Spin filtering and scaling of spin-dependent potentials in quasi-one-dimensional electron liquids with Rashba spin-orbit interaction

N.-Y. Lue* and G. Y. Wu*1,7
Department of Electrical Engineering, National Tsing-Hua University, Hsin-Chu, Taiwan, Republic of China
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We investigate theoretically the spin-filtering effect in a quasi-one-dimensional (Q1D) electron liquid with spin-orbit interaction. The Q1D system considered is formed from a two-dimensional electron-gas (2DEG) subject to both a lateral confining potential and an interface potential perpendicular to the 2DEG. Spin and charge degrees of freedom in the system are mixed by the interface potential through the Rashba mechanism [A. V. Moroz and C. H. W. Barnes, Phys. Rev. B 60, 14272 (1999)]] and we show that when a spin-dependent \( \delta \) potential is further introduced into the system, for example, via implantation of magnetic/ferromagnetic impurities, the mixing leads to the spin-filtering effect which favors electrons with a certain spin orientation to transport through the \( \delta \) potential. In particular, we calculate the scaling dimension of electron scattering both by spin-flip and by spin-independent \( \delta \) potentials when the temperature is varied and show that, in the spin-flip case, the scaling of electron scattering with temperature varies with spin orientation. Conductance is calculated for both spin and charge transport, and the spin-filtering effect is discussed quantitatively in terms of the conductance.

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I. INTRODUCTION

Spintronics deals with the processing of polarized spin instead of charge signals and offers an innovative approach to technological applications including quantum computation.1–4 Various spintronic devices have been proposed based on Fermi-liquid (FL) systems in two or three dimensions.4 But with the advanced technology in fabrication of quantum wires, spintronic devices have also been suggested which are specifically based on the unique properties of quasi-one-dimensional (Q1D) systems.5–11

One distinct class of states in 1D considered for spintronic applications is known as the Tomonaga-Luttinger or simply Luttinger liquid (LL) and can be investigated within the framework of Tomonaga-Luttinger theory.12–22 It has a unique low-energy excitation spectrum drastically different from that of FL. Specifically, the spectrum is linear, of bosonic nature, and consists of two branches describing separately excitations in association with spin or charge fluctuations. As a result of the unique spectrum, LL shows peculiar electronic properties, e.g., electronic density of states with power-law energy dependence. In the case of a spinless LL system, for example, the density of states \( D(E) \propto E^{1/K_\rho} \), where \( K_\rho \) is the so-called Luttinger parameter specifying the electron-electron interaction, with \( K_\rho=1 \) for repulsive interaction, \( K_\rho>1 \) for a noninteracting system and \( K_\rho<1 \) for attractive interaction (we focus on the repulsive case throughout the work). In particular, \( D(E) \) vanishes at the Fermi point as opposed to that of FL which is constant near the Fermi surface. This power-law characteristic of LL also shows up in electrical transport phenomena, via the dependence of transport on the density of states.18,19 For example, in the case where a strong impurity potential is present, it leads to the ohmic tunneling conductance \( G \propto T^{2/K_\rho-2} \), which scales with \( T \) (temperature) and, in particular, vanishes at \( T=0 \). Similarly, in the case where the impurity strength is weak, renormalization-group analysis shows that the effective impurity strength \( V_{e//} \) is a function of temperature which also scales according to the power law

\[
V_{0}(\lambda/k_BT)^{1-K_\rho},
\]

where \( \lambda \) is the upper energy cutoff of the linear region of excitation spectrum. The ohmic conductance in this case is

\[
G = G_0 - O(1)(V_0/\lambda)^2(\lambda/k_BT)^{2-2K_\rho},
\]

where \( G_0=K_\rho e^2/2\pi \ (h=1) \) is the conductance of LL free of impurities.

We are primarily interested, for spintronic applications, in the possibility of spin filtering which utilizes the above scaling property of conductance/impurity potential to produce spin polarization. It has been proposed that spin filtering can be achieved with the LL implanted with a nonmagnetic impurity.7–10 It is shown that in the presence of a magnetic field, spin and charge degrees of freedom are mixed by the associated Zeeman effect and the mixing causes the exponent in Eqs. (1) and (2) for charge transport to become spin dependent, e.g., with new parameters \( K_\uparrow \) and \( K_\downarrow \) replacing \( K_\rho \) for up and down electrons, respectively. Generally, \( K_\uparrow \neq K_\downarrow \) and, according to the equations, electrons of opposite spins thus see different impurity strength, which leads naturally to different conductance for up- and down-electron currents and gives rise to the effect of spin polarization.

In the forgoing proposal of spin-filtering devices, the magnetic field is indispensable and the presence of it satisfies two conditions important for the realization of spin filtering. First, it mixes spin and charge through the Zeeman effect. Second, it also breaks the time-reversal symmetry of the system in order to produce spin polarization. An interesting question arises as to whether it is possible to replace the magnetic field with one of electrical nature. A clue to the foregoing question has indeed been given in the important work of Moroz and Barnes.23 They have shown that when a Q1D system is formed from a two-dimensional electron-gas...
(2DEG) subject to both a lateral confining potential and an interface potential perpendicular to the 2DEG, the Rashba mechanism of spin-orbit interaction due to the interface potential leads to an asymmetric single-particle dispersion and as a result of the asymmetry, it gives rise to the required spin-charge mixing. Thus, the first condition, i.e., spin-charge mixing, for spin filtering is satisfied there without requiring the presence of any magnetic field. From the viewpoint of device implementation, it simplifies the design of spin filters. A further advantage with their system is that a gate voltage may be applied to adjust the interface potential and the required spin-charge mixing. It offers an electrical means of controlling spin filtering as opposed to the magnetic means in the original proposal. On the other hand, it is important to note that the Moroz-Barnes mechanism of mixing alone does not produce spin polarization since the time-reversal symmetry remains unbroken in the Rashba spin-orbit interaction-induced mixing. In order to break the symmetry and satisfy the second condition for spin filtering, a magnetic/ferromagnetic impurity is therefore introduced, in our work, into the system.

In summary, the foregoing reasoning motivates us to study the Q1D system similar to that of Moroz and Barnes but with the addition of a magnetic/ferromagnetic impurity into the system. In particular, we shall calculate the scaling dimension of a weak, spin-dependent impurity potential and show that the potential scales with temperature with different exponents for electrons of opposite spins. It thus provides a mechanism for spin filtering.

Before we present the calculation, we mention that Q1D systems showing strong Rashba spin-orbit interaction have also been proposed as asymmetric spin-polarization filters but the devices are based on a different principle. It requires a magnetic field to be applied along the wire creating a Zeeman splitting of energy bands and opening a gap for spin filtering. We also note that, in addition to spin-charge mixing, spin-orbit coupling can renormalize Luttinger parameters or even affect the phase diagram of a Q1D system. In the present work, we shall assume that the system remains in the LL phase and the Luttinger parameters used in our calculation are already renormalized.

The paper is organized as follows. In Sec. II, we present the calculation of scaling dimension of a spin-flip δ potential which models the spin-flip part of a magnetic/ferromagnetic impurity. The non-spin-flip part is modeled by a spin-independent δ potential and the calculation of corresponding scaling dimension is presented in Sec. III. In Sec. IV, we derive the conductance for both spin and charge transport and discuss the spin-filtering effect quantitatively in terms of the conductance. In Sec. V, we summarize the study.

II. SCALING DIMENSION OF A SPIN-FLIP δ POTENTIAL

In Sec. II A, we discuss bosonization, the bosonized form of a spin-flip δ potential, and the path-integral formalism for our calculation. In Sec. II B, we derive the scaling dimension of the potential.

A. Bosonization, spin-flip δ potential, and path-integral representation of correlators

1. Bosonization

For a Q1D system of electrons, the low-energy spectrum of the system is linear (up to an energy Λ) and consists of only (bosonic) density fluctuations. Following the standard procedure of bosonization in the low-energy sector (see Ref. 22, for example), we express the electron field operator in terms of the fluctuations as follows. We write

\[ \Psi_s(y) = e^{i k_F y} R_s(y) + e^{-i k_F y} L_s(y), \quad s = \uparrow, \downarrow, \]

\[ R_s(y) = \frac{1}{\sqrt{2\pi a}} e^{i \phi_{R,s}(y)} \quad L_s(y) = \frac{1}{\sqrt{2\pi a}} e^{-i \phi_{L,s}(y)}, \]

where we take the Q1D wire to be in the y direction, \( k_F \) is the Fermi wave vector, \( R_s(y) \) and \( L_s(y) \) are the right and left movers, and \( a \) is a length parameter determined by the upper wave vector cutoff of the linear, low-energy spectrum. The phase fields \( \phi_{R,s}(y) \) and \( \phi_{L,s}(y) \) are basically the accumulated fluctuating right- and left-moving charges with spin \( s \), respectively,

\[ \varphi_{R(s),L}(y) = 2\pi \int_0^y \delta \rho_{R(s),L}(y') dy'. \]

The above gives a transformation from the fermion field \( \Psi_s(y) \) to the boson fields \( \{ \varphi_{R,s}(y), \varphi_{L,s}(y) \} \). In order to describe separately charge- and spin-fluctuation sectors (denoted below by \( \rho \) and \( \sigma \), respectively), linear combinations of \( \varphi_{R,s}(y) \) and \( \varphi_{L,s}(y) \) are formed, with

\[ \varphi_\rho(y) = \frac{1}{2\sqrt{2}} [\varphi_{R,\uparrow}(y) + \varphi_{L,\uparrow}(y) + \varphi_{R,\downarrow}(y) + \varphi_{L,\downarrow}(y)], \]

\[ \theta_\rho(y) = \frac{1}{2\sqrt{2}} [\varphi_{R,\uparrow}(y) - \varphi_{L,\uparrow}(y) + \varphi_{R,\downarrow}(y) - \varphi_{L,\downarrow}(y)], \]

\[ \varphi_\sigma(y) = \frac{1}{2\sqrt{2}} [\varphi_{R,\uparrow}(y) + \varphi_{L,\uparrow}(y) - \varphi_{R,\downarrow}(y) - \varphi_{L,\downarrow}(y)], \]

\[ \theta_\sigma(y) = \frac{1}{2\sqrt{2}} [\varphi_{R,\uparrow}(y) - \varphi_{L,\uparrow}(y) - \varphi_{R,\downarrow}(y) + \varphi_{L,\downarrow}(y)], \]

\[ \Pi_\rho(y) = \frac{1}{\pi} \partial_y \theta_\rho(y), \quad \Pi_\sigma(y) = \frac{1}{\pi} \partial_y \theta_\sigma(y). \]

These fields have the following interpretation. Within a multiplicative constant, \( \partial \varphi_\rho(y)/\partial y \) and \( \Pi_\rho(y) \) (or \( \partial_\theta_\rho(y) \)) are the fluctuating charge and charge current densities, respectively. Similarly, \( \partial \varphi_\sigma(y)/\partial y \) and \( \Pi_\sigma(y) \) (or \( \partial_\theta_\sigma(y) \)) are basically the spin and spin current densities, respectively. With the above boson fields, one can transform the Hamiltonian density of a Q1D system written in terms of fermion fields to one in terms of boson fields, e.g.,
h_{0}(\varphi_{\rho},\varphi_{\sigma},\Pi_{\rho},\Pi_{\sigma}) = \text{Hamiltonian density}

\begin{align*}
&= \frac{1}{2\pi} \left\{ v_{\rho}\mathcal{K}_{\rho}(\pi\Pi_{\rho})^{2} + v_{\sigma}\mathcal{K}_{\sigma}(\partial_{y}\varphi_{\sigma})^{2} + v_{\sigma}\mathcal{K}_{\sigma}(\partial_{y}\varphi_{\sigma})^{2}ight\},
&+ v_{\rho}\mathcal{K}_{\rho}(\partial_{y}\varphi_{\rho})^{2},
\end{align*}

where \( K_{\rho} \) and \( K_{\sigma} \) are the Luttinger parameters, and \( v_{\rho} \) and \( v_{\sigma} \) are the velocities of charge and spin excitations, respectively. These parameters depend on the electron-electron coupling constants \( g_{1,\rho}, g_{2,\rho}, g_{4,\rho}, g_{4,\sigma}, \) and \( g_{4,\sigma}, \) with

\begin{align*}
v_{\rho} &= v_{0}\left\{ 1 + \frac{g_{4,\rho} + g_{4,\sigma}}{2\pi v_{0}} \right\}^{2} - \frac{\left( g_{2,\rho} + g_{2,\sigma} - g_{1,\rho} \right)^{2}}{2\pi v_{0}},
K_{\rho} &= \sqrt{\frac{2\pi v_{0} + g_{4,\rho} + g_{4,\sigma} - g_{2,\rho} - g_{2,\sigma} + g_{1,\rho}}{2\pi v_{0} + g_{4,\rho} + g_{4,\sigma} + g_{2,\rho} + g_{2,\sigma} - g_{1,\rho}}},
K_{\sigma} &= \sqrt{\frac{2\pi v_{0} + g_{4,\rho} - g_{4,\rho} - g_{2,\rho} + g_{2,\rho} - g_{1,\rho}}{2\pi v_{0} + g_{4,\rho} - g_{4,\rho} + g_{2,\rho} - g_{2,\rho} - g_{1,\rho}}},
\end{align*}

With the bosonized Hamiltonian density, one can study the dynamics of the system in terms of that of a boson gas, valid up to the energy \( \Lambda \), the upper cutoff of linear region in the low-energy spectrum of the system.

In our case, we consider the Q1D system of length \( L_{0} \), as shown in Fig. 1, formed from a 2DEG subject to both a lateral confining potential and an interface potential perpendicular to the 2DEG. The 2DEG is taken to lie on the \( y-z \) plane with the \( x \) direction being normal to the 2DEG and the \( y \) direction being along the Q1D wire. The single-particle dispersion of the system is asymmetric due to the interface potential-induced Rashba spin-orbit interaction. Correspondingly, this modifies the bosonized Hamiltonian density. Let \( v_{0} \) be the average and \( \delta \theta \) be the difference between the Fermi velocities of left and right branches of the dispersion. We have the following bosonized Hamiltonian density:  

\[ h(\varphi_{\rho},\varphi_{\sigma},\Pi_{\rho},\Pi_{\sigma}) = h_{0}(\varphi_{\rho},\varphi_{\sigma},\Pi_{\rho},\Pi_{\sigma}) + \frac{\delta\theta}{2\pi} (\pi\Pi_{\rho} + (\partial_{y}\varphi_{\rho})(\pi\Pi_{\rho}) \]
where \( Z_0 \) is the partition function, \( (\Pi_\rho, \Pi'_\rho) \) are the canonical conjugates of \((\varphi_\rho, \theta_\rho)\), and \( S_E \) is the action in the Matsubara formalism given below

\[
S_E[\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho] = \int_0^\beta d\tau \int dy(l(\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho)|_{\tau=t})
\]

with \( \beta=1/k_BT \) being basically the inverse temperature. Note that, in Eq. (5), \((\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho)\) are chosen as integration variables instead of \((\varphi_\rho, \varphi_\rho, \varphi_\rho, \varphi_\rho)\). In fact, it follows from the “charge-current” duality\(^{17}\) that the path-integral representation of \( \Pi_\rho \) can be described in terms of the “charge” fields \((\varphi_\rho, \varphi_\rho)\), the “current” fields \((\theta_\rho, \theta_\rho)\), or the hybrid sets \((\varphi_\rho, \theta_\rho)\) or \((\theta_\rho, \varphi_\rho)\), and their corresponding canonical conjugates. The reason we choose the hybrid \((\varphi_\rho, \theta_\rho)\) is that the correlators involve the fields \((\varphi_\rho, \varphi_\rho)\) instead of \((\varphi_\rho, \varphi_\rho)\) and may hence be calculated more conveniently with \((\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho)\) as the integration variables. For this reason we shall now derive the Lagrangian density \( l(\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho) \) which appears in the integral of Eq. (5).

We shall first convert the Hamiltonian density \( h(\varphi_\rho, \varphi_\rho, \Pi_\rho, \Pi'_\rho) \) in Eq. (3) to \( h(\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho) \). We make the transformation from \((\varphi_\rho, \Pi_\rho)\) to the representation \((\theta_\rho, \Pi'_\rho)\) (with the substitution \( \partial_\rho \varphi_\rho \rightarrow \pi \Pi'_\rho \) and \( \pi \Pi_\rho \rightarrow \partial_\rho \theta_\rho \)) in the spin part of \( h_\rho \), and \( h \) becomes

\[
h(\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho) = \frac{1}{2\pi} \{ \nu_\rho K_\rho(\pi \Pi'_\rho)^2 + \frac{\nu_\rho}{K_\rho} (\partial_\rho \varphi_\rho)^2
\]

\[
+ \nu_\rho K_\rho (\partial_\rho \varphi_\rho)^2 + \frac{\nu_\rho}{K_\rho} (\pi \Pi_\rho)^2 \} + \frac{\delta \theta_\rho}{2\pi} (\partial_\rho \varphi_\rho)
\]

\[
\times (\partial_\rho \theta_\rho) + (\pi \Pi'_\rho)(\pi \Pi_\rho).
\]

It yields the following Lagrangian density:

\[
l(\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho) = \Pi_\rho \partial_\rho \varphi_\rho + \Pi'_\rho \partial_\rho \theta_\rho - h(\varphi_\rho, \theta_\rho, \Pi_\rho, \Pi'_\rho)
\]

\[
= \Pi_\rho \partial_\rho \varphi_\rho + \Pi'_\rho \partial_\rho \theta_\rho - \frac{1}{2\pi} \{ \nu_\rho K_\rho(\pi \Pi'_\rho)^2
\]

\[
+ \nu_\rho K_\rho (\partial_\rho \varphi_\rho)^2 + \nu_\rho K_\rho (\partial_\rho \varphi_\rho)^2 + \frac{\nu_\rho}{K_\rho} (\pi \Pi_\rho)^2 \}
\]

\[
+ \frac{\delta \theta_\rho}{2\pi} (\partial_\rho \varphi_\rho)(\partial_\rho \theta_\rho) + (\pi \Pi'_\rho)(\pi \Pi_\rho).
\]

This is the Lagrangian density which enters the action \( S_E \) in Eq. (5). However, we can further simplify it as follows. We note that the field variables \( \Pi_\rho \) and \( \Pi'_\rho \) in Eq. (5) can immediately be integrated out and it reduces to

\[
\frac{1}{Z_0} \int D\varphi_\rho D\theta_\rho \cos(\sqrt{2}(\varphi_\rho + \theta_\rho)|_{y=0, T_n})
\]

\[
\times \cos(\sqrt{2}(\varphi_\rho + \theta_\rho)|_{y=0, T_n}) e^{\int dy dy' d\tau [l(\varphi_\rho, \theta_\rho)|_{\tau=t} - l(\varphi_\rho, \theta_\rho)|_{\tau=t-\tau}]
\]

now with the Lagrangian density dependent only on \((\varphi_\rho, \theta_\rho)\). \( l(\varphi_\rho, \theta_\rho) \) is given in terms of matrices

\[
l(\varphi_\rho, \theta_\rho)|_{\tau=t} = \frac{1}{2\pi} \int dy' d\tau' (\varphi_\rho(y, \tau) \theta_\rho(y, \tau)) g_\rho^{-1}(\varphi_\rho(y', \tau') \theta_\rho(y', \tau'))
\]

\[
\times (y-y', \tau-\tau') (\varphi_\rho(y', \tau') \theta_\rho(y', \tau'))
\]

where the matrix elements of \( g_\rho^{-1} \) are listed below

\[
(g_\rho^{-1})_{11} = \delta(y-y') \delta(\tau-\tau') [\varphi_{\rho}(\partial_\rho \partial_\rho) + b_\rho (\partial_\rho \partial_\rho)]
\]

\[
(g_\rho^{-1})_{22} = \delta(y-y') \delta(\tau-\tau') [\varphi_{\rho}(\partial_\rho \partial_\rho) + b_\rho (\partial_\rho \partial_\rho)]
\]

\[
(g_\rho^{-1})_{12} = (g_\rho^{-1})_{21} = \delta(y-y') \delta(\tau-\tau') [-c_\rho (\partial_\rho \partial_\rho) - d_\rho (\partial_\rho \partial_\rho)]
\]

\[
(\sigma_\rho)^2 = \frac{1}{4} \int dy dy' d\tau d\tau' (\varphi_\rho(y, \tau) \theta_\rho(y, \tau))
\]

(7)

**B. Scaling dimension**

We determine the scaling dimension of the spin-flip potential as follows. We write the binary correlators

\[
\langle \cos(\sqrt{2}(\varphi_\rho + \theta_\rho)|_{y=0, T_n}) \cos(\sqrt{2}(\varphi_\rho + \theta_\rho)|_{y=0, T_n}) \rangle
\]

\[
= \sum_{n,m=1,2} C_{n,m}^{-2}
\]

where

\[
C_{n,m}^{-2} = \frac{1}{Z_0} \int D\varphi_\rho D\theta_\rho e^{\int dy dy' d\tau [l(\varphi_\rho, \theta_\rho)|_{\tau=t} - l(\varphi_\rho, \theta_\rho)|_{\tau=t-\tau}]
\]

\[
\times (y-y', \tau-\tau') (\varphi_\rho(y', \tau') \theta_\rho(y', \tau'))
\]

(8)

where \( g_\rho^{-1} = (g_\rho^{-1})_{11} + 2(g_\rho^{-1})_{12} + (g_\rho^{-1})_{22} \). In order to study its
scaling behavior, we adopt the following approach. We transform $g_{S}(z)$ from $(y, \tau)$ space to $(q, w)$ space and obtain

$$g_{S}(z)(q, w) = N^{-2}(q^{2}, w^{2})/D(q^{2}, w^{2}),$$

where the numerator,

$$N^{-2}(q^{2}, w^{2}) = -[v_{p}K_{p} + v_{q}K_{q}] \pm \delta v^{2}w^{2}$$

and the denominator,

$$D(q^{2}, w^{2}) = w^{2} + v_{p}^{2} + (\delta v^{2}/2)w^{2} + v_{q}^{2}K_{q}^{2} - (\delta v^{2}/4)q^{2}.$$

Being a quadratic function of $w^{2}$, $D(q^{2}, w^{2})$ can be factorized as

$$D(q^{2}, w^{2}) = (w^{2} + v_{p}^{2})/(w^{2} + v_{p}^{2}),$$

with

$$v_{z}^{2} = \frac{1}{2} \left[ (v_{p}^{2} + v_{q}^{2} + (\delta v^{2})/2) \right] \pm \sqrt{\left[ (v_{p}^{2} + v_{q}^{2} + (\delta v^{2})/2) \right]^{2} - 4 \left[ v_{p}v_{q} + v_{p}K_{q}^{2} - (\delta v^{2})/4 \right]}.$$
\[
\langle \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_1}} \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_2}} \rangle \\
= \frac{1}{Z_0} \int D\varphi_{\rho} D\varphi_{\sigma} D\Pi_{\rho} D\Pi_{\sigma} \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_1}} \\
\times \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_2}} e^{S_L[\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma}]} \tag{14}
\]

Here we have
\[
S_L[\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma}] = \int_0^\beta d\tau \int dy l(\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma})|_{it \to \tau}.
\]

Starting from the Hamiltonian density \( h(\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma}) \) in Eq. (3), the Lagrangian density is
\[
l(\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma}) = \Pi_{\rho} \partial_\tau \varphi_{\rho} + \Pi_{\sigma} \partial_\tau \varphi_{\sigma} - h(\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma})
\]
\[
= \Pi_{\rho} \partial_\tau \varphi_{\rho} + \Pi_{\sigma} \partial_\tau \varphi_{\sigma} - \frac{1}{2\pi} \left( v_{\rho} K_{\rho} (\Pi_{\rho})^2 + v_{\sigma} K_{\sigma} (\Pi_{\sigma})^2 + \frac{v_{\rho}}{K_{\rho}} (\partial_\tau \varphi_{\rho})^2 + \frac{v_{\sigma}}{K_{\sigma}} (\partial_\tau \varphi_{\sigma})^2 \right)
\]
\[
- \frac{\delta_0}{2\pi} \left( (\partial_\tau \varphi_{\rho})(\Pi_{\rho}) + (\partial_\tau \varphi_{\sigma})(\Pi_{\sigma}) \right).
\]

Integrating out \( \Pi_{\rho} \) and \( \Pi_{\sigma} \) in Eq. (14), we obtain
\[
\langle \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_1}} \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_2}} \rangle \\
= \frac{1}{Z_0} \int D\varphi_{\rho} D\varphi_{\sigma} \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_1}} \\
\times \cos[\sqrt{2}(\varphi_{\rho} + \varphi_{\sigma})]_{|_{\rho = 0, \tau_2}} e^{S_L[\varphi_{\rho}, \varphi_{\sigma}, \Pi_{\rho}, \Pi_{\sigma}]}|_{it \to \tau},
\]
where
\[
K = \frac{v_{\rho} K_{\rho} + v_{\sigma} K_{\sigma} + (v_{\rho} K_{\rho} v_{\sigma}^2 + v_{\sigma} K_{\sigma} v_{\rho}^2 - (v_{\rho} K_{\rho} + v_{\sigma} K_{\sigma})(\delta_0)^2/4)}{v_{\rho} + v_{\sigma}}
\]

for the nonspin-flip scattering. We note that the scaling dimension of a spin-independent \( \delta \) potential has previously been calculated by Moroz et al. However, the result in Eq. (16) is expressed in the form which treats \( K_{\rho} \) and \( K_{\sigma} \) as independent parameters of the model while that of Moroz et al. is specifically written for the case where the Luttinger parameters \( K_{\rho} \) and \( K_{\sigma} \) are related, e.g., \( K_{\rho}^2 + K_{\sigma}^2 = 2 \). Such a relation follows the assumption in their LL model that electron-electron interactions are of pointlike density-density type only.

As Eqs. (12) and (16) show, for numerical calculation of the scaling dimension (and spin polarization in this work), numerical values of the following parameters are required, namely, \( K_{\rho} \), \( K_{\sigma} \), \( u_{\rho} = \frac{v_{\rho}}{v_0} \), \( u_{\sigma} = \frac{v_{\sigma}}{v_0} \), and the velocity asymmetry \( \varepsilon = \frac{v_{\rho}}{v_{\sigma}} \), where \( v_0 \) is the average of Fermi velocities of left and right branches of the energy bands. The numerical value of \( K_{\rho} \) is discussed in Ref. 27, where it is shown that \( K_{\rho} > 0.5 \) for a wide range of semiconductor quantum wire width and \( K_{\rho} < 0.5 \) for systems with strong electron-electron repulsion. We consider the range of \( K_{\rho} \approx 0.6 \) in the calculation to show the trend of scaling and spin polarization. As for the value of \( K_{\sigma} \), both \( K_{\sigma} > 1 \) and \( K_{\sigma} < 1 \) have been mentioned in the literature. We shall therefore consider both of the possibilities in the calculation. \( u_{\rho} \) and \( u_{\sigma} \) are taken to be 1/\( K_{\rho} \) and 1/\( K_{\sigma} \), respectively, or, equivalently, with \( v_{\rho} K_{\rho} = v_{\sigma} K_{\sigma} = v_0 \), which holds in the special case of pointlike density-density-type interactions. Last, the velocity asymmetry \( \varepsilon \) is estimated to be about 0.1–0.2 according to Ref. 23. Numerical results of scaling dimension based on Eqs. (12) and (16) are listed below for two distinct cases, one with \( K_{\rho} > 1 \) and the other with \( K_{\sigma} < 1 \).

(a) \((K_{\rho}, K_{\sigma}, u_{\rho}, u_{\sigma}) = (0.6, 1.1, \frac{1}{0.6}, \frac{1}{1.1})\). For \( \varepsilon = 0.1 \), scaling dimension 0.2547 \((R \uparrow \leftrightarrow L\downarrow)\); 0.237 \((R \downarrow \leftrightarrow L\uparrow)\); and
IV. CONDUCTANCE AND SPIN FILTERING

Next, we use the Kubo formula to calculate both the charge and spin conductance, as given below.

\[ \sigma_{\alpha} = \lim_{\omega \to 0} \frac{-e^2}{\omega} \]

\[ \times \left[ \frac{1}{L_0} \int_{-L_0/2}^{L_0/2} dy \left( j_{\alpha}(y = 0, -w) j_{\alpha}(y, w) \right) \right] \text{,} \]

with \( j_{\alpha} \) and \( j_{\sigma} \) being charge and spin current densities, respectively, and, in terms of the boson fields, can be represented as

\[ j_{\rho} = \frac{\sqrt{2}}{\pi} \partial_\rho \varphi_\rho, \]

\[ j_{\sigma} = \frac{\sqrt{2}}{\pi} \partial_\sigma \varphi_\sigma \text{ or } \frac{\sqrt{2}}{\pi} v_{\alpha} K_\alpha \partial_\sigma \theta_{\sigma}. \]

With them, the spin conductance in Eq. (17) is written as

\[ G_{\alpha} = \lim_{\omega \to 0} \frac{2e^2}{\pi} v_{\alpha} K_\alpha \left[ \frac{1}{L_0} \int_{-L_0/2}^{L_0/2} dy \left( \partial_\sigma \theta_{\sigma} \right) \times (y, -w) \right]_{y = 0} \text{.} \]

The calculation of spin conductance therefore reduces to the evaluation of the following path integral:

\[ \langle \partial_\sigma \theta_{\alpha} \varphi_\rho \rangle = \frac{1}{Z} \int D\varphi_\rho D\theta_{\alpha} \int D\varphi_\rho e^{\int dtdy \gamma_\alpha'(\varphi_\rho(y, t))} S_\chi \text{.} \]

\[ Z = \int D\varphi_\rho D\theta_{\alpha} e^{\int dtdy \gamma_\alpha'(\varphi_\rho(y, t))} S_\chi \text{.} \]

Here, \( S_\chi \) is the action contributed by the impurity potential scattering, which includes both the spin-flip and nonspin-flip ones as given in Eqs. (4) and (13). In order to simplify the analysis and get semiquantitative insight, we assume we are in the so-called high-temperature/long-system regime where the characteristic energy \( k_B T > v_0 / L_0 \), which permits us to neglect the effect of electrode on the transport properties of LL here16,20,21 and calculate the integral perturbatively treating the impurity potential strength as the small parameter. Detailed calculations are presented in the Appendix. Obviously, the zeroth-order conductance \( G^{(0)}_{\sigma} \) (i.e., the conductance without the presence of any impurity scattering) is zero. It is found that the first nonzero correction \( \delta G^{(1)}_{\sigma} \) derives from the spin-flip \( \delta \)-potential \( \frac{\varphi_\rho}{m} \) and is of \( O(V_3^2) \).
Similarly, the charge conductance in Eq. (17) can be written as

\[ G_p = \lim_{\omega \to 0} \frac{2e^2}{\pi} i(\omega + i\delta) \times \left[ \frac{1}{L_0} \int_{-L_0/2}^{L_0/2} dy \langle \varphi_p(y, w) |_{w=0} \varphi_p(y_1, w) |_{w=\omega + i\delta} \rangle \right]. \] 

The calculation of charge conductance therefore reduces to the evaluation of path integral \( \langle \varphi_p | \varphi_p \rangle \), which is done perturbatively in the Appendix, too. The zeroth-order conductance without the presence of impurity scattering has previously been obtained and is given as \( G_p^{(0)} = \frac{K_p e^2}{\pi} \).

It is found that the correction to the conductance due to the nonspin-flip potential \( \frac{V_p}{m} \) is

\[ \delta G_p^{(C)} = -e^2 \left( \frac{V_p}{m} \right)^2 \frac{1}{(v_+ + v_-)^2} \times \left( v_p K_p + \frac{\gamma_s}{v_+ v_-} v_o K_o \right)^2 \]

\[ \times \left( \frac{\Gamma(K_p + K_o)}{\Gamma(1 + K_p + K_o)} \right) \left( \frac{2\pi}{\beta \lambda} \right)^{K_p + K_o - 2} \] 

\[ \times \left( \frac{\Gamma(K_p + K_o)}{\Gamma(1 + K_p + K_o)} \right) \left( \frac{2\pi}{\beta \lambda} \right)^{K_p + K_o - 2} \] 

\[ \times \left( \frac{\Gamma(K_p + K_o)}{\Gamma(1 + K_p + K_o)} \right) \left( \frac{2\pi}{\beta \lambda} \right)^{K_p + K_o - 2} \] 

\[ \times \left( \frac{\Gamma(K_p + K_o)}{\Gamma(1 + K_p + K_o)} \right) \left( \frac{2\pi}{\beta \lambda} \right)^{K_p + K_o - 2} \] 

The polarization as a function of \( K_p \) and \( K_o \) is plotted in Fig. 2. In this numerical calculation, we take \( e=0.1 \), \( T=10^{-3} \), \( v_0=0.01 \), \( m=0.3 \), and \( \frac{\kappa}{\kappa} = 10^{-3} \). \( v_p K_p = v_o = v_0 \) is also assumed in the calculation with Eqs. (19), (21a), and (21b), although the equations themselves are free from this assumption. The results with \( K_p=0.9 \), 1.0, and 1.1 are shown in Fig. 3. It shows that the polarization increases with increasing \( K_p \) and decreases \( K_o \), and the maximum polarization reaches 19%. In Fig. 4, we set \( e=0.2 \) while keeping all the other parameters the same as those used in Fig. 2. The results with \( K_p=0.9 \), 1.0, and 1.1 are again shown in Fig. 5. It shows that the maximum polarization exceeds 40%. Comparing Figs. 2 and 3 with Figs. 4 and 5, we see that the spin polarization increases with the velocity.
FIG. 4. (Color online) Polarization as a function of $K_p$ and $K_n$. We set $\epsilon=0.2$ and keep the rest of parameters the same as those used in Fig. 2.

asymmetry $\epsilon$. Equation (20) shows that the polarization depends on $\epsilon$ through both the scaling exponent of temperature and the prefactors of power function of temperature. In other words, a change in $\epsilon$ causes not only a corresponding variation in scaling dimensions but it also affects the polarization through the coefficients. In particular, when $\epsilon=0$, $\delta G_{sp}^{(5)}=0$ and the polarization disappears, as Eq. (20) shows. Figure 6 shows the effect of temperature on polarization. Because spin-up and spin-down currents scale differently with negative powers of temperature [as shown in Eq. (12)], the polarization is enhanced when the temperature is lowered. Finally, Fig. 7 shows the variation in polarization with the impurity strength $V_S$ and $V_C$. To keep the calculation within the access of perturbation theory, we take $\frac{V_S}{\Delta_0}=0.3$ and $\frac{V_C}{\Delta_0}=0.3$. The figure shows that the polarization grows with increasing $V_S$ or $V_C$. The increase with $V_S$ is easy to understand since the spin-flip scattering gives rise to spin polarization. On the other hand, when $V_C$ increases, the charge conductance $G_p$ decreases and, therefore, the polarization $P=\frac{\delta G_{sp}^{(5)}}{G_p}$ increases, too.

FIG. 5. Polarization vs $K_p$. The three curves plotted are taken from the result presented in Fig. 4 with $K_n=0.9$, 1.0, and 1.1, respectively.

V. SUMMARY AND CONCLUSION

We have presented a theoretical study of the spin-filtering effect in an interacting Q1D system, which is formed from a constrained 2DEG with the Rashba effect and a magnetic impurity implanted in it. It is shown that the magnetic impurity strength alone scales differently for spin-up and spin-down electrons, with the difference enhanced as the characteristic energy scales down. In contrast, other devices previously suggested, such as those using the Rashba effect to form asymmetric spin filters, or those using nonmagnetic impurities, all need the presence of magnetic fields to break time-reversal symmetry while this is not required in our device with a magnetic impurity. Moreover, our study shows that, with the variation in parameters $(u_p, K_p, V_o, K_n, \epsilon)$ the scaling effect brings the system from the regime of nonspin-flip dominant scattering to one dominated by spin-flip scattering. Last, with the temperature lowering down to

FIG. 6. Polarization as a function of $T$. Three curves are plotted for $\epsilon=0.1$, 0.15, and 0.2, respectively, with $T$ ranging from $T=10^{-3}T_F$ to $T=10^{-2}T_F$, $K_p=0.6$, $K_n=1.1$, and $\frac{\alpha}{\Delta_0}=0.3$.

FIG. 7. (Color online) Polarization as a function of $V_S$ and $V_C$, with $\frac{V_S}{\Delta_0}=0.3$ and $\frac{V_C}{\Delta_0}=0.3$. We set $\epsilon=0.2$, $T=10^{-3}T_F$, $K_p=0.6$, and $K_n=1.1$. 

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$T = 10^{-3} T_F$ according to our conductance calculation, we expect the spin polarization to be in the range of 10% or more.

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**APPENDIX**

In this appendix, we discuss the perturbative calculation of path integrals appearing in the conductance. We treat the impurity potential strength (including both $V_z$ for the nonspin-flip scattering and $V_j$ for the spin-flip scattering) as the small expansion parameter.

1. Calculation of $\langle \delta, \theta_0 (y, -w) | v_0 \rho_0 (y, w) \rangle$

The calculation of $\langle \delta, \theta_0 (y, -w) | v_0 \rho_0 (y, w) \rangle$ is essential for the spin conductance in Eq. (18) and the integral is expanded as follows:

$$\langle \delta, \theta_0 (y, -w) | v_0 \rho_0 (y, w) \rangle = \int dq' \frac{dq}{2\pi} e^{iqy} \langle N_0 | (N_0 + N_1 + N_2) \rangle_{\tau_1} + O(V^2, v^3), \quad (A1)$$

where

$$N_0 = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$N_1 = \int d\tau_1 \sum_{j=1}^{4} V_{jz}(\tau_1),$$

$$N_2 = \frac{1}{4} \int d\tau_1 d\tau_2 \sum_{m=0}^{4} \sum_{j=1}^{4} V_{jz}^{(m)}(\tau_1 - \tau_2),$$

$$V_1 = V_2 = \frac{V}{\pi a}, \quad V_3 = V_4 = \frac{V_C}{\pi a},$$

$$n_{11}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{11}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_1} E_E},$$

$$n_{12}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{12}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_1} E_E},$$

$$n_{13}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{13}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_1} E_E},$$

$$n_{14}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{14}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_1} E_E},$$

$$n_{22}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{22}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_2} E_E},$$

$$n_{33}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{33}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_3} E_E},$$

$$n_{44}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} \theta_{p}(q',-w) \rho_{p}(q,w) e^{E_E},$$

$$z_{44}(\tau_1 - \tau_2) = \int D\phi_{p} D\theta_{p} e^{i\varphi_{p}(\tau_0)_{\eta_4} E_E}.$$
nonspin-flip scattering when \( j = 3 \) or \( 4 \). The action in the integrals is given as
\[
S_{E} = \int_{0}^{\beta} d\tau \int dy l(\varphi_{\mu}, \theta_{\mu})|_{\mu = \tau} \quad \text{(spin-flip scattering)},
\]
\[
S_{E} = \int_{0}^{\beta} d\tau \int dy l(\varphi_{\mu}, \varphi_{\nu})|_{\mu = \tau} \quad \text{(nonspin-flip scattering)}.
\]

The Lagrangian densities \( l(\varphi_{\mu}, \theta_{\mu}) \) and \( l(\varphi_{\mu}, \varphi_{\nu}) \) here are already discussed in Sec. II.

All the integrals in Eqs. (A2a) and (A2b) can be evaluated by the shifting
\[
\Phi(y, \tau) \to \Phi(y, \tau) + \int dy' d\tau' g(y, \tau; y', \tau') I_{j}^{\prime}(y', \tau'),
\]
where \( \Phi(y, \tau) = \pi_{j}(y, \tau) \) for the spin-flip scattering and \( \Phi(y, \tau) = \pi_{j}(y, \tau) \) for the nonspin-flip scattering. The results needed for the correction to the zeroth-order conductance are given below:

\[
j_{j}(\tau_{1} - \tau_{2}) = z_{j}^{(1)}(\tau_{1} - \tau_{2})
\]
\[
\times \left[ N_{0} - (g_{F_{j}})_{2}(q', -w)(g_{F_{j}})_{1}(q, w) \right]
\]
\[j = 1 - 4\] (A3a)

and

\[
z_{11}^{(1)}(\tau_{1} - \tau_{2}) = Z_{0} \left( \frac{2\pi}{\beta \Lambda} \right)^{K_{s}^{z}}
\]
\[
\times \left\{ 2 - 2 \cos \left[ \frac{2\pi}{\beta}(\tau_{1} - \tau_{2}) \right] \right\}^{-(K_{s}^{z} + K_{s}^{x})/2},
\]

\[
z_{22}^{(1)}(\tau_{1} - \tau_{2})
\]
\[
= Z_{0} \left( \frac{2\pi}{\beta \Lambda} \right)^{K_{s}^{x}} \left\{ 2 - 2 \cos \left[ \frac{2\pi}{\beta}(\tau_{1} - \tau_{2}) \right] \right\}^{-(K_{s}^{z} + K_{s}^{x})/2},
\]

\[
z_{33}^{(1)}(\tau_{1} - \tau_{2}) = z_{44}^{(1)}(\tau_{1} - \tau_{2})
\]
\[
= Z_{0} \left( \frac{2\pi}{\beta \Lambda} \right)^{K_{s}^{x}} \left\{ 2 - 2 \cos \left[ \frac{2\pi}{\beta}(\tau_{1} - \tau_{2}) \right] \right\}^{-(K_{s}^{z} + K_{s}^{x})/2},
\]

\[
J_{11} = J_{33} = \sqrt{2\pi} (e^{i\omega(t_{1} - t_{2})} - e^{-i\omega(t_{1} - t_{2})}) \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[
J_{22} = J_{44} = \sqrt{2\pi} (e^{i\omega(t_{1} - t_{2})} - e^{-i\omega(t_{1} - t_{2})}) \begin{pmatrix} 1 \\ -1 \end{pmatrix}.
\] (A3b)

In Eq. (A3a), \( (g_{F_{j}}) \) is a two-component vector and its subscript indicates the component of vector to be taken in the expression there. In particular, \( g \) is the Green’s function and when \( j = 1 \) or \( 2 \), we have

\[
g(q, w) = g_{g}(q, w)
\]
\[
= \begin{pmatrix} (g_{g})_{11} & (g_{g})_{12} \\ (g_{g})_{21} & (g_{g})_{22} \end{pmatrix}
\]
\[
= \frac{\gamma_{s}}{D(q^{2}, w^{2})} \begin{pmatrix} e_{s}w^{2} + f_{s}q^{2} & c_{s}w^{2} - d_{s}q^{2} \\ c_{s}w^{2} - d_{s}q^{2} & a_{s}w^{2} + b_{s}q^{2} \end{pmatrix}.
\] (A4a)

and when \( j = 3 \) or \( 4 \), we have

\[
g(q, w) = g_{c}(q, w)
\]
\[
= \begin{pmatrix} (g_{c})_{11} & (g_{c})_{12} \\ (g_{c})_{21} & (g_{c})_{22} \end{pmatrix}
\]
\[
= \frac{\gamma_{c}}{D(q^{2}, w^{2})} \begin{pmatrix} e_{c}w^{2} + f_{c}q^{2} & -d_{c}wq \\ -d_{c}wq & a_{c}w^{2} + b_{c}q^{2} \end{pmatrix}.
\] (A4b)

Here, \( \gamma_{c} = \nu_{c} \kappa \mu_{c} \kappa_{c} \) and definitions of the symbols \( a_{s} \sim f_{s} \) in \( g_{g}(q, w) \), \( a_{c} \sim f_{c} \) in \( g_{c}(q, w) \), \( \gamma_{c} \), and \( D(q^{2}, w^{2}) \) appearing above are given in Eqs. (6) and (15) in Sec. II.

Then, in the order of \( O(V_{2}^{2}) \), we have

\[
\langle \partial_{y} \theta_{\mu}(y, -w) \rangle_{y = 0} \langle \varphi_{\mu}(y, 1, w) \rangle
\]
\[
= -\frac{1}{Z_{0}} \int \frac{dq'}{2\pi} \frac{dq}{2\pi} e^{iq'y} \left\{ \frac{V_{S}}{4} - \frac{\nu_{a}}{2\pi} \right\}^{2} \int d\tau_{1} d\tau_{2} z_{j}^{(-1)}(\tau_{1} - \tau_{2})
\]
\[
\times (g_{F_{j}})_{2}(q', -w)(g_{F_{j}})_{1}(q, w)
\]
\[
\times (g_{F_{j}})_{2}(q', -w)(g_{F_{j}})_{1}(q, w)
\]
\[
\int d(\tau_{1} - \tau_{2})[1 - \cos[w(\tau_{1} - \tau_{2})]] z_{j}^{(-1)}(\tau_{1} - \tau_{2})
\]
\[
\times \left\{ \frac{\nu_{c}}{2\pi} \right\}^{2} \int \frac{dq'}{2\pi} \frac{dq}{2\pi} e^{iq'y} \left\{ (g_{s})_{11} + (g_{s})_{12} \right\}(q, w)
\]
\[
\times \left\{ \frac{\nu_{c}}{2\pi} \right\}^{2} \int \frac{dq'}{2\pi} \frac{dq}{2\pi} e^{iq'y} \left\{ (g_{s})_{11} + (g_{s})_{12} \right\}(q, w)
\]
\[
\int d(\tau_{1} - \tau_{2})[1 - \cos[w(\tau_{1} - \tau_{2})]] z_{j}^{(-1)}(\tau_{1} - \tau_{2}).
\] (A5)

Moreover, substituting into Eq. (A5) the result below (obtained by contour integration in the complex \( q \) plane),
\[
\int_{-\infty}^{\infty} dq e^{i\eta q} \langle (g_{31})_{11} \pm (g_{32})_{12} \rangle_{(q,w)}
= \frac{\pi \gamma_S}{(v_+^2 - v_-^2) |w|} \exp(-|w_{y_1}|/v_+) \left[ \frac{v_+}{\gamma_S} \left( v_{\rho K_p} - \frac{\delta_0}{2} \right) - \frac{1}{v_+} \left( v_{\sigma K_\sigma} + \frac{\delta_0}{2} \right) \right] - \exp(-|w_{y_1}|/v_-) \left[ \frac{v_-}{\gamma_S} \left( v_{\rho K_p} + \frac{\delta_0}{2} \right) - \frac{1}{v_-} \left( v_{\sigma K_\sigma} + \frac{\delta_0}{2} \right) \right] \right],
\]

(A6a)

and

\[
\int_{-\infty}^{\infty} dq e^{i\eta q} \langle (g_{32})_{21} \pm (g_{33})_{22} \rangle_{(q,w)}
= \frac{\pi \gamma_S}{(v_+^2 - v_-^2) |w|} \exp(-|w_{y_1}|/v_+) \left[ \frac{v_+}{\gamma_S} \left( \pm v_{\rho K_p} - \frac{\delta_0}{2} \right) - \frac{1}{v_+} \left( \pm v_{\rho K_p} + \frac{\delta_0}{2} \right) \right] - \exp(-|w_{y_1}|/v_-) \left[ \frac{v_-}{\gamma_S} \left( \pm v_{\rho K_p} + \frac{\delta_0}{2} \right) - \frac{1}{v_-} \left( \pm v_{\rho K_p} + \frac{\delta_0}{2} \right) \right],
\]

(A6b)

we have, finally,

\[
\langle \partial_1 \theta_\sigma(y, -w) \rangle_{(y_1, w)} = -\frac{\sqrt{\pi \eta}}{2} \left( \frac{V_S}{\pi a} \right)^2 \left[ \frac{v_{\rho K_p} \gamma_S}{\gamma_{y_2}} \left( v_{\rho K_p} - \frac{\delta_0}{2} \right) \right] \exp(-|w_{y_1}|/v_+) \left[ \frac{v_+}{\gamma_S} \left( v_{\rho K_p} - \frac{\delta_0}{2} \right) - \frac{1}{v_+} \left( v_{\sigma K_\sigma} - \frac{\delta_0}{2} \right) \right]
\]

\[
\times \frac{\Gamma\left( \frac{K_1 + K_2}{2} \right)}{\Gamma\left( \frac{1 + K_1 + K_2}{2} \right)} \left( \frac{\pi}{\beta \Lambda} \right)^{K_1 + K_2 - 2} \left[ \frac{v_+}{\gamma_2} \left( v_{\rho K_p} - \frac{\delta_0}{2} \right) - \frac{1}{v_+} \left( v_{\sigma K_\sigma} + \frac{\delta_0}{2} \right) \right] - \exp(-|w_{y_1}|/v_-) \left[ \frac{v_-}{\gamma_S} \left( v_{\rho K_p} + \frac{\delta_0}{2} \right) - \frac{1}{v_-} \left( v_{\sigma K_\sigma} + \frac{\delta_0}{2} \right) \right]
\]

\[
\times \frac{\Gamma\left( \frac{K_1 + K_2}{2} \right)}{\Gamma\left( \frac{1 + K_1 + K_2}{2} \right)} \left( \frac{\pi}{\beta \Lambda} \right)^{K_1 + K_2 - 2} \right].
\]

(A7)

2. Calculations of \( \langle \varphi_{\rho}(y=0, -w) \varphi_{\rho}(y_1, w) \rangle \)

The calculation of \( \langle \varphi_{\rho}(y=0, -w) \varphi_{\rho}(y_1, w) \rangle \) is essential for the charge conductance in Eq. (20). The integral is expanded in terms of the impurity potential as follows:

\[
\langle \varphi_{\rho}(y=0, -w) \varphi_{\rho}(y_1, w) \rangle
= \int dq dq' \frac{1}{2 \pi^2} \int dq dq' \frac{1}{2 \pi^2} \langle \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \rangle_{(q, w)} e^{i \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \lambda_{y_1} \lambda_{y_2} \delta E},
\]

where in the numerator,

\[
N_0 = \int D\varphi_{\rho}D\theta_\sigma\varphi_{\rho}(q', -w) \varphi_{\rho}(q, w) e^{i \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \lambda_{y_1} \lambda_{y_2} \delta E},
\]

\[
n_1(\tau_1) = \int D\varphi_{\rho}D\theta_\sigma\varphi_{\rho}(q', -w) \varphi_{\rho}(q, w) e^{i \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \lambda_{y_1} \lambda_{y_2} \delta E},
\]

\[
n_2(\tau_1) = \int D\varphi_{\rho}D\theta_\sigma\varphi_{\rho}(q', -w) \varphi_{\rho}(q, w) e^{i \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \lambda_{y_1} \lambda_{y_2} \delta E},
\]

\[
N_2 = \frac{1}{4} \int d\tau_1 d\tau_2 \sum_{m=\pm 1}^{4} \int V_j^{n_{m}^{(m)}}(\tau_1 - \tau_2),
\]

\[
n_1(\tau_1) = \int D\varphi_{\rho}D\theta_\sigma\varphi_{\rho}(q', -w) \varphi_{\rho}(q, w) e^{i \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \lambda_{y_1} \lambda_{y_2} \delta E},
\]

\[
n_2(\tau_1) = \int D\varphi_{\rho}D\theta_\sigma\varphi_{\rho}(q', -w) \varphi_{\rho}(q, w) e^{i \varphi_{\rho}(q, w) \varphi_{\rho}(q', w') \lambda_{y_1} \lambda_{y_2} \delta E},
\]

\[
N_1 = \int d\tau_1 \sum_{j=1}^{4} V_j n_j(\tau_1),
\]

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n_3(\tau_1) = \int D\varphi_\rho D\varphi_\rho(q',-w)\varphi_\rho(q,w) e^{i\tilde{Z}(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_1}E_{\varphi_\rho}},

n_4(\tau_1) = \int D\varphi_\rho D\varphi_\rho(q',-w)\varphi_\rho(q,w) e^{i\tilde{Z}(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_1}E_{\varphi_\rho}},

n_1^{(m)}(\tau_1 - \tau_2) = \int D\varphi_\rho D\theta_{\rho} \varphi_\rho(q',-w)\varphi_\rho(q,w)
\times e^{i\tilde{Z}(\varphi_{\rho}^{+}\theta_{\rho})_{0,\tau_1}m(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_2}E_{\varphi_\rho}},

n_2^{(m)}(\tau_1 - \tau_2) = \int D\varphi_\rho D\theta_{\rho} \varphi_\rho(q',-w)\varphi_\rho(q,w)
\times e^{i\tilde{Z}(\varphi_{\rho}^{+}\theta_{\rho})_{0,\tau_1}m(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_2}E_{\varphi_\rho}},

n_3^{(m)}(\tau_1 - \tau_2) = \int D\varphi_\rho D\varphi_\rho(q',-w)\varphi_\rho(q,w)
\times e^{i\tilde{Z}(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_1}m(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_2}E_{\varphi_\rho}},

n_4^{(m)}(\tau_1 - \tau_2) = \int D\varphi_\rho D\varphi_\rho(q',-w)\varphi_\rho(q,w)
\times e^{i\tilde{Z}(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_1}m(\varphi_{\rho}^{+}\varphi_{\rho})_{0,\tau_2}E_{\varphi_\rho}}.

(A9)

Definitions of \(V_j\), \(Z_0\), \(Z_1\), and \(Z_2\) are already given in Eqs. (A2a) and (A2b).

The path integrals in Eq. (A9) can again be evaluated by shifting the variable \(\Phi\) and the results needed for the conductance correction are given below:

\[
\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w) \rangle \quad \text{(spin-flip part only)}
\]

\[
= -\sqrt{\pi\pi\left(\frac{V_5}{\Lambda}a\right)^2} \left\{ \frac{1}{v_+ + v_-} \left[ \frac{v_+K_\rho + \partial_\rho}{2} + \frac{\gamma_5}{v_+v_-} \left( v_+K_\rho - \frac{\partial_\rho}{2} \right) \right] \right\} \frac{1}{(v_+^2 - v_-^2)}
\times \left\{ \exp(-|wy|/v_+) \left[ v_+ (v_+K_\rho + \partial_\rho) - \frac{\gamma_5}{v_+} \left( v_+K_\rho - \frac{\partial_\rho}{2} \right) \right] \right\}
\times \left\{ \exp(-|wy|/v_-) \left[ v_- (v_-K_\rho + \partial_\rho) - \frac{\gamma_5}{v_-} \left( v_-K_\rho - \frac{\partial_\rho}{2} \right) \right] \right\}
\times \left\{ \frac{\Gamma \left( \frac{K_\rho^* + K^*}{2} \right)}{\Gamma \left( \frac{1 + K_\rho^* + K^*}{2} \right)} \left( \frac{\pi}{\beta\Lambda} \right)^{K_\rho^*+K^*-2} \frac{1}{v_+ + v_-} \left[ v_+ (v_+K_\rho - \frac{\partial_\rho}{2}) + \frac{\gamma_5}{v_+v_-} \left( v_+K_\rho + \frac{\partial_\rho}{2} \right) \right] \right\} \frac{1}{(v_+^2 - v_-^2)}
\times \left\{ \frac{\Gamma \left( \frac{K_\rho^* + K^*}{2} \right)}{\Gamma \left( \frac{1 + K_\rho^* + K^*}{2} \right)} \left( \frac{\pi}{\beta\Lambda} \right)^{K_\rho^*+K^*-2} \frac{1}{v_+ + v_-} \left[ v_- (v_-K_\rho + \frac{\partial_\rho}{2}) - \frac{\gamma_5}{v_+v_-} \left( v_-K_\rho - \frac{\partial_\rho}{2} \right) \right] \right\}
\times \left\{ \frac{\Gamma \left( \frac{K_\rho^* + K^*}{2} \right)}{\Gamma \left( \frac{1 + K_\rho^* + K^*}{2} \right)} \left( \frac{\pi}{\beta\Lambda} \right)^{K_\rho^*+K^*-2} \right\}.
\]

(A11)

Here, \(z_{jj}^{(1)}(\tau_1 - \tau_2), J_{jj}^{(1)}\), and \(g(q,w)\) are already given in Eqs. (A3b), (A4a), and (A4b). Again, in the last expression of\( n_j^{(m)}\), when \(j=1\) or \(2\), it corresponds to the contribution from spin-flip potential with \(g(q,w)=g_S(q,w)\), and when \(j=3\) or \(4\), it corresponds to the contribution from nonspin-flip potential with \(g(q,w)=g_E(q,w)\).

With these, we write the contribution from the spin-flip part (with \(j=1\) or \(2\)) in the order of \(O(V_5^2)\),

\[
\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w) \rangle \quad \text{(spin-flip part only)}
\]

\[
= -\frac{\pi^2}{Z_0} \left( \frac{V_5}{n\Lambda} \right)^2 \int \frac{d\phi'}{2\pi} (g_S)_{11} + (g_S)_{12}\langle q',-w \rangle
\times \int \frac{d\phi'}{2\pi} e^{i\phi'} [(g_S)_{11} + (g_S)_{12}]_{q,w}
\int d(\tau_1 - \tau_2) \{ 1 - \cos[\omega(\tau_1 - \tau_2)] \} z_{jj}^{(1)}(\tau_1 - \tau_2)
\]

\[
= -\frac{\pi^2}{Z_0} \left( \frac{V_5}{n\Lambda} \right)^2 \int \frac{d\phi'}{2\pi} (g_S)_{11} - (g_S)_{12}\langle q',-w \rangle
\times \int \frac{d\phi'}{2\pi} e^{i\phi'} [(g_S)_{11} - (g_S)_{12}]_{q,w}
\int d(\tau_1 - \tau_2) \{ 1 - \cos[\omega(\tau_1 - \tau_2)] \} z_{jj}^{(1)}(\tau_1 - \tau_2). \quad \text{(A10)}
\]

Substituting Eq. (A6a) into Eq. (A10), we obtain
Similarly, we can write the contribution from the nonspin-flip part (with $j=3$ or 4) which is of $O(V^2_\sigma)$,
\[
\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w) \rangle \quad \text{(nonspin part only)}
\]
\[
= \frac{-\pi^2}{Z_0} \left( \frac{V_C}{\pi a} \right)^2 \int \frac{dq}{2\pi} [(g_C)_{11} + (g_C)_{12}](q',-w) \times \int \frac{dq}{2\pi} [(g_C)_{11} + (g_C)_{12}](q,w) \times \int d(\tau_1 - \tau_2)[1 - \cos[w(\tau_1 - \tau_2)]]z_{33}(\tau_1 - \tau_2) \]
\[
= \frac{-\pi^2}{Z_0} \left( \frac{V_C}{\pi a} \right)^2 \int \frac{dq}{2\pi} [(g_C)_{11} + (g_C)_{12}](q',-w) \times \int \frac{dq}{2\pi} [(g_C)_{11} + (g_C)_{12}](q,w) \times \int d(\tau_1 - \tau_2)[1 - \cos[w(\tau_1 - \tau_2)]]z_{44}(\tau_1 - \tau_2). \quad \text{(A12)}
\]

Using the following result (obtained by contour integration in the complex $q$ plane):
\[
\int dq e^{iqy}[(g_C)_{11} \pm (g_C)_{12}](q,w) \]
\[
= \frac{v_p v_s K_p K_s}{(v_+^2 - v_-^2) \vert w \vert} \exp(-\vert wy \vert / v_s) \times \left\{ \frac{v_+ - 1}{v_+ K_s} - \frac{\gamma S}{4 v_p K_p} \left( \frac{\delta v}{v_s} \right)^2 \right\} - \exp(-\vert wy \vert / v_+)
\]
\[
= \frac{\pi}{\Gamma(K/2)} \frac{\left( \frac{\gamma S}{4 v_p K_p} \right)}{-\frac{\gamma S}{4 v_p K_p}} \left( \frac{\gamma S}{4 v_p K_p} \right)^{K-2} \quad \text{(A14)}
\]

Finally, with the above results, we can calculate the conductance as follows. Substituting Eq. (A7) into Eq. (18) yields $\partial G_C^{(1)}$ in Eq. (19). Substituting Eq. (A11) into Eq. (20) yields $\partial G_C^{(2)}$ in Eq. (21b). Substituting Eq. (A14) into Eq. (20) yields $\partial G_C^{(3)}$ in Eq. (21a).