Micro Electrode Array - A New Method for the Design of Electrostatic Micro Actuators and Capacitive Micro Sensors

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ABSTRACT

In this paper, a new method which simplifies the design process of micromachined deformable mirrors is presented. By varying the widths of an array of constant-pitched electrodes, the electrostatic-force profile needed to shape the mirror can be precisely controlled using only one voltage input. In the past, either several independent voltages were necessary or, if only one voltage was available, numerical schemes were required to search for the optimal sizes and locations of a few electrodes.

A mirror is formed by a thin membrane micromachined from a silicon wafer and is coated with a thin metallic film. The electrodes are deposited on a ground plane over which the membrane is suspended. Viewing the mirror as a surface composed of many small patches with the same pitch, we can calculate the average force of each patch from the deformed shape using basic elasticity formulae. Using conformal mapping method, we can solve the analytical solution of the electrostatic field between the mirror and the electrode in one pitch. The relationship between the force and the width of the electrode is established. Finally, the widths of all the electrodes are obtained, and this new method applies equally well to the designs of both membrane electrostatic actuators and capacitive sensors.

Keywords: Deformable Mirror, Electrostatic Actuator, Micromachining, Conformal Mapping, Capacitive Sensor, Schwarz Christoffel transformation

1. INTRODUCTION

Deformable mirrors with electrostatic actuators have been widely used in various adaptive optical systems. These mirrors are formed by thin silicon nitride membranes micromachined from silicon wafers and coated with thin metallic films. The actuators correspond to electrodes deposited on a ground plane over which a membrane is attached. A schematic diagram of the flexible mirror is shown in Fig. 1. The desired mirror deformation is achieved by applying appropriate voltages between the electrode and the membrane.

Fig. 1 A schematic diagram of the deformable mirror
The membrane layer of low-stress Si-rich $\text{Si}_x\text{N}_y$ is suspended on the edge of a circular window isotropically etched in a (1 0 0) silicon chip. To make the membrane reflecting, the etched side is coated with a 0.2 $\mu$m thick layer of aluminum. The chip with the flexible membrane is fixed 20 ~ 100 $\mu$m above another chip on which a pattern of metallic electrodes is deposited.

Controlling the deformation of the mirror membrane has been proposed previously. Vdovin fabricated deformable mirrors by depositing patches of electrodes under the mirror membrane, and the potential of the electrodes are controlled by neural network$^{1,2,3}$. Wang developed a method for designing electrode pattern in micromachined deformable mirror$^{4,5,6}$. Wang's method is based on the Green function method, so a optimization procedure to find a set of voltage inputs must be carried out. Hisanaga proposed a diaphragm mirror of nonuniform thickness profile to achieve a parabolic deformation$^7$. In Hisanaga's method, the photoresist must be developed to form a specific curved profile due to the optical energy profile. However, the purpose of our paper is to determine the specific pattern of the electrostatic-actuator electrodes with only one appropriate voltage applied and the specified deformation of the membrane mirror can be achieved.

We begin to apply the conformal mapping method to find the electrostatic field between the mirror and the electrodes, so we can determine the specific pattern of the electrodes to obtain the desired deformation of the mirror. The application of this method to circular deformable mirror and the axisymmetric patterns of the electrodes is discussed in detail.

2. ELECTROSTATIC-ACTUATOR ELECTRODE PATTERN

2.1 ELECTROSTATIC TRACTION ON THE MIRROR DUE TO THE SPECIFIC DESIRED PROFILE OF THE DEFORMABLE MIRROR MEMBRANE

We consider a circular mirror with $N$ electrostatic actuators. The mirror deformation is modelled by an elastic membrane with radius $a$ and surface tension $T$. For simplicity, we assume that the actuator electrodes are composed of concentric rings all centered about the origin.

The undeformed mirror surface is originally flat due to the tensile stress of the nitride, so the actuator gaps $g$ all equal to $g > 0$. The membrane is regarded as being composed of $N$ patches with concentric rings and a circular disk.

Fig. 2 shows a sketch of the radial profile for a circular deformable mirror. The rings are with the same width $2L$ and the radius of the disk is the same as the width $2L$ of the rings. Each ring and the disk correspond to one concentric ring of actuator electrode in a pitch. Assume that the elastic surface deformation is in the linear elastic regime.

The shape of a membrane under an external load is described by the Poisson’s equation with edge conditions. When the thickness-diameter ratio of the mirror surface is small such that the bending energy is negligible compared to the energy due to the tensile stress, the mirror surface may be regarded as an elastic membrane. Assuming that the mirror is clamped to its boundary $Z_c$, the periphery $U_c$ of the membrane must satisfy the boundary condition:

$$U_c = Z_c = 0,$$

and the Poisson’s equation is

$$\nabla^2 U(r, \theta) = \frac{f(r, \theta)}{T} \quad \text{eq. (1)}$$

where $U(r, \theta)$ is the deflection of the membrane represented in polar coordinates, $f(r, \theta)$ is the traction distribution, $T$ is the membrane tension, $U_c$ is the deflection of the membrane edge and $Z_c$ is the profile of the wafer on the edge of the membrane. Surface tension $T$ is determined by the process condition. Substituting a desired shape of a membrane into eq. (1), we will obtain the electrostatic pressure $f(r, \theta)$ directly.

Because all the patterns of the electrodes and the patches of the membrane are in the form of concentric sings, $f(r, \theta)$ is reduced to $f(r)$. The aim of this paper is to find the approximate distribution of the electrode rings corresponding $f(r)$ given.
### 2.2 Mathematical Model in a Pitch for the Deformable Mirror

When the number of the pitches are large enough, we can assume that the difference between the widths of the neighboring electrodes are small enough to let the pitch line be a magnetic boundary, that is, the electric field tangent to the pitch line. Similarly, the symmetric line for a pitch is regarded as a magnetic boundary for the electrostatic field in a pitch.

To simplify the analysis of this electromagnetic field, we take only 1/2 pitch of the region between the mirror and the substrate as shown in Fig. 3 to analyze it. When the number of the pitches is large or the deflection of the membrane is small, the mirror surface in a pitch can be regarded as a conducting plane parallel to the corresponding electrode on the substrate. Because the thickness of silicon nitride is far less than the gap between the mirror and the electrode, the dielectric effect of silicon nitride for this electrostatic field is negligible. For the same reason we also neglect the thickness of the electrodes.

Because the electric field in a pitch mainly distributes in the region between mirror and electrode, the dielectric effect of substrate can be neglected and regarded as that in the air.

Consider the region in a pitch \( j \) with one actuator electrode as shown in Fig. 3. The electrode is specified by a patch \( Q_j \) (a specified open connected subset of \( \Omega \)) and the mirror portion in a pitch \( j \) is specified by a patch \( \sigma_j \). Under the
assumption that the minimal width $2W$ of the electrode patch $\Omega_j$ is large compared to the original gap $g$ between the mirror and the electrode, the electrostatic traction due to the $j$th actuator $\Omega_j$ can be described approximately by

$$f(r) = \frac{\varepsilon \phi_j^2}{2H^2} \psi_j(r) \quad \text{(2)}$$

where $H = g - U(r)$

$\varepsilon$ is the permittivity of free space; and $\phi_j$ may be taken as the characteristic function of $\Omega_j$ (i.e., $\phi_j(r) = 1$ if $r \in \Omega_j$; $\phi_j(r) = 0$ if $r \not\in \Omega_j - \Omega_j$), or a weighting function that models the spatial variation of $f(r)$ due to the fringing electric field near the boundary of $\Omega_j$. In the past no study has been devoted to the fringing electric field between $\sigma_j$ and $\Omega_j$ previously. We will use the conformal mapping method and find an analytical solution of the electrostatic field in a pitch, so the fringing effect is undoubtedly considered and the desired $f(r)$ can be obtained.

2.3 CONFORMAL MAPPING APPLIED IN A PATCH OF THE DEFORMABLE MIRROR

Fig. 4(a) shows a sketch of a half pitch for the deformable mirror. Because the problem of the electrostatic field is to solve the Laplace equation of the electric potentials, we can use the Schwarz - Christoffel (SC) conformal transformation formula, which reads

$$W(z) = M \prod_{i=1}^{n} \left( z - z_i \right)^{-\mu_i} d\zeta + N \quad \text{(3)}$$

providing a general technique for mapping the points on the real axis of the $z$ - plane upon a polygon in the $w$ - plane, and the upper half $z$ - plane to the region enclosed by this polygon. Fig. 5 shows the mapping involved four planes for this problem.

The mapping function $W_1(z)$ as well as $W_2(z')$ in Fig. 4 are given by the Schwarz - Christoffel transformation formula. We obtain
\[ W_1(z) = C_1 \oint \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}} + C_2 \quad \cdots (4) \]
\[ W_2(z') = C_3 \oint \frac{d\eta}{\sqrt{\eta(\eta + 1)(\eta + q)}} + C_4 \quad \cdots (5) \]

and the transformation from z-plane to z'-plane is given by the bilinear transformation as followed:

\[ z' = z \frac{1 + \rho}{\rho - z} \quad \cdots (6) \]

The physical dimensions defined in Fig. 4(a) are used in the W₁-plane in Fig. 4(b). Based on the point A, B, C, D, E on the W₁-plane, we find

\[ L_j = C_1 \oint_0^X \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta - x)(\zeta + 1)(\zeta - \rho)}} + C_2 \quad \cdots (7) \]
\[ 0 = C_1 \oint_1^Y \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}} + C_2 \quad \cdots (8) \]
\[ H = C_2 \quad \cdots (9) \]
\[ H + W_j = C_1 \oint_0^Z \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}} + C_2 \quad \cdots (10) \]
\[ H = C_1 \oint_0^W \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}} + C_2 \quad \cdots (11) \]

By eq. (7)~(11), we obtain

\[ \lambda = \oint_0^X \frac{\zeta \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}} \quad \cdots (12) \]
\[ W = \frac{\oint_1^Y \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}}}{\oint_1^X \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}}} \quad \cdots (13) \]
\[ \frac{H}{L} = -j \frac{\oint_1^Y \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}}}{\oint_1^X \frac{(\zeta - \lambda) \, d\zeta}{\sqrt{(\zeta + x)(\zeta + 1)(\zeta - \rho)}}} \quad \cdots (14) \]

In this problem, there are three physical dimensions L, H, W in the physical plane W₁ as shown in Fig. 4(a), so only two unique parameters can determine the capacitance of this problem. Here we assign W/L and H/L to be the two unique parameters and then W/L are less than one because L is always larger than W. We can use the exact solution of x and \( \rho \), two independent parameters in the Z-plane, from the known values W/L, H/L by the numerical optimization and iteration.
The relation between \((H/L, W/L)\) and \((\alpha, \rho)\) is shown in Fig. 5 calculated by the mathematical software Mathcad 6.0 plus. This figure is composed of constant-\(\rho\) lines and constant-\(\alpha\) lines.

Fig. 5 The relation between \((H/L, W/L)\) and \((\alpha, \rho)\), where \(h_0!\) denotes \(H/L\), \(w_0!\) denotes \(W/L\).

If the values of \(\alpha, \rho\) can be known, the values of \(H/L\) and \(W/L\) and that of \(q\) in the \(Z'\)-plane can be easily figured out. By eq.(6)

\[ q = \frac{x(\rho + 1)}{x + \rho} \quad (15) \]

By the transformation between the \(Z'\)-plane and the \(W2\)-plane, we find

\[ k_q = \left\{ \begin{array}{l} \frac{1}{q} \quad (16) \\ \sqrt{1 - k_q^2} \quad (17) \end{array} \right. \]

\[ C = \varepsilon_s \varepsilon_0 \frac{K(k_q)}{K(k_q')} \quad (18) \]

where \(C\) is the capacitance per unit length between the two conductor strips. \(\varepsilon_0\) is permittivity in free space and \(\varepsilon_s\) is relative permittivity. \(K(k_q)\) is the complete elliptic integral of the first kind\(^{10,13,14}\). By using analytic mapping functions the mapping
will be conformal, which implies that the capacitance between the conductor strips will be equal in all four planes, so this method can be also applied to the analytic calculation of electrostatic sensors.

2.4 ELECTRIC FIELD CORRESPONDING TO SPECIFIED PHYSICAL DIMENSIONS

The gradient of the potential is the electric field. Based on the theory of conformal mapping, we know

$$\nabla_c \phi = \nabla_c \psi \cdot J'(z) \quad \text{(19)}$$

where $\phi (x, y)$ is the potential in the physical plane, $\psi (u, v)$ is the transformed potential in the model plane and

$$\nabla_c = \frac{\partial}{\partial u} + j \frac{\partial}{\partial v} \quad \text{(20)}$$

$$J'(z) = \frac{\partial u}{\partial x} - j \frac{\partial v}{\partial x} \quad \text{(21)}$$

For this problem, we let $\psi$ be the potential in the $W_2$-plane. Through those three transformations described above. We obtain the electric field in the physical plane

$$E = \nabla_c \phi = \nabla_c \psi \cdot \left( \frac{dW_2}{dz} \right) \left( \frac{dz'}{dW_1} \right) = (-j) \frac{V}{H} \cdot N \cdot M(z) \quad \text{(22)}$$

where ($j$) is the unit normal vector in the complex plane, and

$$N = j \int_0^1 \frac{(z' - \lambda)dz'}{\sqrt{(z' + x)(z' + 1)z'(z' - \rho)}} \quad \text{(23)}$$

$$M(z) = \frac{1}{\sqrt{(z' + 1)z'(z' + q)z' + 1} \sqrt{(z + x)(z + 1)(z - \rho)}} \quad \text{(24)}$$

where $z' = z'(z)$ is defined in eq. (6). A finite element method (FEM) software for electromagnetic field called MSC/EMAS was used to verifying our analytical solution. The electric field corresponding to specified physical dimensions calculated by this conformal mapping method is almost equal to that calculated by FEM. One of many results has shown in Fig. 6, and this comparison is shown in the Figure 7. The Fig. 7 shows the magnitude distribution of the electric field along the upper conductor strip $BA$ from B to A under the conditions $\mu = 0.48938$, $\nu = 0.2245$, and potential difference $V = 2$. All variables are nondimensionalized. In Fig. 7 the horizontal axis is the point along $BA$ that means a pitch of the membrane, $E_{map}$ is the normal component of the electric field due to the S-C transformation. $E_n$ is the normal component of the electric field calculated by FEM software. $E_{mean}$ is the mean value of the line integral $E_{map}$ and $E_{fin}$ is the mean value of the line integral of $E_n$. 

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Fig. 6 The electric potential and the electric field simulated by MSC/EMAS

\[ \frac{H}{L} = 0.48938 \quad \frac{W}{L} = 0.2245 \]

Fig. 7 The normal component of the electric field calculated by the conformal mapping method and FEM

Furthermore, we have known that the tangent components of the electric fields to the membrane surface with 3 orders less than the normal components of those, so the electric fields tangent to the two conductor strips can be ignored.

2.5 MAXWELL STRESS TENSOR APPLIED ON THE MEMBRANE SURFACE CORRESPONDING TO SPECIFIED PHYSICAL DIMENSIONS

By the electromagnetic - elasticity\textsuperscript{16,17}, we know the Maxwell stress tensor is
\[
\tau_{ij} = \varepsilon_0 \left[ E_i E_j - \frac{1}{2} \delta_{ij} E_{kk}^2 \right] \quad \text{(25)}
\]

For our problem under the conditions that magnetic fields are zero. \( E_i \) is the electric field and \( \delta_{ij} \) is Kronecker delta. Let's assign the index 1 as the normal direction to the mirror surface. Because the normal component of the electric field is far larger than the tangent component of that, the dominant component of the Maxwell stress \( \tau_{11} \) acts on the membrane defined by eq.(25) as

\[
\tau_{11} = \hat{x}_1 \cdot \nabla \cdot \hat{x}_1 = \frac{\varepsilon_0}{2} \left[ E_1^2 - (E_2^2 + E_3^2) \right] \quad \text{(26)}
\]

where \( \hat{x}_1 \) is the unit vector normal to the surface of the membrane. \( E_1 \) is the normal component of the electric field and \( E_2, \) \( E_3 \) are the tangent component of that to the membrane surface. By eq.(1) and (26), we can assign

\[
f(r, \theta) = \tau_{11} \quad \text{(27)}
\]

to be the traction pressure acting on the membrane surface. We divide one pitch into \( k \) pieces and find \( T_{\text{mean}} \) the mean value of \( \tau_{11} \), defined by

\[
T_{\text{mean}} = \frac{\sum_i \tau_{11, i} d\ell_i}{\sum_i d\ell_i} \quad \text{(28)}
\]

where \( d\ell_i \) is the small line segment on the electrode. The value of \( T_{\text{mean}} \) is normalized to be

\[
T_n = \frac{T_{\text{mean}}}{\varepsilon_0 \left( V \cdot \frac{1}{R^2} \right)} \quad \text{(29)}
\]

where \( T_{\text{mean}} \) is the mean traction applied on the mirror for a pitch by eq.(28). \( T_n \) is the normalized traction. By eq.(22), (26), (28), (29) we obtain the results about the normalized mean - value of Maxwell stress \( \tau \) corresponding to specified physical dimensions shown in Fig. 8. The conditions of Fig. 8 is the same as those of Fig. 7. In Fig. 8 the tractions \( \tau_{m1} \) and \( \tau_{m1p} \) are calculated from \( E_n \) and \( E_{\text{map}} \) in Fig. 7, and the traction \( \tau_{m1\text{mean}} \) and \( \tau_{m1p\text{mean}} \) are the mean value of \( \tau_{m1} \) and \( \tau_{m1p} \) by line integral.

Fig. 9 shows the mean values of the normalized traction in a pitch for the deformable mirror calculated by this conformal mapping. From Fig. 9, we know that the line from \( \left( \frac{W}{L}, \frac{V}{L} \right) = (0,0) \) to \( \left( \frac{W}{L}, \frac{V}{L} \right) = (1,1) \) is the ideal traction without fringing effect, and here we obtain the real traction with fringing effect. Based on the derivations above, we developed conformal mapping to analyze the electrostatic field of the mirror. For a certain specific deformation and the values \( \frac{V}{L} \) of the electrodes are known, and then the required traction to the mirror can be calculated. Finally, we can obtain the corresponding values of \( \frac{V}{L} \) for each pitch. In fact, if a deformation profile, \( H \) and \( L \) are known, through the corresponding traction calculated the width \( W \) can be determined.
3 NUMERICAL EXAMPLE FOR DESIGNING A PARABOLIC DEFORMATION OF A MEMBRANE

When the thickness - diameter ratio of the membrane is small, we can design the parabolic deformation of the membrane assumed as function of the deflection $U(r, \theta)$ in eq.(1)

$$U(r, \theta) = U(r) = U_0 \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \quad (30)$$

Fig. 8. The normal component of the traction to the membrane surface calculated by the conformal mapping method and FEM.

Fig. 9. Normalized traction in a pitch for the deformable mirror.
where R is the radius of the membrane, $U_0$ is the maximum deflection at the center of the membrane. Substituting eq.(30) into eq.(1), we obtain

$$f(r, \theta) = f = 4 \frac{U_0}{R^2} T$$  -----(31)

Because of the parabolic deformation shape, the traction at the periphery must larger than that at the center. By eq.(29) and (31), let $\frac{W}{L} = 1$ at the pitch that is nearest to the periphery of the membrane, and then $T_n = 1$ there, so the unique voltage $V$ applied to the electrodes becomes

$$V = H_{\text{max}} \sqrt{f \times \frac{2}{\epsilon}}$$  -----(32)

where $H_{\text{max}}$ is the gap at the pitch that is nearest to the periphery of the membrane.

Substituting $V$ solved by eq.(32) into eq.(29), the required $T_n$ for the other pitches becomes

$$T_n = f \cdot \frac{2}{\epsilon} \left( \frac{H}{V} \right)^2$$  -----(33)

Even though the value of $H$ changes with the deformation shape along the radius of the membrane, the appropriate $\frac{W}{L}$ can be figured out to interpolate the required $T_n$ in each pitch found in Fig. 9. Here we present one example for a specific parabolic deformation. The gap $g$ between the mirror membrane and the electrode is 25 $\mu$m, the mirror radius is 5000 $\mu$m, the deflection of the membrane center is 10 $\mu$m, the thickness of the membrane is 0.5 $\mu$m, the mean value of the tensile stress of the membrane is $2 \times 10^7$ N/m$^2$, and we take the width of the pitch is 10m, so there are 100 pitches in this region. By eq(32), the voltage required is about 47.2 Volt. By eq(33), we found that $T_n$ is the range from 0.366 to 1, and $h/L$ is from 0.3 to 0.5. Fig.10 shows the quarter part of the electrode pattern for this problem. This experiment is in process now.

![Fig.10 Example of electrodes pattern](image_url)
4. CONCLUSIONS

This proposed method for design actuator-electrode arrays with rings pattern is based on specifying the ring width of the electrode for given only one actuator voltages. By varying the widths of an array of constant - pitched electrodes, the electrostatic - force profile required to shape a deformable mirror can be precisely controlled using only one voltage input. This approach can be applied to the design of electrostatic microactuator and capacitive microsensers.

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6. REFERENCE


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