König, Lin, and MacDonald Reply: In a recent Letter [1] we developed a theory of carrier-induced ferromagnetism in diluted magnetic semiconductors. We analyzed the elementary spin excitations at low temperatures, where spin waves can be approximated as noninteracting Bose particles ("independent spin-wave theory"). In addition, we proposed a simple *ad hoc* "self-consistent spin-wave approximation" for higher temperatures to demonstrate the increasing inadequacy of mean-field theory critical temperature estimates at high carrier densities.

Yang *et al.* [2] adopt a result [3] derived for the Heisenberg model by an equation-of-motion approach under the Tyablikov decoupling scheme to get an alternative "self-consistent spin-wave theory," which is equally *ad hoc* in the regime addressed in Ref. [1]. Furthermore, as we show now, it is straightforward to rederive their scheme within our formulation and thereby provide a clearer physical picture of the nature of their approximation.

At low temperatures, the small amplitude collective fluctuations of the magnetization with dispersion Ω_p are described by the independent spin-wave theory. It yields

$$\langle S^z \rangle = \frac{1}{V} \sum_{|\vec{p}|<p_c} \{S - n(\Omega_p)\}. \quad (1)$$

The Bose function $n(x)$ reflects the fact that the spin waves are approximately independent Bose particles. It is known [4] that this equation yields the correct prefactor of the characteristic $T^{3/2}$ law.

A self-consistent spin-wave theory provides an approximate theory of large amplitude magnetization fluctuations at higher temperatures. Mean-field theory, which is expected to be accurate for a model with static long-range interactions, neglects correlations and, hence, spin-wave dispersion, but treats the problem self-consistently. The constraint on the number of spin waves per impurity spin ($\leq 2S$) leads to $\langle S^z \rangle = cSB_S(\beta S \Omega)$ or, equivalently,

$$\langle S^z \rangle = c\{S - n(\Omega) + (2S + 1)n[(2S + 1)\Omega]\}, \quad (2)$$

where $B_S(x)$ is the Brillouin function, and the energy Ω of an uncorrelated spin flip is independent of momentum $\vec{p}$. By specifying the dependence of $\Omega = J_{pd} n^*$ on $\langle S^z \rangle$, where n^* is the free-carrier spin density, the magnetization can be obtained self-consistently. The second Bose function in Eq. (2) is the correction term from spin kinematics and rules out unphysical states. Because of the neglect of correlation, mean-field theory can strongly overestimate the critical temperature [5] and fails to describe the low-temperature magnetization, even qualitatively.

A self-consistent spin-wave theory should ideally (i) reduce to the independent spin-wave theory at low temperatures, (ii) simplify to Eq. (2) in the Ising limit, $\Omega_p \to \Omega$, and (iii) yield a second-order phase transition by allowing for the trivial solution $\langle S^z \rangle = 0$.

At low temperatures the correction term in Eq. (2) is negligible and Eq. (1) is recovered if Ω is replaced by an effective energy such that

$$n(\Omega) \equiv \frac{1}{cV} \sum_{|\vec{p}|<p_c} n(\Omega_p) \quad (3)$$

(= Φ in Ref. [2]). Equation (2) with Ω given by Eq. (3) is identical to Eq. (3) of Ref. [2] [and to Eq. (52) of Ref. [3]]. All the requirements (i), (ii), and (iii) are satisfied. The scheme proposed by Yang *et al.* is, thus, equivalent to mean-field theory with an effective spin-flip energy Ω.

In Ref. [1] we discussed a simple self-consistent spin-wave scheme in which $\Omega \to \Omega_p$ in Eq. (2) and momenta are averaged over, $(1/cV)\sum_{|\vec{p}|<p_c} \ldots$. This is equivalent to restricting the number of spin waves *at each wave vector* $\vec{p}$. As we pointed out [1], this approximation is too restrictive at low temperatures. Our intention in introducing this scheme was to demonstrate the failure of mean-field theory for the critical temperature at high carrier densities, not to address low-temperature properties. At low temperatures magnetic anisotropy effects [6], not included in these models, have a dominating importance. Similarly, in lower space dimension ferromagnetic semiconductors, magnetic anisotropy is dominant [7], limiting the importance of long-wavelength spin waves. In particular, compatibility with Mermin-Wagner theorem is not an issue in describing these materials.

Finally, we note that in both self-consistent schemes the polarizations of *both* the impurity spins and the free carriers go to zero at the critical temperature. This is not generally the correct physical picture. For example, in the limit of low carrier densities, the local polarization of the free carriers remains finite even above T_c [5], and neither scheme gives reliable T_c estimates.

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