Chaos in pulse-driven Josephson junctions

Roman Sobolewski,* Douglas R. Dykaar, and Thomas Y. Hsiang

Department of Electrical Engineering, The University of Rochester, Rochester, New York 14627

C. Vanneste† and C.-C. Chi

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

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We report experimental and numerical studies of chaotic behavior in Josephson tunnel junctions driven by a train of picosecond electrical pulses. Dramatic changes were observed in both the experimental and simulated current-voltage characteristics, demonstrating that periodic wide-band excitations can temporarily alter the junction dynamics, leading to intermittent chaotic behavior. We relate the observed chaotic behavior to the junction switching processes in the regime of very short current pulses.

Since 1980, when Huberman et al. 1 and Braiman et al. 2 first demonstrated by simulations the presence of chaotic behavior in rf-driven junctions, chaos in Josephson junctions has been extensively studied, both in experiments and in simulations. 3 Josephson junctions have received special attention because they represent an example of a dissipative quantum system that exhibits deterministic chaos. They also are an ideal experimental tool to test the theoretical predictions of the driven damped pendulum model, 3,5 where several routes to chaos have been established. The most common case is intermittent transition to chaos. The bifurcation diagram (see, e.g., Fig. 3 in Ref. 4) shows that intermittency is possible over a wide range of frequencies, starting with values much lower than the pendulum natural frequency. Both intrinsic intermittence 3,4 and noise-induced intermittency 3,6 have been experimentally observed.

In this Rapid Communication we describe our studies of Josephson tunnel junctions driven by electrical pulses that have picosecond duration and subpicosecond rise time. In terms of frequency, these pulses can be regarded as bursts of broadband excitations with nonvanishing frequency components in the gigahertz frequency range. In a series of experiments, we observed radical changes in the junction current-voltage (I-V) characteristics for different values of pulse amplitude, and demonstrated that, under proper conditions, the periodic pulse kicks gave rise to chaotic voltage oscillations. The numerical studies served to confirm most of the observed features, and to establish that our system exhibited chaotic motion.

Our test structure is shown in Fig. 1. The circuit was fabricated on an undoped GaAs substrate, and consisted of a 30×30 µm² Pb-alloy Josephson tunnel junction incorporated into a coplanar transmission line. The junction current density was selected to be 3×10² A/cm², ensuring essentially a uniform current distribution in the oxide barrier. A 10-µm-wide meander (low-pass filter for the high-frequency signal) provided a means for two-wire measurements of the dc I-V characteristics. The displayed I-V curves were "corrected" electronically using an externally controlled voltage signal proportional to the driving current, to compensate for lead resistance. All I-V measurements were preceded by a 1-MHz low-pass filter.

The photoconductive switch (a 50-µm-wide gap on the left-hand side of the junction, Fig. 1) was driven by laser pulses having 80-fs full width at half maximum and a wavelength of 620 nm, generated by using a prism-compensated colliding-pulse mode-locked laser. This arrangement (for more details see Ref. 7) enabled the generation of a train of electrical pulses with a risetime less than 400 fs and a fall time (90%-10%) of about 8 ps, repeated at 100 MHz. The frequency spectrum of the pulse contained nonvanishing components to more than 800 GHz. In terms of a spatial dimension, the transient represented a localized perturbation on the transmission line of the size of about 1 mm—much larger than our entire test structure; thus, we expect a uniform spatial perturbation of the junction. The pulse amplitude was proportional to the dc voltage bias applied to the switch. The output of the junction was properly terminated and then grounded to minimize the reflected signal.

In this work we focus on the case of small input pulse amplitudes—smaller than those necessary to switch the junction into the linear resistive state. Preliminary experimental results in the high pulse regime were presented

Sample Configuration

FIG. 1. Micrograph of the sample used in this work.
FIG. 2. Experimental I-V characteristics ($T=1.8$ K) for six different values of the amplitude of the input pulse. The small noise present on the gap-voltage sections of the quasiparticle I-V curves is caused by the 60-A laser power supply, and is picked up by the oscilloscope amplifier outside of the cryostat. The gap voltages displayed appear to be slightly reduced due to the two-wire measurements.

The experimental I-V characteristics of a typical junction for six different excitation pulse amplitudes (switch voltages, $V_s$) are shown in Fig. 2. In Fig. 2(a), for very small input pulses, only the critical current ($I_0$) on the positive branch of the I-V curve was reduced (here "positive" is defined as the direction of the current pulse). The negative branch was practically unaffected except that the minimum value of the stable voltage state, $V_{c_{\text{min}}}$, was significantly increased [Fig. 2(b)]. Upon increasing the input pulse amplitude to $V_s$ equal to 1.2 V, complete suppression of the positive branch of $I_0$ was reached. For current pulse amplitudes above this threshold [Figs. 2(c)–2(f)] “noisy” (as we prove later, chaotic) behavior set in on the negative branch, while the positive branch displayed the typical quasiparticle tunneling curve. Figures 2(c) and 2(d) represent the situation where the input pulse was too small to compensate fully the negative branch of $I_0$. We observed a novel “reentrance” process [Fig. 2(d)] in which, with increase of the dc bias, the junction switched from the noise regime back to the stable, zero-voltage state. We note that oscillatory behavior shown in Figs. 2(c)–2(f) is a truly low-frequency noise with the frequency bandwidth (1 MHz) much lower than any characteristic frequency in our problem.

To understand the significance of these observations, we have carried out computer simulations based on the resistively and capacitively shunted junction model. The equation governing the time evolution of the phase difference $\Phi$ across a current biased ($I_D$) junction driven by a train of pulses of an arbitrary shape $f(t)$, amplitude $I_p$, duration $\tau_p$, and repetition rate $1/T$ ($\tau_p \ll T$) may be written in normalized units as

$$\dot{\phi} + \Gamma \dot{\phi} + \sin \phi(\tau) = i_b + i_p \sum_{n=0}^{\infty} f(\tau - n \tau_T),$$

where the normalized time $\tau = \omega_0 t$, $\tau_T = \omega_0 \tau_T$, and $\omega_0 = (2eI_d/\hbar C)^{1/2}$, $i_b = I_b/I_0$, $i_p = I_p/I_0$, $\Gamma = 1/\omega_0 RC$, with C and R corresponding, respectively, to the junction capacitance and voltage-dependent shunting resistance (which approximates the quasiparticle I-V characteristic by three sections of piecewise linear curves).

Equation (1) was used to model the dynamics of our system. Quantitative correspondence between experimental and numerical results was obtained using the junction parameters and the shape of the driving current pulses extracted directly from experimental data. The only fitting parameter (see also Fig. 4) was the pulse amplitude at the threshold of complete suppression of the positive $I_0$. The experimental threshold switch voltage $V_s = 1.2$ V was fitted to the numerical pulse current at $I_p = 0.82 I_0$.

The simulated I-V characteristics (averaged over 250 pulses) for several values of $I_p$ are presented in Fig. 3. At a pulse value somewhat above the threshold [Fig. 3(a)]—corresponding to the switch bias 1.4 V, somewhere between Figs. 2(c) and 2(d)]—, the zero-voltage state for small $I_p$ becomes unstable in a similar fashion as in Fig. 2(c). Figures 3(b) and 3(c) correspond almost directly to the experimental situation shown in Figs. 2(e) and 2(f).
Excessive low-frequency noise characteristic of intermittent chaos, is present on the negative branch, while the positive part of the $I-V$ curve represents the quasiparticle branch. Absent from the simulated results is the noise associated with the reentrance phenomenon shown in Fig. 2(d). Simulation did show, however, that such behavior was possible if sufficient external noise (substantial variations in $I_p$—much larger than the 7% laser intensity fluctuations, and the 1.8-K thermal-background noise) was included in the computation of Fig. 3(a). Since the experimental curve represents an average over an enormous number of switching events, we suspect a noise-induced intermittency. A similar effect was observed previously by Iansiti et al., and they attributed the discrepancy between experimental and numerical data to the thermal and shot noise naturally occurring in the experimental system. As was established in Ref. 5, the external noise is much more important in the regime of intrinsic intermittency [Figs. 3(b) and 3(c)].

Experimental and numerical results are summarized on the threshold stability curve in Fig. 4, which describes the dependence of the maximum supercurrent on $I_p$ (or $V_p$), and separates the steady-state, zero-voltage solutions (area inside the curve) from the rotating, nonzero-voltage state. There is a remarkable agreement between experimental (dots) and simulated (solid curve) results. All three regions discussed earlier are indicated: the phase-locked (I in Fig. 4) and chaotic (III) regions, and an intermediate (II) region where both stable and unstable (broken line) zero-voltage solutions coexist.

We want to stress that the corresponding numerical value $I_p = 0.82 I_0$ for the threshold of switching an unbiased junction is less than $I_0$. In fact the entire threshold curve in Fig. 4 deviates significantly from a linear dependence (dot-dashed line) characteristic for long input pulses, where chaotic behavior was not observed.

In the time domain, the long-term behavior is presented in Fig. 5, where we plot the simulated real-time dynamics of the junction. The origin of the noisy behavior can be understood using simple arguments, similar to those given by D’Humiers et al., for very low driving frequencies, and Benza and Koch for transient chaos. The input pulse rise time is so short that the initial junction response is limited by the junction capacitance leading to nonzero voltage across the junction. The bias point sweeps to the positive voltage, and the $\phi(t)$ motion is highly asymmetrical—a precursor of the chaotic regime. Immediately afterwards, the ringing down begins, followed in most cases by dynamic punchthrough 11 causing the bias point not to rest on the zero-voltage branch of the $I-V$ curve, but rather to slip into the negative gap voltage. As was stressed in Ref. 4 and noticed in our simulations, the ringing-down process is critically sensitive to the exact manner in which the ringing down begins. Thus, we observe apparently random (nonperiodic) transitions between the zero- and gap-voltage states. On the long-time scale (averaged over many pulses) these repeated bursts of chaos lead to an extensive low-frequency noise observed on the junction $I-V$ characteristics.

Additional information can be obtained studying voltage-phase evolution diagram. The diagram taken of several plasma oscillations after the onset of the input pulse shows (see also Ref. 6) that chaos starts from breaking up the limit cycles for the two main frequencies, the plasma frequency and the much lower pulse repetition rate, jumping randomly from a wide limit cycle near the zero-voltage to several finite-voltage cycles. For the large time delays only zero- and gap-voltage cycles remain—both with a substantially shrunken phase space. This latter observation supports a transient origin of chaotic behavior and is a natural consequence of damping.

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**FIG. 4.** Threshold current dependence on the normalized amplitude of the picosecond input pulse. The dot-dashed line is the static (long-pulse) limit.

**FIG. 5.** Numerical simulations of the voltage evolution for the first seven 8-ps-wide pulses applied to the junction. For computational simplicity in this particular figure, we used rectangular pulses repeated at every 600 ps. The latter value was not critical, as long as it was longer than 460 ps, the subgap $RC$ time constant. $I_0 = 0.18$, $I_p = 2.0$; other parameters are the same as in Fig. 3.
more complete analysis will be presented elsewhere.

In conclusion, we demonstrated for the first time that Josephson junctions driven by a train of short (wide-band) electrical pulses, yet repeated on a period much longer than the junction damping time can exhibit bursts of chaotic behavior which is manifested experimentally in extensive low-frequency noise on the junction $I-V$ curve. We believe that this represents a new scenario leading to intermittent chaos. Excellent, quantitative agreement between the experiment and simulations proves the intrinsic nature of the observed phenomena. From a practical viewpoint, these observations are of significant importance in high-speed superconducting electronics and should be taken into account, for example, in the design of Josephson sampling systems.

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*Also at the Institute of Physics, Polish Academy of Sciences, Aleja Lotników 32/46, PL-02668 Warszawa, Poland.
†Permanent address: Laboratoire de Physique de la Matière Condensée (L.A. 190), Université de Nice, F-06034 Nice Cedex, France.
3See, e.g., the excellent “Survey of chaos in the rf-biased Josephson junction” by R. L. Kautz and R. Monaco, J. Appl. Phys. 57, 875 (1985), and references therein.
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