Theory of backward distributed-feedback optical parametric amplifiers and oscillators

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We derive a coupled-wave theory for backward phase-matched optical parametric amplifiers and oscillators in a dielectric modulated nonlinear optical material. Unlike a forward phase-matched distributed-feedback optical parametric oscillator, a backward oscillator of this type can have high-threshold gap modes at Bragg resonance and low-threshold detuned modes. In addition to the structure feedback loop, the internal feedback loop in the counterpropagating waves strongly influences the resonant-mode formation in this oscillator. A backward distributed-feedback optical parametric oscillator also exhibits the mode-dependent oscillation threshold similar to that for a distributed-feedback laser and yet has the additional advantage of wavelength selectivity in a much broader parametric bandwidth. © 2005 Optical Society of America

1. INTRODUCTION

A typical distributed-feedback (DFB) laser has periodic dielectric modulation in a laser gain medium. Laser oscillation in a DFB laser is established from distributed optical feedback in a dielectric grating along the laser axis. A semiconductor DFB laser does not require resonator mirrors and can be fabricated with ease from microlithographic techniques. Among all the advantages, the most notable characteristic of a DFB laser is perhaps its mode-selectivity, in addition to having the compact size and simplicity of a DFB laser. An optical parametric oscillator (OPO) is one major step toward this goal. Although implementing reliable dielectric modulation in a bulk or waveguide nonlinear crystal is still under investigation, the demonstrated conversion efficiency in a bulk DFB OPO has been comparable to an ordinary OPO with a similar material and pump configuration. With appropriate dielectric modulation in a nonlinear optical material, a DFB OPO has the advantage of both wavelength selectivity and mode selectivity, in addition to having the compact size and simplicity of a DFB laser. An optical parametric process requires phase matching among the pump, signal, and idler waves. In a DFB OPO, one of the two output waves, defined to be the signal wave in this paper, is further phase matched to the Bragg resonance of the dielectric grating. Because of a symmetric distributed optical feedback, the resonant wave or the signal wave in a DFB OPO travels in both forward and backward directions. Given a forward pump laser and two possible propagation directions for the idler wave, there are three possible phase-matching conditions, one for forward and two for backward optical parametric processes. The forward DFB optical parametric amplifier (OPA) and oscillator governed by a phase-matching condition among forward pump, signal, and idler waves have been discussed elsewhere. I present in this paper the analysis of backward DFB OPAs and OPOs with the forward pump phase matched to a backward signal and a forward idler or with the forward pump phase matched to a forward signal and a backward idler. The former is called a backward-signal OPA and OPO and the latter is called a backward-idler DFB OPA and OPO in this paper.

Without any structure feedback, a backward-wave oscillator builds up oscillation from an internal feedback loop formed by a forward-propagating pump and a backward-propagating wave. For a backward-traveling wave tube or a gyrotron backward-wave oscillator, the forward pump is an electron beam and the backward signal is a microwave. For a backward-wave OPO, the forward components include a pump laser and a low-frequency output wave, and the backward component is another low-frequency output wave. Difference-frequency generation between the forward pump wave and the lower-frequency backward wave provides an optical feedback to the backward wave; in turn, different frequency generation between the forward pump and the backward wave also provides an optical feedback to the lower-frequency forward wave. Because of this internal feedback loop, a backward-wave oscillator does not rely on electromagnetic feedbacks from physical boundaries and permits continuous frequency tuning through the phasematching bandwidth. With dielectric modulation in a nonlinear gain material, both the internal and the structure feedbacks can affect the oscillation conditions in a backward DFB OPO. As shown in this paper, the DFB structure provides another feedback path and therefore additional flexibility for threshold and mode selections in such an oscillator.

This paper is organized as follows. In Section 2 we derive the coupled-wave equation for the mixing waves in a dielectric-modulated OPA or OPO. In Section 3 we show the solutions to the coupled-wave equation for a lossless
backward DFB OPA. In Section 4 we discuss the threshold condition and the mode formation of a lossless backward DFB OPA subject to various boundary-reflection conditions. Because of the backward phase-matching condition, the wavelength of the backward wave is usually very different from the other two and could be in the absorption spectrum of a nonlinear optical material. Therefore we discuss in Section 5 the effect of optical loss in a backward DFB OPA or OPO. We finally propose an experiment and conclude this study in Section 6.

2. COUPLED-WAVE THEORY

In this paper all laser beams propagate and are phase matched along the DFB grating vector directions ±z. The forward and backward directions are defined in the +z and −z directions, respectively. In an optical parametric process the high-frequency output photon is usually called the signal output and the lower-frequency photon is called the idler output. In a DFB OPO, either of the two output waves could resonate in the DFB structure. To facilitate the discussion in this paper, we call the photon resonating in the DFB structure the signal and the other photon the idler. The polarization directions of the pump, signal, and idler waves are arranged properly according to the phase-matching requirement in a nonlinear optical material. Given a phase-matching condition and polarization arrangement, the effective nonlinear coefficient is denoted by \( d \). Figure 1 depicts the configuration of a forward-pumped DFB OPA or OPO, where \( \beta_0 \) is the Bragg wave vector of the signal wave and \( k_p \) and \( k_i \) are the pump and idler wave vectors, respectively. The wave-vector-matching diagram (a) corresponds to a forward optical parametric process, and (b) and (c) correspond to the backward-signal and backward-idler optical parametric processes, respectively. In a double-side pumped device, another set of three phase-matching diagrams can be obtained by reversing the direction of the z axis. The dielectric modulation has a fundamental period of \( L_\lambda \) in a nonlinear optical material of length \( L \). The dimensionless coordinate \( \bar{z}=z/L \) is introduced for subsequent analysis.

where the subscripts \( p, s, \) and \( i \) denote pump, signal, and idler, respectively; \( \omega \) is the angular frequency of the electromagnetic field; and \( E(z) \) is the complex amplitude of the electric field. For most OPAs and OPOs operating far away from degeneracy, the wavelengths of the mixing waves are fairly different. Here we assume that only the signal wavelength is close to the Bragg resonance and has both forward and backward propagation components, defined by

\[
E_s(z) = R(z) \exp(-j\beta_s) + S(z) \exp(j\beta_s),
\]

where \( \beta_0 = 2\pi/\lambda_0 \); \( \lambda_0 \) is the Bragg wavelength to be determined from the Bragg condition; and \( R(z) \) and \( S(z) \) are the slowly varying field envelopes of the forward and backward components of the signal wave, respectively.

The pump and idler waves, for instance, in an intracavity-pumped OPO, could still have both forward- and backward-propagating components. Therefore the complex amplitudes of the pump and idler waves have the general form

\[
E_{p,i}(z) = A^+_p(z) \exp(-jk_{p,i}) + A^-_{p,i}(z) \exp(jk_{p,i}),
\]

where \( A^\pm(z) \) is the slowly varying field envelope with the superscripts + and − denoting the forward- and backward-propagating directions, respectively; \( k=2\pi/\lambda \); and \( \lambda \) is the wavelength in an unperturbed nonlinear medium.

In a second-order nonlinear optical material with weak dielectric perturbation, the total mixing field \( E = E_p + E_s + E_i \) satisfies the wave equation driven by the nonlinear polarization \( P_{NL} = 2dE_sE_i^*P_{NL}(z,t) \), where \( d \) is the effective nonlinear coefficient and \( e_0 \) is the vacuum permittivity. Under the energy conservation condition \( \omega_p = \omega_s + \omega_i \), three independent sets of coupled-wave equations can be found in association with three different sets of phase-matching conditions. The simultaneous wave-vector-matching conditions \( 2\beta_0 - k_{p,i} = 0 \) and \( k_p - \beta_0 - k_i = 0 \) yield the coupled-wave equations for the forward DFB OPA and OPO. The simultaneous wave-vector-matching conditions \( 2\beta_0 - \lambda_0 \).
\[ \begin{align*}
\frac{dA^t_p(z)}{dz} &= j\kappa_p A^t_p(z)R(z) + \alpha_p A^s_p(z), \\
\frac{dA^t_s(z)}{dz} &= j\kappa_s A^t_s(z)S(z) - \alpha_s A^s_s(z), \\
\frac{dA^t_i(z)}{dz} &= j\kappa_i A^t_i(z)R(z) + \alpha_i A^s_i(z), \\
\frac{dA^s_p(z)}{dz} &= j\kappa_p A^s_p(z)S(z) - \alpha_p A^t_p(z), \\
\frac{dA^s_s(z)}{dz} &= j\kappa_s A^s_s(z)S(z) - \alpha_s A^t_s(z), \\
\frac{dA^s_i(z)}{dz} &= j\kappa_i A^s_i(z)S(z) + \alpha_i A^t_i(z).
\end{align*} \]

where \( \delta = (\beta^2 - \beta_0^2)/2\beta_0 - \beta - \beta_0 \) is the detuning parameter, \( \beta = \omega_0 n_1/c_0 \) is the effective propagation constant of the signal wave, \( c_0 \) is the speed of light in vacuum, \( \kappa = 2\pi n_l/\lambda_0 \) is the DFB coupling coefficient between the forward and backward signal waves, and \( \kappa_{p,s,i} = \alpha_{p,s,i}/(n_{p,s,i} c_0) \) are the coupling coefficients for the pump, signal, and idler waves. The Bragg condition \( 2\beta_0 - l\kappa_p = 0 \) defines the Bragg wave number \( \beta_0 \) or the Bragg resonant wavelength \( \lambda_0 = 2n_s \lambda_p / l \). The simultaneous wave-vector-matching conditions \( 2\beta_0 - l\kappa_p = 0 \) and \( k_p - \beta_0 + k_i = 0 \) give the coupled-wave equations for the backward-idler DFB OPA and OPO:

\[ \begin{align*}
\frac{dR(z)}{dz} &= - (j\delta + \alpha_s) R(z) - j\kappa_i A^s_s(z) A^t_i(z), \\
\frac{dS(z)}{dz} &= j\kappa R(z) + (j\delta + \alpha_s) S(z) + j\kappa_i A^s_s(z) A^t_i(z), \\
\frac{dA^t_p(z)}{dz} &= j\kappa_p A^t_p(z) S(z) - \alpha_p A^s_p(z), \\
\frac{dA^t_s(z)}{dz} &= - j\kappa_p A^t_s(z) R(z) + \alpha_p A^s_p(z), \\
\frac{dA^t_i(z)}{dz} &= j\kappa_i A^t_i(z) R(z) + \alpha_i A^s_i(z), \\
\frac{dA^s_p(z)}{dz} &= - j\kappa_i A^s_p(z) S(z) - \alpha_i A^t_i(z). \\
\end{align*} \]

From Eqs. (6) and (7) and the intensity expression \( I = nEE'/2\eta_0 \), where \( \eta_0 \) is the vacuum wave impedance, it is straightforward to show

\[ \begin{align*}
\frac{1}{\omega_s} \left( \frac{dI_R}{dz} + 2\alpha_i I_R \right) &= \left( \frac{dI_S}{dz} - 2\alpha_s I_S \right) \\
\frac{1}{\omega_p} \left( \frac{dI'_R}{dz} + 2\alpha_i I'_R \right) &= \left( \frac{dI'_S}{dz} - 2\alpha_s I'_S \right),
\end{align*} \]

where \( I_R \) and \( I_S \) are the forward and backward signal intensities, respectively. For a lossless system with \( \alpha_s = \alpha_p = 0 \), Eq. (8) is consistent with the well-known Manley–Rowe relation \(^9\) or photon number conservation for nonlinear frequency conversion.

In practice, single-side forward pumping is the most commonly adopted configuration for an OPA or OPO. To simplify the analysis, we further assume an undepleted forward pump wave. This assumption is valid for the following OPO mode analysis because the pump wave remains nearly unchanged before the device starts oscillating. Substituting \( A^t_1 = 0 \) and \( A^t_p(z) = A^t_s(z) = 0 \) into Eqs. (6) and (7), one obtains the simplified coupled-wave equation

\[ \begin{align*}
\frac{dR(z)}{dz} &= - j\delta - \alpha_s - j\kappa \left[ R(z) \right], \\
\frac{dS(z)}{dz} &= j\kappa \left[ j\delta + \alpha_s \right] \left[ S(z) \right], \\
\frac{dA^t_s(z)}{dz} &= j\kappa_i \left[ -\alpha_i \right] \left[ A^t_i(z) \right],
\end{align*} \]

for a backward-signal DFB OPA or OPO and

\[ \begin{align*}
\frac{dR(z)}{dz} &= j\kappa \left[ j\delta - \alpha_s \right] \left[ R(z) \right], \\
\frac{dS(z)}{dz} &= j\kappa \left[ j\delta + \alpha_s \right] \left[ S(z) \right], \\
\frac{dA^t_i(z)}{dz} &= j\kappa_i \left[ -\alpha_i \right] \left[ A^t_s(z) \right],
\end{align*} \]

for a backward-idler DFB OPA and OPO, where the dimensionless quantities \( \bar{z} = z/L, \bar{\delta} = \delta L, \bar{\kappa} = \kappa L, \bar{\alpha} = \alpha L \), and \( \bar{\kappa}_{p,s,i} = \kappa_{p,s,i} A^t_0 L \) are introduced. Therefore \( \bar{z} = 0 \) and 1 correspond to the locations of the entrance and exit faces of the device, respectively. Without parametric coupling, \( \bar{\kappa}_{p,s,i} = 0 \), Eqs. (9) and (10) are reduced to the coupled-wave equations for a DFB laser \(^{10}\) with a laser gain coefficient equal to \( -\alpha_c \). Without DFB coupling, \( \bar{\kappa} = 0 \), Eqs. (9) and (10) are reduced to the coupled-wave equations for an ordinary backward OPO. \(^{11}\)

It is interesting to note that Eqs. (9) and (10) are dual equations. In other words, replacing the symbols \([R(\bar{z}), S(\bar{z}), A^t_s(\bar{z}), A^t_i(\bar{z}), \bar{z}, \bar{\delta}, \bar{\kappa}, \bar{\alpha}, \bar{\kappa}_{p,s,i}, \bar{\kappa}_{p,s,i} \] with \([S(\bar{z}), R(\bar{z}), A^t_s(\bar{z}), d(1-\bar{z})] \) in Eq. (9) transforms Eq. (9) into Eq. (10). Given a set of device parameters, \( \bar{\kappa}, \bar{\kappa}_{p,s,i}, \bar{\kappa}_{p,s,i} \), the field-envelope solution to a backward-signal DFB OPA and OPO is the same as that of a backward-idler DFB OPA and OPO under the systematic interchange of the symbols. This duality property results from the fact that the internal feedback loop of a backward OPO is symmetric to the counterpropagating signal and idler waves, yielding the same threshold condition for the signal and idler waves [see Eqs. (17)].

For different device parameters, the performance of a backward OPA or OPO could be quite different. The device parameters are all functions of wavelength. For example, among the device parameters \( \bar{\kappa}, \bar{\kappa}_{p,s,i}, \bar{\kappa}_{p,s,i} \), the loss...
coefficient \( \tilde{\alpha} \) can strongly depend on wavelength, and the choice of the resonant wavelength in a DFB structure can result in different solutions from Eq. (9) or Eq. (10). In the following, I first analyze the gain and mode characteristics of a lossless backward-signal OPA and OPO based on Eq. (9) and infer the solution of a lossless backward-idler OPA and OPO from the concept of duality in Appendix A. Finally we study the influence of distributed loss in a backward DFB OPO governed by Eqs. (9) and (10).

For a lossless source material with \( \tilde{\alpha}=0 \), Eq. (9) can be recast into the matrix form

\[
\begin{bmatrix}
\frac{dR(\bar{z})}{d\bar{z}} \\
\frac{dS(\bar{z})}{d\bar{z}} \\
\frac{dA^*_i(\bar{z})}{d\bar{z}}
\end{bmatrix} = \begin{bmatrix}
-j\delta - j\bar{k} & 0 \\
 j\bar{k} & j\delta + j\kappa_i \\
 0 & j\kappa_i
\end{bmatrix} \begin{bmatrix}
R(\bar{z}) \\
S(\bar{z}) \\
A^*_i(\bar{z})
\end{bmatrix},
\]

(11)

where \( \kappa = 2\pi n L / \lambda_0 \). The characteristic equation of the system governed by Eq. (11) is given by

\[
D^3 + (\delta^2 + \bar{\kappa}^2 - \kappa^2)D + j\delta \bar{\kappa}^2 = 0,
\]

(12)

where \( \bar{\kappa} = (\kappa, \bar{\kappa})^{1/2} = (\kappa, \kappa)^{1/2} | A^*_i(0)| L = G L \) and \( \Gamma = \kappa, \bar{\kappa}^{1/2} | A^*_i(0)| \) is the parametric gain defined in an ordinary OPA or OPO.\(^{12}\) The eigenvalues of Eq. (11) or the roots of Eq. (12) have the form of \( -2j \text{Re}(D) \), \( jD_r, jD_i \), where \( D_r \) is a complex number. Given a set of design parameters, the specific values of the eigenvalues solved from Eq. (12) determine whether the mixing waves grow, attenuate, or oscillate in the DFB structure.

3. BACKWARD DISTRIBUTED-FEEDBACK OPTICAL PARAMETRIC AMPLIFIER

Usually it is difficult for a DFB laser to function as an amplifier because of source reflection near the Bragg wavelength. However, a DFB OPA can be realized by seeding the device with an idler wave. Without dielectric modulation or feedback mirrors, a backward OPA can start oscillation at a certain pump threshold. To investigate a backward DFB OPA at zero detuning, we first set \( \bar{z}=0 \) in Eq. (11) and solve the signal and idler waves subject to the initial condition \( R(0)=0, S(1)=0, \) and \( A^*_i(0) \). The solutions to Eq. (11), \( R(\bar{z}), S(\bar{z}), \) and \( A^*_i(\bar{z}) \), are divided into three sets according to the relative magnitude between \( \bar{k} \) and \( \bar{\kappa} \). In the low-gain limit \( \bar{k} < \bar{\kappa} \), one obtains

\[
R(\bar{z}) = A^*_i(0) \frac{-\bar{\kappa}^2}{\kappa_i (\bar{k}^2 - \bar{\kappa}^2)} \left\{ 1 - \frac{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)} \right\},
\]

(13a)

\[
S(\bar{z}) = A^*_i(0) \frac{j\bar{\kappa}^2}{\kappa_i (\bar{k}^2 - \bar{\kappa}^2)1/2} \frac{\sinh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)},
\]

(13b)

\[
A^*_i(\bar{z}) = A^*_i(0) \frac{\bar{\kappa}^2}{\bar{k}^2 - \bar{\kappa}^2} \left\{ \frac{\bar{\kappa}^2}{\bar{k}^2} - \frac{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)} \right\}.
\]

(13c)

This solution set does not contain any pole that makes the field amplitudes unbounded. Therefore, under the condition \( \bar{k} < \bar{\kappa} \), this device cannot function as an oscillator. Although this device is a backward-wave amplifier, some energy in the signal wave is coupled to the forward direction due to the dielectric modulation.

When the parametric gain is equal to the DFB coupling, \( \bar{k} = \bar{\kappa} \), the eigenvalues of the characteristic equation are degenerate, all equal to zero, and we obtain the solution set:

\[
R(\bar{z}) = A^*_i(0) \frac{\bar{k} \bar{k}}{\kappa_i (\bar{\kappa}^2 - \bar{k}^2/2 - \bar{z})},
\]

(14a)

\[
S(\bar{z}) = j A^*_i(0) \frac{\bar{k}_i (\bar{k} - 1)},
\]

(14b)

\[
A^*_i(\bar{z}) = A^*_i(0) \frac{-\bar{k}^2}{\bar{k}^2 - \bar{\kappa}^2} \left\{ 1 - \frac{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)} \right\},
\]

(14c)

Interestingly, if the DFB coupling is strong enough, \( \bar{k} > 2 \) in this solution set, it is possible to propagate more signal power in the forward direction than in the backward direction, despite the backward phase-matching condition. Again, there is no pole in Eqs. (14) and this device cannot function as an oscillator at \( \bar{k} = \bar{\kappa} \).

When the parametric gain is larger than the DFB coupling, \( \bar{k} > \bar{\kappa} \), the signal and idler fields become

\[
R(\bar{z}) = A^*_i(0) \frac{-\bar{k}^2}{\kappa_i (\bar{\kappa}^2 - \bar{k}^2)1/2} \left\{ 1 - \frac{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)} \right\},
\]

(15a)

\[
S(\bar{z}) = A^*_i(0) \frac{j \bar{k}^2}{\kappa_i (\bar{k}^2 - \bar{\kappa}^2)1/2} \frac{\sinh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)},
\]

(15b)

\[
A^*_i(\bar{z}) = A^*_i(0) \frac{-\bar{k}^2}{\bar{k}^2 - \bar{\kappa}^2} \left\{ \frac{\bar{k}^2}{\bar{k}^2} - \frac{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2(\bar{z} - 1))}{\cosh((\bar{k}^2 - \bar{\kappa}^2)1/2)} \right\}.
\]

(15c)

This solution set predicts oscillation modes with the threshold gain of the \( m \)th mode, \( \Gamma_{th,m} \), satisfying

\[
\frac{\bar{k}^2 - \bar{\kappa}^2}{\bar{k}^2 - \bar{\kappa}^2} = (2m - 1) \frac{\pi}{2}
\]

(16)

where \( m \) is a positive integer. Because the oscillation modes have a resonant frequency at Bragg resonance, we name those modes the Bragg modes in this paper. Without the DFB coupling, \( \bar{k} = 0 \), the solution set, Eqs. (15), is reduced to that of an ordinary backward OPO,\(^{11}\) given by

\[
R(\bar{z}) = 0,
\]

(17a)

\[
S(\bar{z}) = A^*_i(0) \frac{j \bar{k} \sin[\bar{\kappa}(\bar{z} - 1)]}{\kappa_i \cos(\bar{\kappa})},
\]

(17b)

\[
A^*_i(\bar{z}) = A^*_i(0) \frac{\cos[\bar{\kappa}(\bar{z} - 1)]}{\cos(\bar{\kappa})}.
\]

(17c)

From Eqs. (17), the threshold gain of the \( m \)th mode for a backward OPO is clearly \( \Gamma_{th,m} = (2m - 1) \pi/2 \). A symmetric
DFB oscillator, establishing oscillation from the structure feedback loop only, usually has a bandgap near the Bragg resonance $\delta = 0$ due to destructive optical phases at zero detuning. At $\delta = 0$ the effect of dielectric modulation in a backward DFB OPO does not completely remove the oscillation modes commonly seen in a backward OPO but increases the threshold of these modes. The threshold increase of the gap modes at $\delta = 0$ can be viewed as a consequence of the competition between the internal feedback loop and the structure feedback loop.

At the device output, $\bar{z} = 0$ for the backward signal wave or $\bar{z} = 1$ for the forward signal and idler waves, all three solution sets of Eqs. (13)–(15) satisfy the condition

$$\frac{I_g(1) + I_s(0)}{\omega_s} = \frac{I'_g(1) - I'_s(0)}{\omega_i},$$

as required by the Manley–Rowe relation [Eq. (8)]. Usually it is difficult to extract signal power in the backward direction from a backward-wave amplifier. A backward DFB optical parametric amplifier has the advantage of coupling some signal power in the forward direction. Figure 2 shows the ratio of the forward and backward signal intensities at the outputs of a backward-DFB OPA, $I_g(1)/I_s(0)$, as a function of the dimensionless gain $\bar{\Gamma}$ for the DFB coupling coefficients $\bar{\kappa} = 1, 2, 3$. The thin solid line marks the level at which the forward signal output power is equal to the backward signal output power. It can be seen from this plot that strong DFB coupling helps forward power extraction in a backward DFB OPA and weak DFB coupling gives a lower threshold gain for the Bragg mode in a backward DFB OPO. Given a finite DFB coupling coefficient, it is possible to produce more forward signal power than backward signal power in a backward-signal DFB OPA.

In the above we have limited our OPA analysis at the Bragg resonance $\delta = 0$. The analytic solutions at zero detuning are useful for comparison with some known properties of an ordinary backward OPO and a DFB laser. For $\delta = 0$, we show in the following that a backward DFB OPA and OPO can be high-gain and low-threshold coherent light sources. By using numerical techniques, we can solve Eq. (11) to obtain

$$\begin{bmatrix} R(1) \\ S(1) \\ A_1^+(1) \end{bmatrix} = B \begin{bmatrix} R(0) \\ S(0) \\ A_1^+(0) \end{bmatrix},$$

with $B = \begin{bmatrix} b_{11}(1) & b_{12}(1) & b_{13}(1) \\ b_{21}(1) & b_{22}(1) & b_{23}(1) \\ b_{31}(1) & b_{32}(1) & b_{33}(1) \end{bmatrix}$.

(19)

Under the OPA boundary condition, $R(0) = 0, S(1) = 0$, and $A_1^+(0)$, the output signal and idler waves are given by

$$R(1) = \frac{b_{12}(1) b_{13}(1)}{b_{22}(1)} A_1^+(0), S(0) = -\frac{b_{23}(1)}{b_{22}(1)} A_1^+(0),$$

$$A_1^+(1) = \frac{b_{22}(1) b_{23}(1)}{b_{23}(1)} A_1^+(0).$$

(20)

Figures 3(a)–3(c) are the contour plots of the OPA intensity amplification ratio $I'_g(1)/I'_s(0)$ in the $\bar{\Gamma} - \bar{\delta}$ plane for the DFB couplings $\bar{\kappa} = 1, 3$, and 5, respectively. The contours with labels from $a$ to $h$ show the $\bar{\Gamma}$ and $\bar{\delta}$ values corresponding to the amplification ratios $I'_g(1)/I'_s(0) = \sec^2(0.2), \sec^2(0.4), \sec^2(0.6), \sec^2(0.8), \sec^2(1.0), \sec^2(1.2), \sec^2(1.4)$, and $\sec^2(1.53)$, respectively. The intensity amplification ratio in an ordinary backward OPA is $I'_g(1)/I'_s(0) = \sec^2(\bar{\Gamma})$ from Eq. (17c), the contours relative to the horizontal lines $\bar{\Gamma} = (m - 1) \pi + (0.2, 0.4, \ldots, 1.53)$ give a qualitative gain comparison between a backward DFB OPA and an ordinary backward OPA. It is evident from the plots that a backward DFB OPA operating near $\bar{\delta} = 0$ and large $\bar{\kappa}$ is unfavorable in gain when compared with an ordinary backward OPA, as noted previously in a comparison of Eqs. (15) and (17). For $\bar{\kappa} = 1$ in Fig. 3(a), only the Bragg modes, marked by $h$ at $\bar{\delta} = 0$, are present in the range of the parametric gain $\bar{\Gamma} < 13$. As $\bar{\kappa}$ is increased to 3 in Fig. 3(b), the thresholds of the Bragg modes are increased according to Eq. (16), but detuned modes are created symmetrically about the $\bar{\delta} = 0$ line. When the DFB coupling coefficient $\bar{\kappa}$ is increased to 5 in Fig. 3(c), the thresholds of the Bragg modes become even higher, but those of the detuned modes are further reduced. It is interesting to see from Fig. 3 that increasing $\bar{\kappa}$ moves up the Bragg modes, branches off the oscillation modes from the $\bar{\delta} = 0$ line, lowers the threshold of the detuned modes, and introduces more abrupt changes to the amplification ratio in the $\bar{\Gamma}_{th} - \bar{\delta}$ plane. Because $\bar{\kappa}$ is an index of the DFB structure feedback strength, the detuned modes, which are gradually generated with increasing $\bar{\kappa}$, result primarily from the DFB structure feedback. In Section 4, however, we show the evidence that, in addition to the structure feedback loop, the internal feedback loop also has a significant influence on those detuned modes. In Fig. 3(c) the two lowest threshold modes at $\bar{\delta} = \pm 5.65$ have a threshold gain of $\bar{\Gamma}_{th} = 1.15$ that is...
smaller than that of the first oscillation mode of an ordinary backward OPO or \( \bar{\Gamma}_{th,1} = \pi/2 \). To achieve \( \bar{k} = 5 \), the index modulation of the DFB structure is approximately \( \Delta n_I \sim 10^{-4} \) for a 1-\( \mu \)m signal wavelength in a 1-cm OPO crystal. The mode distribution in Fig. 3 clearly shows a mode-dependent oscillation threshold. This property is known to be the primary cause of the single-frequency output from a DFB laser.

For comparison, we plot in Fig. 4 the intensity amplification ratio \( I_i I_i^* \) of a lossless single-side pumped forward DFB OPA with \( \bar{k} = 5 \). The contour values marked from \( a \) to \( h \) are the same as those given in Fig. 3. The two lowest threshold modes at \( \bar{\delta} = \pm 5.9 \) have a threshold value of \( \bar{\Gamma}_{th} = 1.34 \) that is slightly higher than that in Fig. 3(c). The range of the frequency detuning \( \bar{\delta} \) between the two lowest threshold modes defines the bandgap or the stop band of this device. It is evident from the plot that the parametric gain is relatively small in the bandgap, as the contours are bent upward near \( \bar{\delta} = 0 \) to access more pump power for a given OPA intensity amplification ratio \( I_i I_i^* \). As expected, no oscillation modes are found in the bandgap for a forward DFB OPO. It becomes clear that in a backward DFB OPO, the formation of the stop band from the dielectric modulation elevates the mode thresholds near \( \bar{\delta} = 0 \), whereas both the internal feedback loop and the distributed feedback from the dielectric modulation lower the threshold of those detuned modes. The interplay between the internal feedback and the structure feedback of a backward DFB OPO determines the overall mode distribution in Fig. 3.

**4. BACKWARD DISTRIBUTED-FEEDBACK OPTICAL PARAMETRIC OSCILLATOR**

In the above we have tried to identify the OPO mode formation from the singular points in the OPA gain contour plots in Figs. 3 and 4. According to Eqs. (20), the parametric gain becomes unbounded at the condition

\[ \Delta n_I \sim 10^{-4} \]
from which one can calculate the detuning frequency $\bar{\delta}$ and the threshold gain $\Gamma_{th}$ of a resonant mode. The matrix element $b_{22}(1)$ can be solved from the inverse Laplace transform,

$$b_{22}(z) = L^{-1}\left[\frac{\bar{s}(s+j\bar{\delta})}{s^2 + (\bar{\delta}^2 + \bar{\Gamma}^2 - \bar{\kappa}^2)s + j\bar{\delta}^2}\right],$$

evaluated at $\bar{z} = 1$, where $L^{-1}$ is the inverse Laplace transform operator and $s$ is the Laplace transform variable. Since only the parametric gain from which one can calculate the detuning frequency $\bar{\delta}$, the threshold gain $\Gamma_{th}$, the relation $b_{22}$ can be solved.

The above mode calculation does not include endface reflections from the device. To include signal reflections from the boundaries, we impose the conditions for steady-state resonance:

$$R(0) = r_1S(0) = \sqrt{R_1}\exp(j\phi_1)S(0) \quad \text{at} \quad \bar{z} = 0,$$

$$S(1) = r_2R(1) = \sqrt{R_2}\exp(j\phi_2)R(1) \quad \text{at} \quad \bar{z} = 1,$$

where the subscripts 1 and 2 denote the quantities associated with the upstream and downstream endfaces, respectively; $r$ is the reflection coefficient of the signal field; $R$ is the reflectance of the signal intensity; and $\phi$ is the reflection phase of the signal field. Assuming $A_r^* (0) = 0$ for self-started oscillation and substituting the boundary condition of Eqs. (23) into Eq. (19), one obtains the steady-state resonance condition with end reflections:

$$\frac{b_{22}(1)\sqrt{R_1}\exp(j\phi_1) + b_{22}(1)}{b_{21}(1)\sqrt{R_1}\exp(j\phi_1) + b_{22}(1)} = \sqrt{R_2}\exp(j\phi_2).$$

For the case of $R_{1,2} = 0$, Eq. (24) is reduced to the resonance condition without end reflections $|b_{22}(1)| = 0$. In a DFB laser, the end reflections have profound influence on the mode threshold and the mode frequency because the longitudinal modes are a consequence of longitudinal boundary conditions. The symmetry of the mode locations with respect to Bragg resonance $\bar{\delta} = 0$ can also be modified by the optical reflections. While keeping $\phi_1 = 0, r_2 = 0$, and $\bar{\kappa} = 5$, we show in Fig. 5 the mode locations in the $\Gamma_{th}-\bar{\delta}$ plane for $R_1 = 0$, 12.4%, 50%, and 99%. $R_1 = 12.4\%$ is the optical reflectance at a lithium niobate crystal surface in air. The arrows in Fig. 5 indicate the moving directions of the resonant modes when $R_1$ is varied from 0 to 99%. The above mode calculation does not include endface reflections from the device. To include signal reflections from the boundaries, we impose the conditions for steady-state resonance:

$$R(0) = r_1S(0) = \sqrt{R_1}\exp(j\phi_1)S(0) \quad \text{at} \quad \bar{z} = 0,$$

$$S(1) = r_2R(1) = \sqrt{R_2}\exp(j\phi_2)R(1) \quad \text{at} \quad \bar{z} = 1,$$

where the subscripts 1 and 2 denote the quantities associated with the upstream and downstream endfaces, respectively; $r$ is the reflection coefficient of the signal field; $R$ is the reflectance of the signal intensity; and $\phi$ is the reflection phase of the signal field. Assuming $A_r^* (0) = 0$ for self-started oscillation and substituting the boundary condition of Eqs. (23) into Eq. (19), one obtains the steady-state resonance condition with end reflections:

$$\frac{b_{22}(1)\sqrt{R_1}\exp(j\phi_1) + b_{22}(1)}{b_{21}(1)\sqrt{R_1}\exp(j\phi_1) + b_{22}(1)} = \sqrt{R_2}\exp(j\phi_2).$$

For the case of $R_{1,2} = 0$, Eq. (24) is reduced to the resonance condition without end reflections $|b_{22}(1)| = 0$. In a DFB laser, the end reflections have profound influence on the mode threshold and the mode frequency because the longitudinal modes are a consequence of longitudinal boundary conditions. The symmetry of the mode locations with respect to Bragg resonance $\bar{\delta} = 0$ can also be modified by the optical reflections. While keeping $\phi_1 = 0, r_2 = 0$, and $\bar{\kappa} = 5$, we show in Fig. 5 the mode locations in the $\Gamma_{th}-\bar{\delta}$ plane for $R_1 = 0$, 12.4%, 50%, and 99%. $R_1 = 12.4\%$ is the optical reflectance at a lithium niobate crystal surface in air. The arrows in Fig. 5 indicate the moving directions of the resonant modes when $R_1$ is varied from 0 to 99%. Because of the additional feedback from the endfaces, the mode locations in Fig. 5 are no longer symmetric with respect to the zero detuning line. If the additional feedback phase does not favor the formation of a particular mode (for example, the Bragg mode), the mode threshold grows higher because of the reflection feedback. However, with a correct phase, a small end reflection can greatly reduce the mode threshold.

From Eqs. (11) and (19), it is straightforward to obtain the relation $b_{12}(\bar{z}) = -b_{21}(\bar{z})$. From Eq. (24), $|b_{21}(1) r_1 + b_{22}(1)| = 0$ for $r_2 = 0$ and $|b_{22}(1) - r_2 b_{12}(1)| = 0$ for $r_1 = 0$ are symmetric in $r_1$ and $r_2$ under the substitution $b_{12} = -b_{21}$. Consequently, the plot in Fig. 5 is also valid for the boundary condition $\phi_2 = 0, r_1 = 0$, and $R_2 = 0, 12.4\%, 50\%, 99\%$. This property clearly results from a DFB structure that is symmetric to the forward and backward signal waves.

It can be seen from Fig. 5 that the boundary reflectance provides only limited tuning to a resonant mode. In a resonator having longitudinal modes, the resonance frequency of one mode can be tuned to the next one by varying the longitudinal phase from 0 to $2\pi$. For example, introducing a longitudinal phase of $\pi/2$ can create a gap mode at the Bragg resonance in both a DFB laser and a forward DFB OPO. However, a backward DFB OPO behaves differently because of the interplay between the internal feedback loop and the structure feedback loop. Figure 6 shows the mode loci of a backward-signal DFB OPO in the $\Gamma_{th}-\bar{\delta}$ plane with $\bar{\kappa} = 5, R_2 = 0, R_1 = 12.4\%$, and a
variable $\phi_t$. Surprisingly the continuous longitudinal mode-tuning property commonly seen in a DFB laser or a forward DFB OPO does not appear in a backward DFB OPO. Varying the longitudinal phase $\phi_t$ only moves each resonance mode over a localized area in the $\Gamma_d-\delta$ plane. Each phase tuning curve does not connect one mode to another but forms a close loop for each oscillation mode over a $2\pi$ phase change in the longitudinal phase $\phi_t$. This phenomenon implies that the detuned modes are not pure longitudinal modes formed from the DFB structure feedback loop. Not only the Bragg modes but also the detuned modes are strongly influenced by the internal feedback loop in the counterpropagating waves. Therefore, compared with a forward DFB OPO or a DFB laser, a backward DFB OPO is less sensitive to boundary reflections and has better mode stability.

A homogeneously gain-broadened optical oscillator builds up single-mode oscillation from the lowest threshold mode. Because of the symmetry breaking in Figs. 5 and 6, a backward DFB OPO retains the mode selectivity or single-mode oscillation property of a DFB laser.

5. EFFECT OF DISTRIBUTED LOSS

All the backward DFB OPA and OPO analysis carried out so far was based on the lossless backward-signal coupled-wave equation [Eq. (11)]. The physics of a backward-idler DFB OPA or OPO is the same as that of a backward-signal OPA or OPO under the duality concept. However, when optical loss is present in the device, the choice of the resonance wavelength can significantly alter a device's performance. Like an ordinary backward OPO, a backward-signal DFB OPO has the frequency relation between the signal wave and the pump wave, $\omega_s/\omega_p=(n_i-n_p)/(n_i+n_p)$, derived from the energy and momentum conservations. For a backward DFB OPO, the frequency relation becomes $\omega_s/\omega_p=(n_i-n_p)/(n_i+n_p)$. For most materials, the consequence of the frequency conditions implies $\omega_s<\omega_p, \omega_s$ for a backward-signal DFB OPO and $\omega_s<\omega_p, \omega_s$ for a backward-idler DFB OPO. In other words, the wavelength of the backward wave is usually much longer than those of the other two mixing waves. Because of the vast difference in the mixing wavelengths, the long-wavelength component in a backward DFB OPO could have significant optical loss in the nonlinear optical material. In the following we consider a nontrivial attenuation coefficient of $\bar{\alpha}_s$ for the long-wavelength component and zero loss for the short-wavelength components in two possible backward DFB OPO configurations.

If the long-wavelength output photon is in resonance with the DFB structure, the configuration is a backward-idler DFB OPO, and from Eq. (10) the coupled-wave equation is given by

$$\begin{bmatrix}
    dR(\bar{z})/d\bar{z} \\
    dS(\bar{z})/d\bar{z} \\
    dA_i^+ (\bar{z})/d\bar{z}
\end{bmatrix} =
\begin{bmatrix}
    -j\delta - j\bar{\alpha}_L & -j\bar{\kappa}_S & 0 \\
    j\bar{\kappa}_L & j\delta + j\bar{\alpha}_L & j\bar{\kappa}_S \\
    0 & j\bar{\kappa}_S & 0
\end{bmatrix} \begin{bmatrix}
    R(\bar{z}) \\
    S(\bar{z}) \\
    A_i^+ (\bar{z})
\end{bmatrix}, \quad (25)
$$

subject to $k_p-\beta_0S+k_L=0$, where the subscripts $S$ and $L$ denote parameters associated with the short-wavelength and long-wavelength output photons, respectively. If the short-wavelength output photon is in resonance with the DFB structure, the configuration is a backward-idler DFB OPO, and from Eq. (10) the coupled-wave equation is given by

$$\begin{bmatrix}
    dR(\bar{z})/d\bar{z} \\
    dS(\bar{z})/d\bar{z} \\
    dA_i^+(\bar{z})/d\bar{z}
\end{bmatrix} =
\begin{bmatrix}
    -j\delta - j\bar{\kappa}_S & -j\bar{\kappa}_S & 0 \\
    j\bar{\kappa}_S & j\delta & 0 \\
    0 & -1/\bar{\alpha}_L & 0
\end{bmatrix} \begin{bmatrix}
    R(\bar{z}) \\
    S(\bar{z}) \\
    A_i^+(\bar{z})
\end{bmatrix}, \quad (26)
$$

subject to $k_p-\beta_0S+k_L=0$. With the condition $\omega_s=\omega_p+\omega_i$, the relation $\Gamma=(\bar{\kappa}_L\bar{\kappa}_S)^{1/2}=\left(\bar{\kappa}_L\bar{\kappa}_S\right)^{1/2}$ holds for a given material. The quasi-phase-matching technique can be employed to obtain phase matching for either the case of Eq. (25) or of Eq. (26).

Given the OPA boundary condition, $R(0)=0$, $S(1)=0$, and $A_i^+(0)$, and a DFB coupling coefficient $\bar{\kappa}=5$, Figs. 7(a) and 7(b) show contours of the OPA intensity amplification ratio $I_i^+ (1)/I_i^+(0)$ with $\bar{\alpha}_s=1$ in Eqs. (25) and (26), respectively. By assuming small loss modulation $\Delta \alpha_t$.

Fig. 7. Intensity amplification ratio contours $I_i^+ (1)/I_i^+(0)$ with $\bar{\kappa} =5$ and $\bar{\alpha}_s=1$ for (a) a lossy backward-signal DFB OPA governed by Eq. (25) and (b) a lossy backward-idler DFB OPA governed by Eq. (26). The labeling of the contour values is consistent with that in Fig. 3. The signal loss in both the forward and the backward directions in Eq. (25), compared with the idler loss in only the backward direction in Eq. (26), results in a lower parametric gain and higher oscillation threshold for a backward-signal OPO with long-wavelength loss.
<2\pi\Delta n_l/\lambda_0, we kept only the real part of \kappa=2\pi\Delta n_l/\lambda_0−j\Delta \alpha_l, the gain and mode dynamics of a backward DFB OPA and OPO are apparently more complicated. Compared with Fig. 3(c), Fig. 7 clearly shows reduction in the OPA intensity amplification ratio for a given pump intensity in \Gamma. The thresholds of the resonant modes also become higher in Fig. 7. In Fig. 7(a) the signal loss \tilde{\alpha}_l associated with Eq. (25) tends to smooth out the OPA gain distribution in the \Gamma−\delta plane and slow down the division of the Bragg modes into detuned modes. In Fig. 7(b) the idler loss \tilde{\alpha}_l associated with Eq. (26) tends to localize the OPA gain and isolate the newly formed detuned modes in the \Gamma−\delta plane. The first oscillation mode in Fig. 7(b) can be seen to have a threshold lower than that in Fig. 7(a). Since the abrupt intensity increase near an oscillation mode is a characteristic of a DFB oscillator and a low oscillation threshold is always desirable for an oscillator, it is therefore apparent that the device configuration associated with Eq. (26) is a preferred design for a backward DFB OPO having long-wavelength loss. The low intensity gain or the high oscillation threshold resulting from Eq. (25) is a consequence of the optical loss to the signal amplitudes in both forward and backward directions in the DFB structure. On the other hand, the optical loss in Eq. (26) is applied to the idler amplitude in only the backward direction. The device configuration associated with Eq. (25), although having a higher pump threshold, could still be useful for applications requiring a broad amplifier bandwidth.

6. DISCUSSION AND CONCLUSION

Material dispersion for most known materials imposes a severe constraint on the mixing frequencies \omega_o<<\omega_p, \omega_i for a backward-signal DFB OPO or \omega_o<<\omega_p, \omega_i for a backward-idler DFB OPO. A possible experiment to demonstrate a backward DFB OPO is to employ the periodically poled lithium niobate (PPLN) crystal.\textsuperscript{15} For a 1064-nm pumped backward OPO in a 40-\mu m period PPLN crystal, the forward and backward wavelengths can be 1068 nm and 300 \mu m, respectively, at 25°C. The refractive index of lithium niobate\textsuperscript{16} at 1068-nm wavelength is 2.16 and that at 300 \mu m is \sim5. For \i=1 in the Bragg condition 2\beta_0−lk_p=0, the DFB structure period is 247 nm to oscillate the 1068-nm wavelength and is 30 \mu m to oscillate the 300-\mu m wavelength. In lithium niobate, the Bragg grating can be implemented by the photorefractive, electro-optic, or acousto-optic technique.\textsuperscript{2} At a 300-\mu m wavelength, an electromagnetic wave is strongly absorbed in lithium niobate with a field attenuation coefficient\textsuperscript{17} of 4.1 cm\textsuperscript{−1}. Therefore a DFB period of 247 nm to oscillate the 1068 nm is the preferred design for this backward DFB OPO. Using \kappa=2.7 scaled from the demonstrated photorefractive DFB grating in Ref. 1, we calculated a threshold gain of \Gamma=2.7 or a threshold intensity of 100 MW/cm\textsuperscript{2} for a 1-cm-long PPLN crystal. As a comparison, a threshold intensity of 130 MW/cm\textsuperscript{2} is required to achieve backward oscillation in the same PPLN crystal without a DFB grating. Apparently, the threshold intensity can be further reduced by use of a longer crystal.

For terahertz nonlinear frequency mixing in lithium niobate, noncollinear phase matching has been a popular configuration in which the near-infrared output is oscillated at an angle with respect to the near-infrared pump direction.\textsuperscript{18} By using a DFB structure in a nonlinear optical material, one is able to collinearly oscillate either the near-infrared signal or the terahertz wave to maximize the parametric gain length.

Previously the perfect phase-matching conditions 2\beta_0−lk_p=0 and \beta_p±\beta_0+k_i=0 were both assumed to obtain the stationary solutions above. In practice, mode detuning \delta could cause a k vector mismatch \Delta k=\overrightarrow{\beta}_p±\overrightarrow{\beta}_0+k_i in the parametric process, and a finite bandwidth is imposed to the range of frequency detuning. In general, a finite bandwidth will further affirm the mode-dependent oscillation threshold derived above because an oscillation mode outside the system bandwidth has an even higher threshold. On the other hand, as long as the amount of mode detuning \delta is well within the system bandwidth, the assumption that \Delta k=0 is insensitive to \delta is still valid. The system bandwidth is a combination of intrinsic parametric bandwidth, pump bandwidth, DFB grating bandwidth, and material bandwidth. In the following, we use the proposed backward-idler DFB OPO experiment as an example and show how \Delta k=0 can be satisfied in a typical experiment.

If mode detuning \delta causes a signal frequency shift of \Delta \omega_o from the Bragg resonance, the corresponding shifts in the idler and pump frequencies \Delta \omega_i and \Delta \omega_p, respectively, satisfy the expression \Delta \omega_o+\Delta \omega_i=\Delta \omega_p. The resulting k vector mismatch due to the frequency shifts in a backward-idler DFB OPO is

\[ \Delta k = n_p \Delta \omega_o/c_0 - \pi \Delta \lambda_p/\lambda_p^2 + n_i \Delta \omega_i/c_0. \]  

(27)

Suppose that the pump is a single-frequency source \Delta \omega_p =0 and the mode frequency is detuned by approximately one free spectral range \Delta \omega_o = 2\pi c/2n_pL from the Bragg resonance. From Eq. (27), the DFB grating bandwidth required to ensure \Delta k=0 is

\[ \Delta \lambda_p/\lambda_p = (n_p/n_s)(\Delta \lambda_p/L) = 5.6 \times 10^{-5}, \]  

(28)

with \lambda_p=0.25 \mu m, L=1 \text{ cm}, and n_s=2.2 and n_i=5 for the proposed experiment in lithium niobate. \Delta \lambda_p\sim1.4 \times 10^{-2} \text{ nm} yielded from Eq. (28) is probably smaller than any dimension fluctuation in a microfabrication process, and the intrinsic DFB grating bandwidth should be large enough to ensure \Delta k=0. In addition, for lithium niobate near 298 K, the DFB grating period varies with temperature T according to\textsuperscript{19} \Delta \lambda_p/\lambda_p = -1.1 \times 10^{-5}(T-298), which allows temperature adjustment within a degree to keep \Delta k=0 for a crystal length of L=5 \text{ cm}.

The pump laser for an OPO usually has multiple longitudinal modes. Assume that a perfect DFB grating is made in a nonlinear optical material and \Delta \lambda_p=0 in Eq. (27). It is straightforward to show that the following two equations have to be satisfied to ensure \Delta k=0:

\[ \Delta \omega_i = -\frac{n_p}{n_i+n_p} \Delta \omega_o, \]  

(29)
\[ \Delta \omega_p = \frac{n_i}{n_i + n_p} \Delta \omega_s. \]  
(30)

Because the detuned signal frequency \( \Delta \omega_s \) is only approximately a free spectral range and \( n_i \gg n_p \approx n_s \), \( \Delta \omega_s \) and \( \Delta \omega_p \) in Eqs. (29) and (30) can be well within the material bandwidth and the pump bandwidth, respectively.

In summary, without pump depletion a backward-signal and a backward-idler DFB OPA and OPO are governed by dual coupled-wave equations. The field-envelope solutions of one OPA and OPO can be readily obtained from the solutions of the other by a systematic variable substitution. Therefore the mode and threshold properties of a backward-signal DFB OPA and OPO can be inferred from those of a backward-idler DFB OPA and OPO and vice versa. In a lossy material, choosing the absorbing wavelength as the idler wavelength gives a higher parametric gain and a lower oscillation threshold in a backward DFB OPA and OPO.

Like a forward DFB OPA, a backward DFB OPA can be seeded by an idler wave to generate an amplified idler wave from parametric amplification and a signal wave from difference-frequency generation. This optical amplification scheme does not have the source feedback problem associated with an ordinary DFB laser amplifier. As expected in a DFB structure, the parametric gain can be greatly enhanced near resonant-mode frequencies. Despite the backward phase-matching condition, the signal power in a backward DFB OPA can flow primarily in the forward direction with some proper design parameters, as shown by Fig. 2.

A DFB laser has a mode-dependent oscillation threshold and is capable of single-frequency oscillation within the laser gain bandwidth. A backward-wave DFB OPO also retains the mode selection property of a DFB laser and yet has the advantage of wavelength selectivity in a much broader parametric bandwidth. Because of the internal feedback loop, a backward DFB OPO has oscillation modes at Bragg resonance, in addition to those discrete ones at some detuned frequencies. The forming of a stop band in an ordinary DFB structure does not completely remove the Bragg modes in a backward DFB OPO but increases their thresholds. However, the resonant modes of a backward DFB OPO are not simply the superposition of those of an ordinary backward OPO and the longitudinal modes of a DFB oscillator. The most striking result is revealed in Fig. 6, where the longitudinal phase at a boundary face has only a limited frequency-tuning effect on the detuned oscillation modes. On the contrary, a forward DFB OPO or a DFB laser can be tuned continuously in mode frequencies by varying the longitudinal phase. Therefore a backward DFB OPO is fundamentally different from a backward OPO and a forward DFB OPO.

The primary focus of this study is on the amplification gain of idler-seeded backward DFB OPAs and the mode characteristics of backward DFB OPOs. Future work includes the study of pump-depleted and double-side pumped backward DFB OPAs and OPOs.

APPENDIX A

We show in this appendix some field-envelope solutions to Eq. (10) in comparison with the solutions derived from Eq. (9). In this comparison, we limit the discussion to a lossless source material and thus \( \alpha = 0 \). Under the OPA boundary condition, \( R(0) = 0 \), \( S(1) = 0 \), and \( A_{m}^{\pm}(1) \), the solutions \( R(\xi), S(\xi) \), and \( A_{m}^{\pm}(\xi) \) to Eq. (10) can likewise be divided into three sets according to the relative magnitude between \( \tilde{k} \) and \( \tilde{\Gamma} \). In the low-gain limit, \( \tilde{\Gamma} \ll \tilde{k} \), we obtained

\[
R(\xi) = A_{m}^{\pm}(1) - \frac{\tilde{\Gamma}^2}{\tilde{k}^2} \left\{ \frac{1 - \cosh[(\tilde{k}^2 - \tilde{\Gamma}^2)\xi/2]}{\cosh[(\tilde{k}^2 - \tilde{\Gamma}^2)\xi/2]} \right\}, \tag{A1a}
\]

\[
S(\xi) = A_{m}^{\pm}(1) - \frac{\tilde{\Gamma}^2}{\tilde{k}^2} \frac{\sinh[(\tilde{k}^2 - \tilde{\Gamma}^2)\xi/2]}{\cosh[(\tilde{k}^2 - \tilde{\Gamma}^2)\xi/2]}, \tag{A1b}
\]

\[
A_{m}^{\pm}(\xi) = A_{m}^{\pm}(0) - \frac{\tilde{\Gamma}^2}{\tilde{k}^2} \left\{ \frac{\cosh[(\tilde{k}^2 - \tilde{\Gamma}^2)\xi/2]}{\cosh[(\tilde{k}^2 - \tilde{\Gamma}^2)\xi/2]} \right\}. \tag{A1c}
\]

This set of solutions gives parametric amplification only. When the parametric gain is equal to the DFB coupling, \( \tilde{\Gamma} = \tilde{k} \), we obtained the solutions

\[
R(\xi) = -jA_{m}^{\pm}(1)\tilde{k}\xi, \tag{A2a}
\]

\[
S(\xi) = -A_{m}^{\pm}(1)\tilde{k}\xi/2 + A_{m}^{\pm}(1)\tilde{k}\xi/2, \tag{A2b}
\]

\[
A_{m}^{\pm}(\xi) = A_{m}^{\pm}(0)(1 + \tilde{k}\xi/2) - A_{m}^{\pm}(1)\tilde{k}\xi/2. \tag{A2c}
\]

Again, at \( \tilde{\Gamma} = \tilde{k} \) this device functions only as a parametric amplifier. With the parametric gain larger than the DFB coupling, \( \tilde{\Gamma} > \tilde{k} \), the signal and idler fields become

\[
R(\xi) = -jA_{m}^{\pm}(1)\tilde{k}\xi, \tag{A3a}
\]

\[
S(\xi) = -A_{m}^{\pm}(1)\tilde{k}\xi/2 + A_{m}^{\pm}(1)\tilde{k}\xi/2, \tag{A3b}
\]

\[
A_{m}^{\pm}(\xi) = -A_{m}^{\pm}(0)\tilde{k}\xi/2 - A_{m}^{\pm}(1)\tilde{k}\xi/2. \tag{A3c}
\]

This set of solutions predicts oscillation modes with the threshold gain of the nth mode, \( \tilde{\Gamma}_{th,n} \), satisfying \( (\tilde{\Gamma}_{th,n}^2 - \tilde{k}^2)^{1/2} = (2m - 1)\pi/2 \). It is straightforward to show that Eqs. (A1)–(A3) can be obtained by replacing the symbols \( R(\xi), S(\xi), A_{m}^{\pm}(\xi), A_{m}^{\pm}(0) \) in Eqs. (13)–(15) with \( [S(\xi), R(\xi), A_{m}^{\pm}(\xi), A_{m}^{\pm}(0), \xi] \) in Eqs. (A1)–(A3) with \( \bar{S}(\xi), \bar{R}(\xi), A_{m}^{\pm}(\xi), A_{m}^{\pm}(0) \). Equations (A1) can also be solved numerically to obtain

\[
\begin{bmatrix}
R(1) \\
S(1) \\
A_{m}^{\pm}(1)
\end{bmatrix} = \mathbf{B}
\begin{bmatrix}
R(0) \\
S(0) \\
A_{m}^{\pm}(0)
\end{bmatrix}, \text{with } \mathbf{B} =
\begin{bmatrix}
b_{11}(1) & b_{12}(1) & b_{13}(1) \\
b_{21}(1) & b_{22}(1) & b_{23}(1) \\
b_{31}(1) & b_{32}(1) & b_{33}(1)
\end{bmatrix}.
\tag{A4}
\]

The backward transmittance of the idler wave is
\[
\left| \frac{A_i^{-}(0)}{A_i^{-}(1)} \right|^2 = \left| \frac{b_{22}(1)}{b_{22}(1) b_{33}(1)} \right|^2 \quad \text{for } R(0) = 0 \text{ and } S(1) = 0.
\]

Therefore the resonance condition for a backward-idler DFB OPO is given by

\[
\left| \frac{b_{22}(1) b_{33}(1)}{b_{22}(1) b_{33}(1)} \right| = 0.
\]

We numerically compared the mode locations governed by Eqs. (21) and (A6) in the \( \bar{\delta} - \bar{\Gamma}_{th} \) plane and found that both gave the same answers for a specified \( \bar{\pi} \) value.

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