Observation of Al₂O₃:Cr³⁺ magnetic resonance via solitons in long Josephson junctions

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(Received 24 June 1987)

We have detected the zero-field electron-spin resonance signal of Al₂O₃ with ~1000 ppm Cr³⁺ using an edge Josephson junction. The sample comprises the substrate upon which the junction was fabricated and the coupling to the electron-spin resonance is via the solitons which exist in these “long” devices.

Some time ago Barnes suggested that the ac Josephson effect might be used as a method of performing in situ electron-spin resonance (ESR). Later Pelisson, Delescluse, and Barnes demonstrated that superconductor-normal metal-superconductor (SNS) junctions could be fabricated in which the “sample” comprised the N layer in a new type of point-contact junction. Based upon this work, Bures and co-workers were the first to perform successful ESR experiments on Au:Gd and Au:Er. Recently Goldman, Koper, and Valls and Barnes and Mehran have shown how the same technique might be extended to make q- and ω-dependent determinations of the full dynamical susceptibility χ(q,ω). While it has been argued that this technique, like many Josephson-based techniques, might be limited in sensitivity by only intrinsic limitations, it is far from clear that it represents a realistic alternative technique for performing ESR.

In this Rapid Communication we show that it is possible to easily detect the ESR signal of ~1000 ppm of Cr³⁺ in corundum using a conventional edge junction fabricated on the sample. Also new is the fact that the ESR is detected via the coupling of the solitons to the magnetic system. This demonstration is important because it shows for the first time that this technique can be applied to materials without the need to incorporate the sample into the junction itself and that the coupling to solitons is possible.

The Cr³⁺ ion in corundum has a narrow zero-field ESR transition at 11.447 GHz which via the Josephson relation 2 eV = hv corresponds to a voltage of 23.667 μV. It is this same transition which is used in a ruby maser.

We have used a “long” Josephson junction. When the length L exceeds the Josephson penetration length λ_J ≈ (h/2edμ₁Jₑ)₁/², where Jₑ is the critical current density and d is the magnetic thickness of the insulating (I) layer, the basic excitations of a simple superconductor-insulator-superconductor junction change from being the Fiske (or Eck) modes considered in the earlier theories to solitons.

The basic excitation consists of a single soliton, or vortex, which propagates along the junction with a velocity u determined by the average voltage V via V = Φ₀u/L, where Φ₀ ≈ h/2e is the flux quantum. In zero external magnetic field the soliton is reflected at the end of the junction as an antisoliton, i.e., a flux vortex with the magnetic field in the opposite sense. A second reflection recovers the original vortex. Thus, the period T = 2L/u and

\[ hν = hT = hμ₁/2L = \Phi₀u/L = eV, \]

i.e., the relation between the frequency of the magnetic field and the dc voltage is hν = eV, which differs from the usual Josephson relation by a factor of exactly 2. The largest voltage associated with this basic excitation is obtained by observing that the soliton cannot travel faster than the speed of light in the junction, c = (2a/d)¹/²c, where 2a is the thickness of the I layer and the magnetic thickness d = 2a + λ₁ + λ₂, where λ₁ and λ₂ are the London penetration depths of the two S layers. The corresponding voltage is Vₙ = nΦ₀c/L. In addition to the branch associated with this simplest excitation are branches which asymptotically reach Vₙ = nΦ₀c/L and are associated with n solitons propagating along the junction. These voltages Vₙ correspond to the position of the even order Fiske modes. A typical set of such branches is shown in Fig. 2(b).

The junctions were fabricated using a process developed for Nb:Pt alloys based on seven-level integrated circuit photolithographic methods. The junction illustrated in Fig. 1 has the length L determined by the thickness of the lower superconducting film and a width W. The I layer makes an angle of roughly 45° to the substrate which means that the flux which passes through the junction also passes through the substrate, i.e., with such a system there is essentially no difference between having the ESR ions situated in the substrate or within the magnetic thickness of the junction. We will analyze the system as if the latter were the case.

The resonance of Cr³⁺ has been detected in a some...

FIG. 1. The edge-junction geometry.
what different fashion in two different junctions. The junctions are (current) fed from a voltage source via a 1-kΩ resistor. A small field \( \sim 1 \) G is applied in the plane of the substrate using a pair of coils.

In one junction (see Table I for parameters) the signal at \( \sim 24 \) μV appears as a vertical step in a soliton branch, this is shown in Fig. 2(a). The feature is very similar to the microwave induced steps. For another junction, fabricated on the same sample, a quite separate branch associated with the resonance appeared but only in finite, small, magnetic fields \((H \sim 0.1-1 \) G) and now at \( 47 \) μV, i.e., twice the voltage predicted by the Josephson relation. The relevant branch is labeled \( A-B \) in Fig. 2(b). In both cases, the position and height of the steps on the current axis but not the position on the voltage axis was sensitively dependent upon the field. In addition to these expected steps, the former junction exhibited a doubling of some of the soliton branches which we also associate with the magnetic substrate [Fig. 2(c)]. One branch is simply a replica of the other displaced along the voltage axis by \( 47 \) μV.

These observations can be explained by adding the

\[
\phi = 4 \sum_{n=-\infty}^{\infty} \left[ \tan^{-1} \left( \exp \left[ \frac{x - 2nL - u^+ t - L/2}{\lambda(u^+)} \right] \right) + \tan^{-1} \left( \exp \left[ \frac{-x + 2nL + u^- t + L/2}{\lambda(u^-)} \right] \right) \right],
\]

where \( \lambda(u) = \lambda_0 (1 - u^2/\epsilon^2)^{1/2} \) and where the \( u^\pm \) are the velocities of the soliton and the antisoliton. The presence of a magnetic field not only modifies the boundary conditions\(^9\) but also, and more importantly, changes the kinetic energy \( K \) by an amount \( \Delta K = \pm H_{\text{ext}} \). As a result the velocity of the soliton \( u^+ \) and antisoliton \( u^- \) are different. It is this which introduces the important field dependence to our theory.

The magnetic field associated with the solitons is obtained by differentiation with respect to distance, i.e., \( B(z) = \frac{\hbar}{2ed} \frac{\partial \phi}{\partial x} \). The result for \( B_z \) is straightforward to obtain but rather complicated. Each soliton is associated magnetic coupling to the theory of resistively damped solitons. In the absence of either resistive or magnetic damping it is easy to combine Maxwell and Josephson equations to obtain the sine-Gordon equation,\(^8\)

\[
\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\epsilon^2} \frac{\partial^2 \phi}{\partial t^2} = \frac{1}{\lambda_0} \sin \phi,
\]

where \( \phi \) is the phase difference of the superconducting order parameter across the junction. An approximate periodic solution to this equation for a junction of length \( L \) is obtained by summing the solutions for an infinite junction, i.e.,

\[
B_z(x,t) = \frac{\Phi_0}{d} \sum_m \left[ \delta(x - 2mL - u^+ t - L/2) - \delta(x - 2mL - u^- t - L/2) \right],
\]

where we have used \( \delta \) functions to represent the solitons. It is important to note that this expression for \( B_z \) is odd, i.e., changes sign when \( x \rightarrow -x \).

FIG. 2. (a) Shown is the steplike feature lying between \( A \) and \( B \) which we identify as the \( n=2 \) 24-μV ESR signal. (b) The curve to the left is for zero and to the right for finite field. The first vertical step also labeled \( A-B \) on the right and which is only present for finite fields, is identified as the \( n=1 \), 47-μV signal. The other branches which appear with the field might be associated with the existence of a magnetic soliton. (c) A pair of branches which differ by a shift of \( \sim 47 \) μV on the voltage axis.
If the susceptibility of interest were not resonant, its effect could be included in the definitions of $\bar{c}, \lambda_j$, and the effective damping constant. Because the susceptibility of the magnetic system is resonant the change $\phi \to \phi + \Phi$ caused by the magnetic system involves only relatively low frequencies and wavelengths of the order of the junction length. It is, therefore, important in calculating $\Phi$ to correctly account for the boundary conditions $[\partial \phi/\partial x = -(2\pi d/h)B_{max}]$ at the ends of the junction. In the usual way, we accomplish this by expanding $\Phi$ or the magnetic perturbation $B$ in terms of Fiske modes. In complex notation, the expansion for the field is

$$B = e^{i\omega t} \left( \sum_{n=\text{even}} B_n \sin(k_n x) + \sum_{n=\text{odd}} B_n \cos(k_n x) \right); k_n = \frac{n\pi}{L}.$$  \hspace{1cm} (4)

$$B_n = \frac{2\lambda_{\text{even}} \omega_0 k_n^2 \Phi_0}{I_d} \frac{1}{(n^2 \omega_0^2 / \bar{c}^2 - k_n^2)} \times \left[ \sin \left( \frac{1}{2} \Delta \omega T \right) / \left( \frac{1}{2} \Delta \omega T \right) \right], \quad n = \text{even},$$  \hspace{1cm} (7)

where $\Delta = \frac{1}{2} k_n (u^+ - u^-)$ determined by the magnetic field $B_{\text{sat}}$ and $\omega_0 \equiv eV/h$ measures the applied voltage. The current response associated with the ESR is then calculated by equating the time derivative of the magnetic energy to the electrical power, i.e.,

$$P = I_m \bar{V} = \frac{4WL}{\mu_0} \frac{1}{T} \int_0^T \left( \frac{\partial \Phi}{\partial t} \right) dt.$$  \hspace{1cm} (8)

The final result for the current $I_n$ associated with the $n$th Fiske mode is

$$I_n = 2n \pi S_n \frac{\lambda_j}{L} \left( \frac{k_n^2}{(neV/h \bar{c})^2 - k_n^2} \right) \chi''(neV/h),$$  \hspace{1cm} (9)

where $I_n$ is the critical current and

$$S_n = \left( \frac{\Delta \omega \omega_0}{\Delta \omega^2 + 4\omega_0^2} \right) \left( \sin \left( \frac{1}{2} \Delta \omega T \right) / \left( \frac{1}{2} \Delta \omega T \right) \right), \quad n = \text{odd},$$  \hspace{1cm} (10)

The results, Eqs. (9) and (10), have several interesting properties. First the coupling of the solitons to the ESR is strong. If the junction is not too long so that $\lambda_j$ is sufficiently smaller than $L$ that the solitons are well-defined excitations but still of the same order as $L$ then $I_n$ is of order $I_c \chi''$ which, since $\chi''$ is of order unity on resonance, gives a large signal of the order of the critical current. The “fundamental” signal only occurs for finite fields. It is associated with the $n=1$ Fiske mode and occurs when $h \omega_0 = eV$ where $\omega_0$ is the ESR resonant frequency; this corresponding here to 47 $\mu$V. The $n > 1$ signals appear as subharmonics and are associated with the corresponding order Fiske mode. The even (or odd) subharmonics are of comparable magnitude, however, the theory assumes $\lambda_j k_n < 1$ and when this inequality fails the coupling will become small. This limits the number of subharmonics which might be observed. To the sum of

The magnetic response is only important when $\omega$ is near that of the ESR resonant frequency. Using this, Eq. (1) can be rewritten to

$$[1 + \chi'(\omega)] \frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\bar{c}^2} \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^2 \phi}{\partial \xi^2} - \frac{1}{\bar{c}^2} \frac{\partial^2 \phi}{\partial t^2} = \frac{1}{\lambda_j} \sin \phi,$$  \hspace{1cm} (5)

where $\chi'(\omega) = \chi''(\omega)/i\chi''(\omega)$ is the complex susceptibility associated with the ESR and where it has been assumed that $\Phi$ is sufficiently small that it can be neglected in the argument of the sine. $B$ is then given by

$$\frac{\partial^4 B}{\partial x^2} - \frac{1}{\bar{c}^2} \frac{\partial^2 B}{\partial t^2} = -\chi'(\omega) \frac{\partial^4 B}{\partial x^2}.$$  \hspace{1cm} (6)

Solving this equation gives

$$I_n = (2\pi L^2 / \Delta \omega \bar{c}) \left( \frac{\partial \Phi}{\partial \xi} \right) \left[ \frac{1}{1 - (\bar{V}/V_1)} \right]^{1/2},$$  \hspace{1cm} (10)

due to resistive damping of the solitons; here $\sigma$ reflects the normal conductivity and $V_1 = h \bar{c} \omega_0 / eL$ is the voltage expected for the second Fiske mode.

The magnetic susceptibility

$$\chi''(\omega) = \chi_0 \omega \delta / ((\omega - \omega_0)^2 + \delta^2),$$  \hspace{1cm} (11)

where $\delta$ is the width for Cr$^{3+}$ resonance, is very strongly peaked, i.e., $\delta << \omega_0$. It follows that, in a current-fed junction, the resonance will appear as an almost vertical step. We identify the vertical region from $A$ to $B$ in Fig. 2(a) at 24 $\mu$V as such an $n=2$ step in the lowest soliton branch. The lowest vertical, finite field branch at 47 $\mu$V, and also labeled $A-B$ in Fig. 2(b), is identified with the fundamental $n=1$ step. Lower subharmonics are not observed because the lower parts of the soliton branches are not stable at the small current end.

So much is in accord with our expectations. What is surprising is that the other branches for junction of Fig. 2(b) and the doubling of the branches seen in Fig. 2(c) imply the existence of what might be called a “magnetic soliton.” We speculate that the higher voltage branches shown in Fig. 2(b) and which appear with a finite field are associated with this magnetic soliton. These branches are certainly not vertical as would be the case if they were a harmonic of the ESR signal. In fact they coincide with a part of a zero-field soliton branch moved down the voltage axis by 47 $\mu$V. The paired branches shown in Fig. 2(c) would appear to be the same branch shifted by 47 $\mu$V. These observations are consistent with the existence of a magnetic soliton which has its velocity “locked” to that associated with the ESR resonant frequency. The extra branches are then explained as being due to the addition of one slow-moving magnetic soliton which coexists and passes through the faster-moving regular solitons. The magnetic soliton sits at some point on its vertical branch and simply adds 47 $\mu$V and a small current to the branch which would exist without this soliton.
We would like to thank A. Davidson, J. L. Mauer, M. Moldovan, N. F. Pedersen, W. J. Gallagher, C. D. Jessen, A. W. Kleinsasser, L. Kristianson, M. R. Scheuermann, and P. P. Sorokin for their contributions to this work and one of us (S.E.B.) wishes to thank IBM for their hospitality.

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