{m_1^2} - \frac{g^4}{8} \frac{m_X^2}{m_W^2} - \frac{g^4}{16} (1 + r_\perp^2) \frac{M_Z^2}{m_Z^2} \right].
\]

Here \( \lambda_0 = 1/\sqrt{2} m_f/v \) and \( \Lambda \approx 4 f \). The negative mass-squared term for the Higgs doublet in Eq. (10) generates the VEV for the Higgs boson as \( \langle h \rangle = v_0/\sqrt{2} \), which then triggers the EWSB and generates the Higgs boson mass \( m_{H0} \), given by

\[
v^2 = \frac{m_0^2}{\lambda_0}, \quad m_{H0}^2 = 2m_0^2.
\]

This CW potential alone has been considered insufficient to explain the EWSB, due to excessively large soft mass-squared \( m_0^2 \). If \( f = 2 \text{ TeV} \) and \( x_\perp = t_\beta = 2 \), for example, \( m_0 \approx 710 \text{ GeV} \) and thus \( m_{H0} \approx 1 \text{ TeV} \). In addition, the quartic coupling \( \lambda_0 \) is also small since it is generated by logarithmically divergent diagrams, not by quadratically divergent ones. In the ordinary parameter space of \( t_\beta \) and \( x_\perp \) of the order of 1, the \( v \approx 246 \text{ GeV} \) condition cannot be satisfied. However, this flaw in the original model without the \( \mu \) term is not as serious as usually considered in literature. If we extend the parameter space allowing \( x_\perp \) and \( t_\beta \) up to \( \sim 10 \), the \( v \approx 246 \text{ GeV} \) condition can be easily met. Reducing \( m_0^2 \) in Eq. (11) is possible if the heavy \( T \) mass decreases. As discussed in Ref. [17], the heavy \( T \) mass is minimized when \( t_\beta = x_\perp \) and \( t_\beta \) increases. Larger \( t_\beta \) helps to satisfy \( v \approx 246 \text{ GeV} \) in addition, large \( t_\beta \) suppresses the new contributions to the EWPD [17].

When we require that the radiatively generated Higgs VEV be equal to the SM Higgs VEV, ‘what is the SM Higgs VEV in this model’ is an important question. A definite way is to require that the SM Higgs VEV \( v \) should explain the observed SM \( W \) gauge boson mass. In this model, the \( W \) gauge boson mass is modified into

\[
m_W = \frac{g_v}{2} \left[ 1 - \frac{v^2}{12f^2} \left( \frac{t_\beta^2 - t_\beta^2 + 1}{t_\beta^2} \right) + \frac{1}{180f^4} \left( \frac{v^4}{f^4} \right) \right].
\]

The Higgs boson VEV explaining \( m_W \), which we denote by \( v_W \), is
\[
v_W = v_0 \left[ 1 + \frac{v_0^2}{12 f^2} t_\beta^4 - \frac{t_\beta^2 + 1}{180 f^4} - \frac{v_0^4}{4 f^4} \right]
\]

\[
\times \left[ t_\beta^4 - t_\beta^2 + \frac{v_0^2}{f^2} + \mathcal{O}(1) \right]
\]

\[
= v_0 \left[ 1 - \delta_v^{(2)} v_0^2 + \delta_v^{(4)} + \cdots \right].
\]  

(15)

where \( v_0 = 2m_w / g = 246.26 \) GeV. With the observed \( m_w \), the \( v_W \) in this model depends on \( t_\beta \) and \( f \). Therefore the \( v(f, t_\beta, x_\lambda) = v_W(f, t_\beta, x_\lambda) \) condition determines one of the model parameters.

We have brief comments on the concern about the perturbative expansion with large \( t_\beta \). The correction to the Higgs boson VEV is proportional to \( r_\beta^2 v_0^2 / f^2 \) in the large \( t_\beta \) limit. Even though the large value of \( t_\beta \sim 10 \) suppresses extra contributions to EWPD, it questions the validity of perturbative expansion particularly when \( f \) is not very high compared to \( v_0 \approx 246 \) GeV. We examine the parameter space of \((t_\beta, f)\) for perturbation validity, based on the Higgs VEV correction in Eq. (15). We require that the next-to-leading order (NLO) correction should be suppressed by a factor of 1/10 relative to the LO correction, i.e., \( \delta_v^{(4)} / \delta_v^{(2)} < 0.1 \) in Eq. (15):

\[
\begin{array}{c|cc}
   & f = 2 \text{ TeV} & f = 3 \text{ TeV} & f = 4 \text{ TeV} \\
\hline
\delta_v^{(4)} / \delta_v^{(2)} < 0.1 & t_\beta < 9.9 & t_\beta < 14.9 & t_\beta < 19.9 \\
\end{array}
\]

(16)

For \( f = 3, 4 \text{ TeV} \), the whole parameter space \( t_\beta \in [1, 15] \) under consideration supports the perturbative expansion. If \( f = 2 \text{ TeV} \), however, \( t_\beta > 10 \) does not guarantee high accuracy of results based on a truncation of the expansion.

In Fig. 1, we present the contours of \( m_0^2 = 0 \) and \( \nu = v_W \) (thin lines for \( \mu = 0 \)). In the upper right corner, \( m_0^2 \) becomes negative such that the EWSB is not possible. This is because too large of \( t_\beta \) and thus too small of \( M_T \) makes \( m_0^2 \) negative. The \( \lambda_0 < 0 \) region is contained in the excluded region by \( m_0^2 < 0 \). We do have considerably large parameter space, particularly around \( t_\beta \approx 10 \), to explain appropriate EWSB.

In spite of this successful EWSB, the SU(3) simplest little Higgs model without the \( \mu \) term has some problems. The most serious one is the presence of massless pseudoscalar \( \eta \). Any term in the CW potential, proportional to \( |\Phi_1^+ \Phi_2^-|^2 \) or \( |\Phi_1^0 \Phi_2^0|^2 \), cannot accommodate the \( \eta \) dependence. Even though lower bounds on CP-odd scalar masses from the \( b \)-physics signal [18] and cosmology [19] are not very stringent, no pseudoscalar particle is massless: The \( \eta \) mass should be above \( O(100) \) MeV from the \( b \)-physics signal such as rare \( K, B \) and radiative \( Y \) decays with the \( \eta \) in the final state, \( B_s \rightarrow \mu^+ \mu^- \) and \( B-B \) mixing; the cosmological bound is also weak but finite, as low as 10 MeV. Another unsatisfactory aspect, which matters for the \( f = 2 \text{ TeV} \) case, is that the allowed parameter space by EWSB has a non-negligible NLO correction to \( v \) (or \( m_v \)) compared to the LO one. For reliable results, we need to include higher order corrections. In the next section, we will show that the so-called \( \mu \) term, introduced by hand, gives a mass to the pseudoscalar and moves the theory to a region consistent with the perturbative expansion.

**III. SU(3) MODEL WITH THE \( \mu \) TERM**

One of the simplest solutions to the massless \( \eta \) problem as well as generically large \( m_0^2 \) problem is to introduce a new term of \( -\mu^2 (\Phi_1^+ \Phi_2 + \text{H.c.}) \) into the scalar potential by hand [9,12,20]. Unfortunately, this explicitly breaks the global SU(3) symmetry. The little Higgs mechanism is lost as the Higgs loop generates the one-loop quadratically divergent corrections to the Higgs mass. Since this correction is numerically insignificant, we adopt this extension.

Using Eq. (9), we can express the \( \mu \) term in a closed form of, without any expansion,
The scalar potential becomes
\[ V = -m^2 h^\dagger h + \lambda (h^\dagger h)^2 - \frac{1}{2} m^2 \eta^2 + \lambda' h^\dagger h \eta^2 + \cdots, \]
(18)

where
\[ m^2 = m_0^2 - \frac{\mu^2}{s_\beta c_\beta}, \quad \lambda = \lambda_0 - \frac{\mu^2}{12 s^3 c^3 \beta f^2}, \]
\[ \lambda' = -\frac{\mu^2}{4 f^2 s^3 c^3 \beta}. \]
(19)

Note that the results are the same as those from the expansion of \( \Phi_{1,2} \) in Ref. [12]. The Higgs VEV \( v \), the Higgs mass \( m_H \), and \( \eta \) mass \( m_\eta \) are then

The CW potential as well as the masses of new heavy particles depend on the following four parameters:
\[ f, \ x, \ t_\beta, \ \mu. \]
(21)

As before, the \( v = v_W \) condition removes one parameter. In Fig. 1, we present the contours of \( v = v_W \) for \( \mu = 30 \text{ GeV} \) and \( f = 2 \text{ TeV} \). Increasing \( \mu \) reduces the allowed value of \( t_\beta \) by \( \sim 10\% \). This helps the validity of perturbative expansion. With \( \mu = 30 \text{ GeV} \), the parameter space of \( x_\lambda > 5 \) supports the perturbative expansion.

Unfortunately there is no prior information about \( \mu \). Nevertheless the upper bound on \( \mu \) can be imposed since \( \mu \) contributes negatively to the Higgs mass-squared parameter \( m^2 \). If \( m^2 \) becomes negative due to too large of \( \mu \), the EWSB cannot occur. In Fig. 2, we present the allowed parameter space of \( (t_\beta, \mu) \) for \( x_\lambda = 3, 6, 10 \) and \( f = 2, 4 \text{ TeV} \) by requiring \( m^2 > 0 \). The upper right corner where

![FIG. 2 (color online). Allowed parameter space of \((t_\beta, \mu)\) for \(x_\lambda = 3, 6, 10\) by requiring a positive Higgs mass-squared parameter \(m^2\). We consider \(f = 2 \text{ TeV}\) and \(f = 4 \text{ TeV}\). The upper right corner is excluded since \(m^2 < 0\).](image)

The masses of the Higgs boson (solid line) and \( \eta \) (dashed line) as a function of \( x_\lambda \) for \( f = 2 \text{ TeV} \) and \( f = 4 \text{ TeV} \). The value of \( t_\beta \) is determined by the \( v = v_W \) condition.

![FIG. 3 (color online).](image)
present, with $f = 2, 4$ TeV, the parameter space of $(\mu, x_A)$ where $2m_\eta < m_H$ (to the left-hand side of the contours). If $\mu$ is too large, the $H \to \eta \eta$ decay is kinematically prohibited unless $x_A$ is smaller than a certain value.

IV. $H \to \eta \eta$ Decay and LEP Implications

A. Branching ratios

In this model, major decay modes of the Higgs boson are SM-like ones with the partial decay rates as

$$\Gamma(H \to f \bar{f}) = \frac{N_c g^2 m_f^2}{32\pi m_W^2} \left(1 - x_f\right)^{3/2} m_H,$$

for $f = b, c, \tau$.

$$\Gamma(H \to W^+ W^-) = \frac{g^2 m_H^3}{64\pi m_W^2} \left(\frac{g_{WWH}^{SM}}{g_{WWH}}\right)^2 \sqrt{1 - x_W}$$

$$\times \left(1 - x_W + \frac{3}{4} x_W^2\right).$$

$$\Gamma(H \to ZZ) = \frac{g^2 m_H^3}{128\pi m_Z^2} \left(\frac{g_{ZZH}^{SM}}{g_{ZZH}}\right)^2 \sqrt{1 - x_Z}$$

$$\times \left(1 - x_Z + \frac{3}{4} x_Z^2\right),$$

where $x_f = 4m_f^2/m_H^2, N_c = 3(1)$ for $f$ being a quark (lepton). The detailed expressions for $y_i$ are referred to in Ref. [11]. In this model, $g_{WWH}$ and $g_{ZZH}$ deviate from the SM value by

$$\frac{g_{WWH}^{SM}}{g_{WWH}} = 1 - \frac{v_0^2}{4f^2} \left(\frac{t_\beta}{f} - 1 + \frac{1}{t_\beta}\right) + \frac{v_0^4}{36 f^4} \left(\frac{t_\beta}{f} - 1\right)^2$$

$$+ O\left(\frac{v_0^6}{f^6}\right).$$

(23)

$$\frac{g_{ZZH}^{SM}}{g_{ZZH}} = 1 - \frac{v_0^2}{4f^2} \left(\frac{t_\beta}{f} - 1 + \frac{1}{t_\beta}\right) + (1 - \frac{t_\beta}{f})^2$$

$$+ \frac{v_0^4}{36 f^4} \left(\frac{t_\beta}{f} - 1\right)^2 + O\left(\frac{v_0^6}{f^6}\right).$$

(24)

Note that the next-to-leading order corrections to $g_{WWH}$ and $g_{ZZH}$ are proportional to $t_\beta^2 (v_0/f)^4$. Even for the $f = 2$ TeV case with $t_\beta \sim 10$, these NLO corrections are negligible.

New decay channels are

$$\Gamma(H \to \eta \eta) = \frac{\lambda^2}{8\pi} \frac{v^2}{m_H^2} \sqrt{1 - x_\eta} = \frac{m_\eta^4}{8\pi v^2 m_H^2} \sqrt{1 - x_\eta},$$

$$\Gamma(H \to Z\eta) = \frac{m_H^3}{32\pi f^2} \left(\frac{t_\beta}{f} - \frac{1}{t_\beta}\right) \frac{\lambda^{3/2}}{f} \left(1 - m_\eta^2/m_H^2\right).$$

(25)

where $\lambda(1, x, y) = (1 - x - y)^2 - 4xy$. The last decay

FIG. 4 (color online). The contours of $m_H = 2m_\eta$ in the parameter space $(\mu, x_A)$ for $f = 2, 4$ TeV. To the left-hand side of the contour, $2m_\eta < m_H.$
A search strategy of the Higgs boson depends sensitively on its branching ratios (BR): In the SM, the major decay mode for $m_H < 2m_W$ is into $b\bar{b}$ while that for $m_H \approx 2m_W$ is into $W^+W^-$. In this model, there are two new decay modes for the Higgs boson, $H \rightarrow \eta\eta$ and $H \rightarrow Z\eta$. In Fig. 5, we present the contours of $B(H \rightarrow \eta\eta) = 0.3, 0.5, 0.7$, in the parameter space $(x_\lambda, \mu)$ for $f = 2, 4$ TeV. Quite sizable portions of the parameter space can accommodate dominant decay of $H \rightarrow \eta\eta$. For $f = 2$ TeV, $B(H \rightarrow \eta\eta) > 0.5$ requires $x_\lambda \in [6, 14]$ and $\mu \in [16, 30]$ GeV. Note that this $x_\lambda$ range is indeed preferred by the validity of perturbative expansion (see Fig. 1).

The optimal $\mu$ for large $B(H \rightarrow \eta\eta)$ is around 20 GeV. A smaller $\mu$ increases the 2-body phase-space factor since $\mu$ is proportional to the produced $\eta$ mass, while it reduces the $H-\eta\eta$ coupling. The size of parameter space for $f = 4$ TeV is relatively smaller with $x_\lambda \in [5.6, 6.6]$ and $\mu \in [10, 22]$ GeV. In this case, the optimal $\mu$ is also around 20 GeV.

Figure 6 shows the same contours for $B(H \rightarrow Z\eta)$, which depend quite sensitively on $f$. For $f = 2$ TeV, sizable parameter space of $x_\lambda \simeq 6$ and $\mu \lesssim 10$ GeV can allow dominant decay of $H \rightarrow Z\eta$. When $f = 4$ TeV, only a small region around $x_\lambda \simeq 6$ and $\mu \lesssim 15$ GeV can accommodate dominant $H \rightarrow Z\eta$. This is mainly due to the $\eta$ mass. As can be seen in Fig. 3, $\eta$ for $f = 4$ TeV is relatively heavier than that for $f = 2$ TeV.

Figures 7 and 8 show the branching ratios as a function of $m_H$. In Fig. 7, we first fix $\mu = 20$ GeV and $f = 2, 4$ TeV for a generic illustration. We change $x_\lambda$ to generate various $m_H$. A different distribution of BRs for $f = 2$ TeV from that for $f = 4$ TeV is mainly due to the Higgs mass range. In the $f = 2$ TeV case, the $Z\eta$ mode is solely dominant for $m_H$ from the $Z\eta$ threshold to $2m_W$. Even for $m_H > 2m_W$, $B(H \rightarrow Z\eta)$ is almost the same as $B(H \rightarrow W^+W^-)$. In the $f = 4$ TeV case, the $H \rightarrow \eta\eta$ is dominant for $140 \leq m_H \leq 160$ GeV, but the $H \rightarrow b\bar{b}$ becomes dominant if $m_H$ is below about 140 GeV. For $m_H$ above WW threshold, $H \rightarrow WW$ is the leading decay mode, but not as dominant as in the SM because of the presence of the $Z\eta$ mode. The second important decay mode is into $Z\eta$, which is very different from a SM-like Higgs boson [12]. These BR patterns can be dramatically changed if we take more fine-tuned parameters. Figure 8 shows the same plots for $f = 3$ TeV with $\mu = 14$ GeV, and $f = 4$ TeV with $\mu = 15$ GeV. These are more illustrative for the LEP2 results. Even for the Higgs mass accessible at the LEP2 ($m_H < 115$ GeV), $H \rightarrow \eta\eta$ is dominant or compatible with $H \rightarrow b\bar{b}$. A detailed study on its impact on the LEP search is to be presented below.

Brief comments on the decay of $\eta$ are in order here. If $m_\eta < 2m_W$, the decay pattern is very similar to that of the...
FIG. 7 (color online). Branching ratios of the Higgs boson in the simplest little Higgs model with the $\mu$ term as a function of $m_H$ for $f = 2$ TeV and $f = 4$ TeV. We fix $\mu = 20$ GeV but vary $\alpha$. SM Higgs boson with the main decay mode into a SM fermion pair via the coupling $\epsilon(m/f)i\bar{f}\gamma_5f$, where $c \sim O(1)$ and $m_f$ is the mass of the fermion. Although this coupling is suppressed by $1/f$, the decay is still prompt in collider experiments for $f \sim O$(TeV) unless $\eta$ is extremely light. Therefore, the light $\eta$ boson mainly decays into a $b\bar{b}$ pair [12] if kinematically allowed. This characteristic feature of $\eta$ decay is useful to probe $\eta$ at high energy colliders.

B. LEP bound on $m_H$

Because of the presence of the dominant decay of $H \to \eta\eta$, one expects that the LEP bound on the Higgs mass can be loosened to some extent. The four LEP collaborations [2] searched for the Higgs boson via

$$e^+ e^- \to ZH \to (l^+ l^-, q\bar{q}, \nu\bar{\nu}) + b\bar{b}.$$  

(26)

Here the main decay mode of the SM Higgs boson into $b\bar{b}$ dominates the width of the Higgs boson, with a branching fraction about 90% for most of the mass range and down to about 74% at $m_H = 115$ GeV. There is also a search using a minor mode of $H \to \tau^+\tau^-$. Nevertheless, the combined limit is almost the same as that using just the $b\bar{b}$ mode. The mass bound on the SM Higgs boson is 114.4 GeV [2]. For model-independent limits the LEP collaborations presented the upper bound on $[g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \to b\bar{b})$ at the 95% C.L., as shown by the rugged curve in Fig. 9.

In the simplest little Higgs scenario with the $\mu$ term, one anticipates that the LEP bound on $m_H$ would be reduced, because of (i) sizable decay rate of $H \to \eta\eta$ such that $B(H \to b\bar{b})$ is substantially reduced as shown in Fig. 7, and (ii) the reduced coupling $g_{ZZH}$ as shown in Eq. (24), especially when $t_\beta$ is large.

In Fig. 9, we present the prediction of $[g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \to b\bar{b})$ for $f = 2, 3, 4$ TeV, and compare to the 95% C.L. upper limit obtained by the LEP collaborations. We found the best value of $\mu = 14(15)$ GeV for $f = 3(4)$ TeV such that the prediction of $[g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \to b\bar{b})$ for $f = 2, 3, 4$ TeV is the smallest. The $f = 2$ TeV case is safe because the minimum value of $m_H$ predicted is already above 114 GeV. For $f = 3, 4$ TeV, however, the Higgs boson mass bound is restricted by the data as follows:
$m_H > 109 \text{ GeV}$ for $f = 3 \text{ TeV}$,

$m_H > 111 \text{ GeV}$ for $f = 4 \text{ TeV}$.  \hfill (27)

The lowering of the Higgs mass limits is a combined effect of reduction of Higgs production and the reduction of $B(h \to b\bar{b})$. For example, at $f = 4 \text{ TeV}$ with $\mu = 15 \text{ GeV}$ and $m_H = 114 \text{ GeV}$, the size of the coupling $g_{ZZH}$ is only 80% of the SM value while the branching ratio $B(h \to b\bar{b}) = 0.45$.

C. LEP limit on $C_{Z(AA\to 4b)}^2$

The four LEP collaborations [13] have searched for the process $e^+e^- \to ZH \to Z(AA) \to Z + 4b$ for $m_H > 2m_A$. Here $A$ is a $CP$-odd scalar particle, for which $\eta$ is a good candidate. The cross section is parametrized by

$$\sigma_{(AA)Z\to 4b+\text{jets}} = \sigma_{SM}^{ZH} \times B(Z \to \text{hadrons}) \times C_{Z(AA\to 4b)}^2. \hfill (28)$$

where

$$C_{Z(AA\to 4b)}^2 = \left( \frac{g_{ZH}}{g_{SM}} \right)^2 \times B(H \to AA) \times B(A \to b\bar{b})^2. \hfill (29)$$

As no convincing evidence for a signal was found, the upper bound on $C_{Z(AA\to 4b)}^2$ was presented [13].

We show the values of $C_{Z(AA\to 4b)}^2$ predicted in our model for $f = 3, 4 \text{ TeV}$ in Fig. 10. The $f = 2 \text{ TeV}$ case is not constrained because the Higgs boson mass is already above the lower bound of 114.4 GeV. Here we fix $\mu = 14, 15 \text{ GeV}$ for $f = 3, 4 \text{ TeV}$, respectively. We also show the upper bounds on $C_{Z(AA\to 4b)}^2$ for various combinations of $m_H$ and $m_A$ presented in Table 15 of Ref. [13]. For both cases, the $C_{Z(AA\to 4b)}^2$ values in this model are smaller than the experimental upper bound. These LEP searches do not constrain the model for these choices of parameters. For $f = 3, 4 \text{ TeV}$, smaller $m_H$ can evade the LEP search since $g_{ZZH}$ decreases substantially for large $t_\beta$ and $H \to b\bar{b}$ is still dominant for $m_H \leq 100 \text{ GeV}$ as discussed before. The kinks in the curves are due to the onset of the $Z\eta$ mode when $m_H > m_Z + m_\eta$.

V. CONCLUSIONS

Little Higgs models provide a very interesting perspective on answering the little hierarchy problem. As attributing the lightness of Higgs boson to its being a pseudo-Nambu-Goldstone boson, the collective symmetry breaking mechanism removes the quadratically divergent radiative-corrections to the Higgs mass at one-loop level. As a perfect type of “simple group” models, the SU(3) simplest little Higgs model has drawn a lot of interest due to its lowest fine-tuning associated to electroweak symmetry breaking [21]. In the original framework, this simplest model cannot avoid the presence of massless pseudoscalar particle $\eta$. A cosmological lower bound on the axion mass requires to extend the model. One of the simplest choices is to add the so-called $\mu$ term in the scalar potential by hand. Then $\eta$ acquires a mass of order $\mu$, and the $H-\eta-\eta$ coupling is also generated of the order of $\mu^2/f^2$. In order to accommodate the EWSB, this $\mu$ has a natural scale of a few ten GeVs, which leads to relatively light $\eta$. It is possible to allow a substantial branching ratio for the $H \to \eta\eta$ decay. In addition, the $H-Z-\eta$ coupling, which is present in the original model without the $\mu$ term, leads to $H \to Z\eta$ decay.

We found that the $H \to \eta\eta$ decay can be dominant for $m_H$ below the WW threshold for $\mu \approx 15-20 \text{ GeV}$, while $H \to Z\eta$ can be dominant if $140 \text{ GeV} \leq m_H \leq 2m_W$. For
$m_H$ even above $2m_w$, the $H \rightarrow Z\eta$ decay can be as important as $H \rightarrow W^+ W^-$. We have investigated the LEP bound on $[g_{ZZH}/g_{ZHH}^\text{SM}] B(H \rightarrow b\bar{b})$ in the search for the SM Higgs boson. In the $f = 2$ TeV case, the model restricts $m_H$ above the LEP bound. For the $f = 3(4)$ TeV cases, a lowering in the Higgs boson mass bound occurs: $m_H > 109(111)$ GeV, respectively. This is the main result of our work.

A few comments are in order here.

(i) This new and dominant decay channel can lead to important implications on the LEP search for the neutral Higgs boson. The four LEP collaborations examined, in extended models, the process of $e^+e^- \rightarrow HZ \rightarrow (AA)Z \rightarrow (b\bar{b}b\bar{b})Z$, and presented the upper bound on $[g_{ZZH}/g_{ZHH}^\text{SM}] B(H \rightarrow \eta\eta) B(\eta \rightarrow b\bar{b})^2$. Our models with $f = 2, 3, 4$ TeV are not constrained by this bound.

(ii) Further probes of the scenario are possible at LEP, at the Tevatron, and at the LHC. The LEP collaborations can investigate the scenario by searching for

$$e^+e^- \rightarrow ZH \rightarrow Z(\eta\eta) \rightarrow Z(4b, 2b2\tau, 4\tau),$$

where $Z \rightarrow \ell^+\ell^-, \nu\bar{\nu}, q\bar{q}$. This mode may suffer from the fact that the coupling $g_{ZZH}$ is reduced relative to the SM one because of the little Higgs corrections. At the Tevatron, similar channels such as

$$p\bar{p} \rightarrow WH, \quad ZH \rightarrow W/Z + (4b, 2b2\tau, 4\tau)$$

can be searched for. At the LHC, the two-photon decay mode of the intermediate Higgs boson will suffer because of the dominance of the $H \rightarrow \eta\eta$ mode in that mass range. Thus, the branching ratio into $\gamma\gamma$ reduces. On the other hand, $gg \rightarrow H \rightarrow \eta\eta \rightarrow 4b, 2b2\tau, 4\tau$ open, which may be interesting modes to search for the Higgs boson. However, a detailed study is needed to establish the feasibility.

(iii) The $Z\eta$ decay mode of the Higgs boson is very unique in this simplest little Higgs model. In fact it dominates for 140 GeV $< m_H < 2m_w$. Even for $2m_w < m_H$ the $Z\eta$ mode is as important as the WW mode. It is very different from an SM-like Higgs boson, which usually has the ZZ mode in the second place. Since the $ZZ \rightarrow \ell^+\ell^-\ell^+\ell^-$ is the golden mode for Higgs discovery, the emergence of the $Z\eta$ mode will affect the Higgs detection significantly. Careful studies of $Z\eta$ mode are therefore important for Higgs searches.

(iv) Another possibility to probe the $\eta$ is the direct production of the $\eta$ boson in $gg$ fusion [17] or the associated production with a heavy quark pair. Although the production is suppressed by $1/f$ in the coupling of the $\eta$ to the SM fermion pair, this remains as an interesting possibility because the coupling to the heavy top quark is not suppressed.

We end here with an emphasis that $4b, 2b2\tau, 4\tau$ modes should be seriously searched for in the pursuit of the Higgs boson, which we have clearly demonstrated that it is possible in the simplest little Higgs models for $H \rightarrow \eta\eta$ and $H \rightarrow Z\eta$ to be dominant.

ACKNOWLEDGMENTS

We thank the Physics division of the KIAS for hospitality during the initial stage of the work. K. C. also thanks K.S. Cheng and the Centre of Theoretical and Computational Physics at the University of Hong Kong for hospitality. We would like to also express our special gratitude to Alex G. Dias for correcting our mistakes. We also appreciate the valuable comment from Juergen Reuter. The work of J.S. is supported by KRF under Grant No. R04-2004-000-10164-0. The work of K.C. is supported by the National Science Council of Taiwan under Grant No. 95-2112-M-007-001 and by the National Center for Theoretical Sciences.


[16] Alex Dias (private communication).


