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Ultrasonic measurement of the critical exponent $\beta$ for the spin glass $\text{CuMn}$

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Ultrasonic measurements are presented providing direct measurement of the critical exponent $\beta$ for a spin glass. Sound velocity measurements were made on $\text{Cu}_{0.95}\text{Mn}_{0.05}$ and $\text{Cu}_{0.976}\text{Mn}_{0.024}$ single crystals which exhibited the characteristic spin glass cusp in low-field ac susceptibility. A sound velocity anomaly was observed in the vicinity of the susceptibility peak temperature. We attribute this anomaly to critical fluctuations. Using perturbation theory we show that for a spin glass such as $\text{CuMn}$, the critical part of sound velocity is related to that of the Edwards-Anderson spin glass order parameter $q$. We are able to determine the critical exponent $\beta$ associated with this order parameter by fitting the theory to the measured critical change in sound velocity in $\text{CuMn}$ samples. By combining our values of $\beta$ with the values of $\alpha'$ associated with the critical change in specific heat below the spin glass transition temperature, and Suzuki's scaling laws, we find the values of exponent $\gamma$, $\alpha'$, and $\gamma'$, associated with the zero field susceptibility, the magnetic field dependent part of specific heat, and the field dependent part of susceptibility.

PACS numbers: 64.60.Fr, 75.50.Kj, 75.80.+q

INTRODUCTION

Spin glasses have a sharp peak in the low field ac magnetic susceptibility\(^1\) at the spin glass transition temperature $T_c$. The specific heat,\(^2\) however, contains an anomalously larger linear contribution and is smooth and rounded at the transition. The spin-glass phase transition was proposed by Edwards and Anderson (EA).\(^3\) According to EA, the phase is characterized by a nonvanishing value of the different time correlation of a spin at a given site so that the spin may be visualized as being frozen in time. Hertz et al.\(^4\) studied propagation of sound with a view to investigating spin dynamics through a time-dependent Landau-Ginzburg model coupled to phonons. They found that the sound speed decreases linearly with $T-T_c$, and the sound damping diverges as $(T-T_c)^{-1}$, when the spin glass transition $T_c$ is approached from high temperature. Our data show the sound speed changing follows a more complicated function. In this letter we simply use the perturbation theory to show how the EA order parameter for an ideal spin glass is formally related to the observed change in the sound speed for the temperature range above and below $T_c$, and we report on our attempt to measure it for $\text{Cu}_{0.976}\text{Mn}_{0.024}$ and $\text{Cu}_{0.95}\text{Mn}_{0.05}$ using phase comparison method.\(^5\) We have succeeded in obtaining $\beta$, the critical exponent of the order parameter, directly from 30-MHz longitudinal and shear sound velocity measurements.

EXPERIMENT

Measurements were on $\text{Cu}_{0.976}\text{Mn}_{0.024}$ and $\text{Cu}_{0.95}\text{Mn}_{0.05}$ single crystal of irregular cross section. The 5-at. % crystals were grown at the Ford Scientific Laboratory by a Bridgman technique in argon at 1 at. The 2.4-at. % sample was made at Purdue Central Materials Preparation Facility. Two samples exhibited the characteristics spin-glass cusp at 16.5 and 23.5 °K in the low-field ac susceptibility.

The shift of freezing temperature\(^6\) $T_c$ at low field in the frequency range from 5 to 100 Hz is approximately 0.2$T_c$. So we take these values as critical temperature of the samples. A phase comparison method\(^5\) was used to measure the temperature dependence of the sound velocity $v$. Evaporated CdS transducers were employed, and 30-MHz longitudinal and shear waves propagated along [110] direction. In this experiment the best resolution for $\Delta T/v$ was $\sim 10^{-6}$.

![Figure 1](image_url)
THEORY

The basic physical idea involves the fact that a sound wave will be perturbed by the thermal fluctuation of the spin system due to spin-phonon coupling. Since the thermal spin fluctuations increase rapidly near the critical temperature, there will also be an anomalous increase in the attenuation and a corresponding decrease in sound velocity. We take the Hamiltonian of the system as

\[ H = \sum_i \sum_q \hbar \omega_q b_i^q b_i^q + \sum_i \sum_q J(R_i, R_j) S_i S_j, \]

where the first term is the elastic energy and the second term is the Heisenberg exchange interaction of the spin system. The modulation of the Heisenberg exchange interaction due to the lattice vibration is perturbing Hamiltonian, so that one-phonon and two-phonon terms are involved in the spin-phonon interaction.\(^7\) Treating the spin-phonon interaction as a perturbation in the process of adiabatic switching, we calculate the energy shift \( \Delta E \). Also, we identify the perturbed phonon energy shift \( \hbar \omega_q \) with the energy required to increase \( n_q \) by unity. We have in the long-wavelength limit

\[ \hbar \Delta \omega_q = \frac{d \Delta E}{d n_q} = \sum_{q, \rho} \frac{\delta}{\partial N_{\omega_q}} \hat{\varepsilon}_q(q, \rho) \hat{\varepsilon}_q(p, q, s) \]

\[ \times \frac{\partial^2 J(R_i^0, R_j^0)}{\partial R_i^0 \partial R_j^0} (q R_i) (s R_j) \]

\[ - \Im \sum_{q, \rho} \frac{\delta}{\partial N_{\omega_q}} \hat{\varepsilon}_q(q, \rho) \hat{\varepsilon}_q(q, s) \frac{\partial J(R_i^0, R_j^0)}{\partial R_i^0} \]

\[ \times (1 - e^{-\hbar \omega_q}) \int_0^\infty e^{-\hbar \omega_q t} \langle S_i(t) S_j(t) \rangle dt, \]

where \( R_i = R_j - R \), and \( R^0 \) is the equilibrium position of spin \( i \), and \( \hat{\varepsilon}(q, s) \) is the polarization vector of a phonon with wave vector \( q \) and \( s \). (...) denotes the statistical thermal average, and \( \beta = 1/kT \). Subscript \( \delta \) and \( \epsilon \) represent cartesian components. The first and second term in Eq. (2) are two-phonon and one-phonon processes. The imaginary part of \( \Delta \omega_q / \omega_q \) gives us the fractional change in sound velocity. We adopt the EA model and neglect short range correlations between spins, i.e., the two-spin spatial correlations function \( \langle S_i S_j \rangle \) may be ignored. So \( \Delta v / v \) for \( \hbar \omega < kT \) in the long wavelength limit is given

\[ \frac{\Delta v}{v} = -\sum_{q, s, i, j, \delta, l, m, e} C(q, s, i, j, \delta, l, m, e) \hbar \omega_q / kT \]

\[ \times \Im \int_0^\infty e^{-\hbar \omega_q t} \langle \sum_p S_p(t) S_p^\dagger(t) S_0(t) S_0^\dagger(0) \rangle dt, \]

where \( C(q, s, i, j, \delta, l, m, e) \) is the coefficient. Following Bennett,\(^8\) we introduce an approximation for the four spin correlation function by factorizing the four-spin correlation function into terms containing products of time-dependent and time-independent pair correlation functions. All the equal time correlations can be neglected and correlation function \( \langle S_i^\dagger(t) S_j(t) \rangle \) has been set equal to zero except \( \mu = \nu \). This self-correlation time of a dipole was obtained by E-A.\(^9\)

\[ q(t) = \langle S_i(t) S_j(0) \rangle = \exp \{-2kT[1 - T_c / T_v] / v \}, \]

\[ T > T_c, \]

\[ q(t) = q [1 - \exp(-qJt / v_c)] + \exp(-qJt / v_c), \quad T < T_c, \]

where \( q = \langle S_i^\dagger(t) S_j(t) \rangle \), and \( v_c \) represents a friction coefficient. The critical exponent \( \beta \) is defined by \( \Delta v / v = \{1 - T / T_c \}^\beta \). The change in sound velocity is rewritten as

\[ \Delta v / v = A_1 \left( \frac{\omega^2}{T} \right) \left[ \omega^2 + \frac{4T}{A_2} \left(1 - \frac{T}{T_c} \right)^2 \right], \quad T > T_c \]

\[ \Delta v / v = A_1 \frac{\omega^2}{T} \left[ \omega^2 + \frac{4q^4(A_3)^2}{T} \right], \quad \omega < \omega_c. \]

FIG. 2. Critical change in sound velocity as a function of reduced temperature for 30-MHz shear wave along [110]. The inset shows fractional change in sound velocity vs temperature.
RESULTS AND DISCUSSION

Measurements at 30 MHz of the sound velocity along the [110] direction in Cu$_{0.976}$Mn$_{0.024}$ sample (longitudinal and shear) are shown in the inset of Figs. 1 and 2. The anomalously large linear contribution to the specific heat of CuMn alloys implies a $T^2$-magnetic contribution to the sound velocity background which is much larger than the usual $T^2$-electronic contribution. This is confirmed by a computer fit of the data to the thermodynamically expected form.\(^1\)

\[ \Delta v/v = B_0 - B_1 T^2 - B_4 T^4, \]

(7)

for temperatures sufficiently above the critical region. This fit is shown by the solid curves in the inset of Figs. 1 and 2. The difference, $\Delta v/v$, between the data and this background is plotted in Figs. 1 and 2. We compare $\Delta v/v$ to Eq. (5) for $T > T_c$ and Eq. (6) for $T < T_c$. Using $A_1$, $A_2$, $A_3$, and $\beta$ as fitting parameters, we fit Eq. (5) and Eq. (6) to the data. The values of these fitting parameters are collected in Table I and the fitted curves are also shown in Figs. 1 and 2.

According to our derivation, the constant $A_1$ depends on the polarization of the sound wave and on the sample composition, but should be the same above and below $T_c$. As may be seen from Table I, $A_1$ is indeed significantly different for the two samples, and for the shear and longitudinal waves. However, it is essentially the same above and below $T_c$. The constant $A_2$, $A_3$, and $\beta$ should also depend on the sample composition, but not on the polarization. These predictions are also borne out by Table I. It is interesting to note that our values of $\beta$ are close to the renormalization group predictions for the $X-Y$ model.\(^2\) A glance at the fits of Figs. 1 and 2 reveals that the agreement between our theory and experiment is good except very close to $T_c$. We must attribute this to the approximation made in our calculation, the most important one being the factorization of the four-spin correlation functions and the neglect of spatial spin-spin correlations, resulting in an underestimate of the effect of fluctuations, particularly at $T_c$.

Combining our values of $\beta$ (Table I), Hawkins’ values of $\alpha^1$,\(^3\) where $\alpha$ is associated with $\Delta C_v$ below $T_c$, and Suzuki’s scaling laws,\(^4\) such as $\alpha + 2\beta + \gamma^2 = 2$, $\gamma = -\beta$, and $\alpha + \beta = -2$, we are now in a position to predict the critical exponents $\gamma$, $\alpha^*$, and $\gamma^*$ associated with the magnetic field independent susceptibility, the field dependent part of the specific heat, and the magnetic field dependent part of the susceptibility. The results are tabulated in Table II.

**TABLE I.** Parameters obtained from fitting the critical sound velocity change.

<table>
<thead>
<tr>
<th>$\text{Cu}<em>{0.99} \text{Mn}</em>{0.01}$</th>
<th>$\text{Cu}<em>{0.976} \text{Mn}</em>{0.024}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Longitudinal</strong></td>
<td><strong>Longitudinal</strong></td>
</tr>
<tr>
<td>$T &gt; T_c$</td>
<td>$T &lt; T_c$</td>
</tr>
<tr>
<td>$T &gt; T_c$</td>
<td>$T &lt; T_c$</td>
</tr>
<tr>
<td>$T &gt; T_c$</td>
<td>$T &lt; T_c$</td>
</tr>
<tr>
<td>$10^{-4} A_1$</td>
<td>$1.41$</td>
</tr>
<tr>
<td>$1.9$</td>
<td>$1.85$</td>
</tr>
<tr>
<td>$10^{-2} A_3$</td>
<td>$6.83$</td>
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<tr>
<td>$2.00$</td>
<td>$8.00$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$8.12$</td>
</tr>
</tbody>
</table>

**TABLE II.** Critical exponents calculated from the text.

<table>
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<tr>
<th>$\text{Cu}<em>{0.99} \text{Mn}</em>{0.01}$</th>
<th>$\text{Cu}<em>{0.976} \text{Mn}</em>{0.024}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^*$</td>
<td>$-1.9$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$2.0$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$-2.0$</td>
</tr>
<tr>
<td>$\alpha^*$</td>
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<tr>
<td>$\gamma^*$</td>
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**CONCLUSIONS**

In this paper, the investigation of the effects of critical fluctuations on the sound velocity in a spin glass has been done. We have developed a perturbation theory treating the spin–phonon interaction as a perturbation and expressed the critical part of sound velocity in terms of the spin glass order parameter. The critical exponent $\alpha$ associated with the order parameter can be determined by fitting experimental data. Furthermore, we are able to predict three more critical exponents, namely $\gamma$, $\alpha^*$, and $\gamma^*$, from the measurements of changing in sound velocity. Meanwhile, average $\alpha$, for 0.279-at. % Mn which was obtained from the results of Fogles et al.\(^5\) is about less than 1. The value of $\beta$ is compatible with electrical resistivity measurement\(^6\) and theoretical prediction.\(^7\) Critical exponent $\gamma^*$ is small, which means the $T$ dependence is weak. The work of Malozemoff et al.\(^8\) gives $\gamma^* = 0.07$ for 4.6% Mn in Cu. So the accurate measurements of the zero-field susceptibility, of the field dependence of the specific heat, and of the field dependence of the susceptibility are highly desirable to measure these exponents directly.

**ACKNOWLEDGMENTS**

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