國立清華大學100學年度碩士班入學考試試題

系所班組別：動力機械工程學系碩士班 乙組(電控組)

考試科目（代碼)：控制系統 （1101）

1. For the mechanical system shown in the following figure, a force input f is applied to a mass m. The spring element has a spring coefficient k. There are also two linear dampers which can generate mechanical forces in proportional to their respective velocities. The damping coefficients of the dampers are respectively equal to b_1 and b_2.

(a) Determine the differential equations for the system using x and y as the variables. (8 %)

(b) Derive the transfer function from f to x. (7 %)

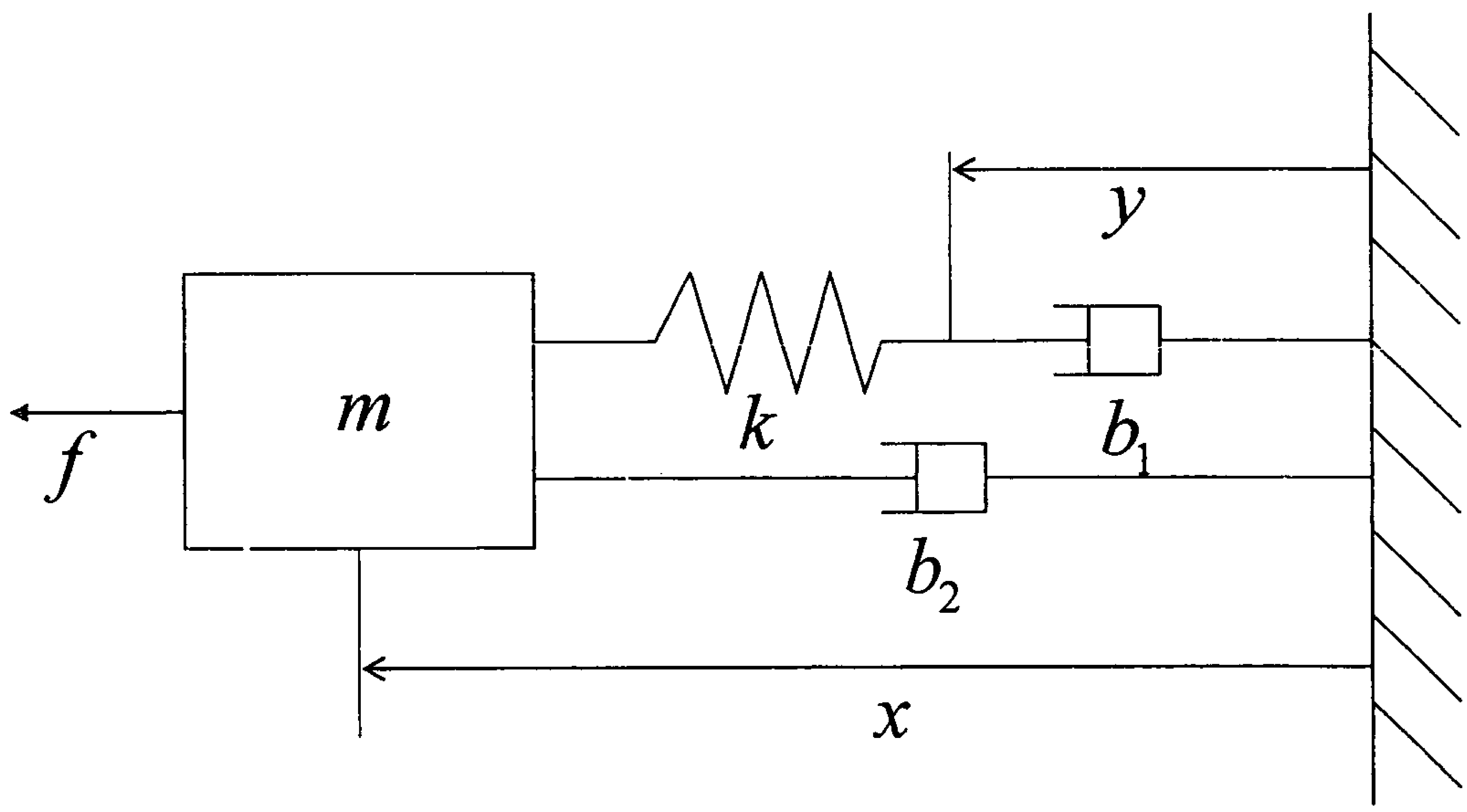

Fig. 1

2. Consider the following nonlinear system:

$$\ddot{y} + 3\left(y + 0.2y^3\right)\dot{y} + 2y = 2\dot{u} + u.$$

(a) When input u is a constant with $u \equiv 2$ and the system achieves steady state, the output y is also a constant with $y_{steady-state} = C$. $C = ?$ (5%)

(b) Based on the operating point you obtain in part (a), linearize the nonlinear system and derive a linear model. (10%)

3. Estimate the transfer function represented by the Bode plot in Figure 2 below.

(a) At low frequency the gain is constant, breaking upwards at around ω_b(rad/s). What is the frequency of ω_b? (2%)

(b) There is a marked resonance at ω_n(rad/s), after which the gain drops away. What is the frequency of ω_n? (2%)

(c) Describe how the phase changes from low frequencies to around ω_n? (2%)

The general form of the transfer function is thus a first order zero and second order under-damped denominator term:

$$G(s)=\frac{a(s+0.1)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

(d) What is the value of a? (2%)

(e) What is the value of ζ? (2%)

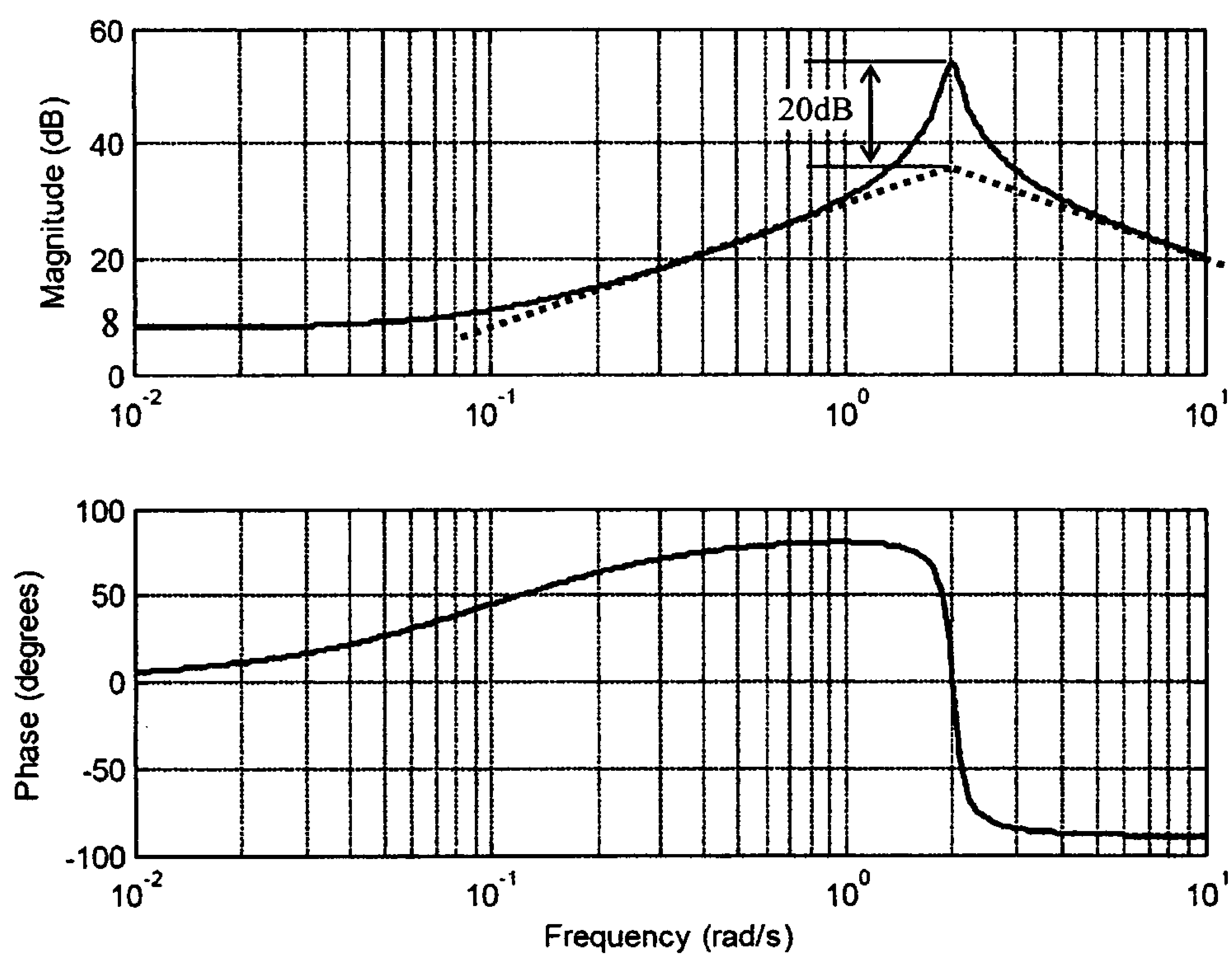

Figure 2

4. A third-order plant is described by the transfer function:

$$G_p(s) = \frac{6}{(s+1)(s+2)(s+3)} \qquad (1)$$

and is to be controlled via a *proportional-plus-integral* scheme.

(a) Write the controller transfer function. (2%)

(b) Determine the fixed relationship between the proportional (k_p) and integral (k_i) gains, that ensures exact cancellation of the least-dominant plant pole. (3%)

(c) Following (a) and (b), write the closed-loop characteristic equation as a function of k_p. (3%)

Based on the result of (c), sketch the roots' loci of the closed-loop characteristic equation, when the parameter along the curve is a simple function of k_p.

(d) Determine the three asymptote angles. (3%)

(e) Determine the interception point of the asymptotes. (2%)

(f) Determine the relevant break point. (2%)

(g) Sketch the complete roots' loci and show the asymptotes and the break point. (5%)

5.

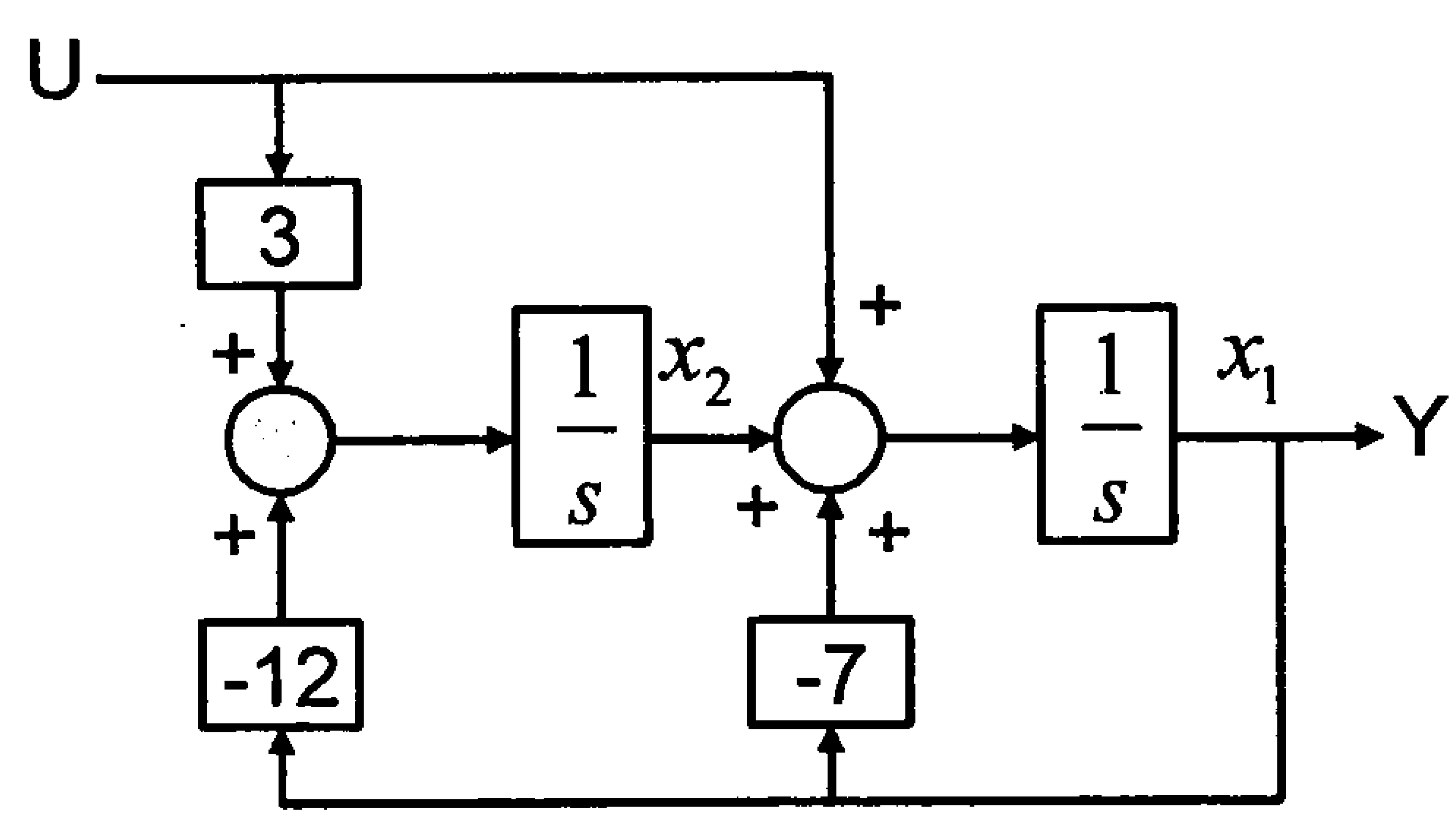

Fig. 3

(a) Represent the above system in the state-space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = F \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + G \cdot u$$

$$y = H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + J \cdot u$$

and find the matrix *F, G, H, and J.* (5%)

(b) Check the controllability of the individual closed-loop ($\frac{Y}{U}$) poles. (controllable/or uncontrollable mode for each closed-loop pole) (5%)

(c) Check the observability of the individual closed-loop ($\frac{Y}{U}$) poles. (observable/ or unobservable mode for each closed-loop pole) (5%)

6. The equation of motion for a mechatronic system in state space representation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$; (w is the process noise)

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v$$ (v is the sensor noise)

Use state feedback control $u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Here the performance cost function is $J = \int_0^\infty [\rho y^2(t) + u^2(t)] dt$

Use Linear Quadratic Regulator (Symmetric Root Locus) to find the optimal control ($\begin{bmatrix} k_1 & k_2 \end{bmatrix}$), which minimizes the cost function J, with the fixed weighting factor $\rho = 64$. (10%)

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7. For a plant with the transfer function $\frac{Y}{U}=\frac{1}{s^2}$, the corresponding state-space equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = F\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + G\cdot u\,; \qquad y = H\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here $F=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $G=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $H=\begin{bmatrix} 1 & 0 \end{bmatrix}$

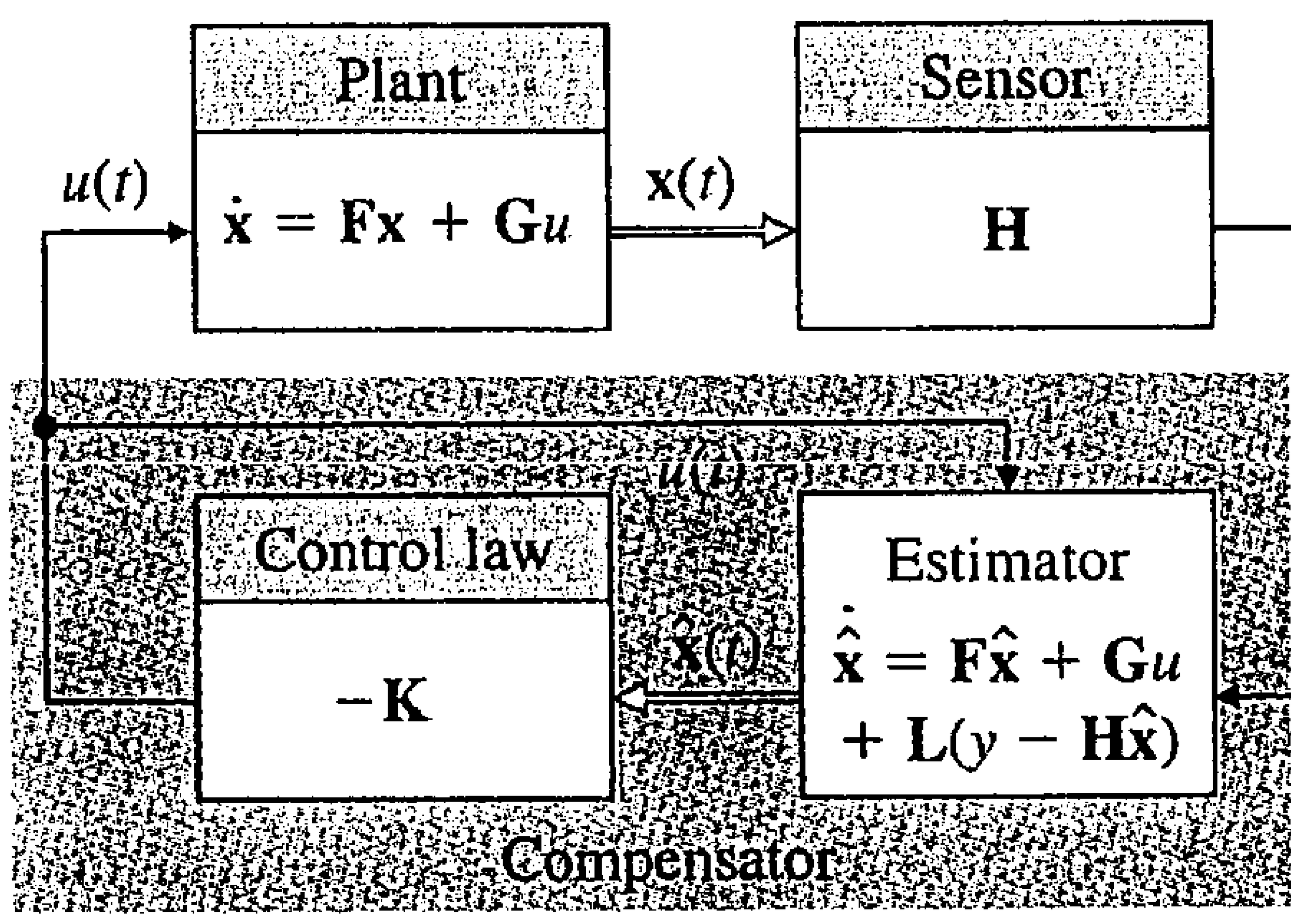

Fig. 4

(a) Design a compensator as shown in above figure by using pole placement. Place control poles at $\omega_n = 1$ $rad/\sec$ and $\zeta = \frac{1}{\sqrt{2}}$. Find the state feedback gain K. (5%)

(b) Place estimator poles at -2.5 + 4.3j & -2.5 - 4.3j. Find the estimator gain L.(5%)

(c) Find the transfer function of the compensator. (5%)