Internal and External Relations.

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It is not easy to say what relations are. They are probably too fundamental to be defined. At any rate, we are as yet unable to define them. However we are directly presented with such situations as involve "left and right", "before and after", etc, etc. Relations are all pervasive. The idea of unrelated entities, though in itself logically, free perhaps from contradiction, is as a matter of fact untenable. We do sometimes say with regard to certain things that they are totally unrelated, but what is implied in such a statement is merely a negation of certain specific relation or relations which happens to be the subject of our attention. To be unrelated in any specific way is itself a relation, and from this point of view nothing is unrelated.

Any and every entity can become a relatum, logical propositions and relational complexes included. These latter are simply terms in relation. But relations themselves cannot become relata. Thus, a relational complex such as "I am taller than you" may bear the relation "Therefore" to another relational complex "I am heavier than you". But the relation "Taller than" cannot bear the relation "Therefore" to the relation "Heavier than". Bradley's infinite regress so far as relations are concerned is to us meaningless.

Bradley seems to have entertained another notion that is equally untenable. Not only did he conceive of relations as capable of being related, but also as capable of relating. Without involving ourselves in verbal niceties, we may say that relations do not relate in the sense that relational complexes are created through their activities. They are not active, they simply are; there is no sense in which they can be said to have either succeeded or failed in relating. The question is not whether a certain relation succeeds or fails in relating certain entities, it is rather whether those entities are or are not in a relational complex characterized by that
specific relation. Relations therefore are not relata, nor do they of their own choice relate.

II

We propose for consideration the following three relational complexes to be known as \(a, b, c\), relational complexes embodying \(a, b, c\), relations.

\[a, \quad \text{This book is on the table.}''
\]

\[b, \quad \text{``H}_2\text{O'' as relational complex.}
\]

\[c, \quad \text{``He knows Physics.''}
\]

Is a relation in the \(a\) relational complex the same as the \(b\) relation in the \(b\) relational complex? If so, are relations all of one kind? If all relations are \(b\) relations, then the \(c\) relation in the \(c\) relational complex is also a \(b\) relation. If relations are of different kinds, then the \(c\) relation may be either an \(a\) or a \(b\) or something different from both, and it remains for us to ascertain which and what it is.

From the common sense point of view, the above does not present much of a problem, since, at a glance, a good many vague and perhaps obvious differences may be pointed out. But philosophical sophistry has made out of the situation a problem of the first importance. It has been maintained that all relations are "internal". It is difficult to find out what an "internal" relation is, but if it supposed to be anything in the nature of a \(b\) relation, then we are confronted with very grave problems. For in this latter case the \(c\) relation subsisting between the knower and the known is also internal and therefore has properties which are the characteristics of the \(b\) relation. Knowing would then make a difference to the thing known, and no form of knowledge, science included, could claim objectivity. While it is not easy to say exactly what "objectivity" means, it is easy to see that if knowing, being internal, modifies the thing known, then knowledge is not objective at least in the following senses.

\[a, \quad \text{If all relations are \(b\) relations, then the relation between the knower and the known is a \(b\) relation. If so, the known is always a relatum, an entity that is}
\]
not its original self, because it is modified by knowing. It follows that we can
never know any entity other than as relatum. Hence "Noumenon" emerges.
Whatever the differences may be does not matter in the least, the point is that
there are differences between an entity as such and an entity as relatum, and what
is even more significant, we are never in position to know what these differences
are.

b, But under the above assumption we are not in position to say that there
are no entities other than relata, for in that case how could they be said to be
different from relata? If we admit differences between the two, then they either
exist or simply are, and the denial of either or both would be contrary to the
premises. If entities in any knowledge relation were external objects, then these
can not be known as they are, they can only be known as they are known.

c, Since what we know is but knowledge-relata, there can hardly be any
standard for the validity of knowledge. What we know is simply "what we know". If "what we know" about a certain thing is again questioned, we can only answer
by a further assertion of knowledge possibly in the form: "I know that I know", etc, etc. The question can be asked ad infinitum and any answer other than the
first is neither better nor worse than the first.

d, The above may be said just as well about the knower who in any capacity
other than a knowing faculty or organism would be just as elusive as any external
object.

The impossibility of knowing anything as itself under our assumption may be
approached from another angle.

e, If all relations are internal, then such relations as "to the right of", "to the
left of", "before" and "after", "above" and "below" are all internal relations. The
knower and the known not only bear a specific relation to each other, namely that
of knowing, but also involve spatial relations. Here is a table: if all relations are
internal in the sense of a b relation, then this table to the east of me is different
from this very table when it is placed to the west of me. I myself change with a
change of spatial relations. Thus, when I was to the east of the table, my knowledge of it was such as was appropriate to a specific place, a knowledge that should not be confounded with anything I may know of the table when I shifted to the west of it.

Temporal relations would be internal under the same assumption. The relational complex known as knowledge is an event that takes place in finite time. But in finite time, however short, both the table and I change our temporal relations. Thus my knowledge of the table at one time must not be confounded with my knowledge of that very table at any other time. Time, it is poetically claimed, flows, and our temporal relations change with it; the only knowledge that can be said to be valid is the knowledge that is appropriate to instants, just as in the previous paragraph the only knowledge that can be said to be valid is the knowledge that is appropriate to points. But since knowledge as a relational complex and an event takes place in finite time and place, no knowledge is valid, if all relations are internal in the sense of a b relation.

g. At the same time all entities are related and in no time are they free from relational changes. Under the assumption we are making in the present section, if anything involves any kind of relational changes, then it is impossible for us to know anything about it. My little desk here in my room bears some kind of relation to a twinkling star ten thousand light years away; if that star changes in any way whatsoever, even to the slightest degree, the desk becomes different in some way from what it was. Thus, if we want to know this little desk, we have to know that little star as well as the pyramids of Egypt and the trees of Labrador. Since my desk bears relation to anything and everything in this universe, then on the basis of our supposition we have to know the universe in order to know the desk. But we do not know the universe, hence we do not know this desk. And yet with a suitable definition of knowledge corresponding perhaps inadequately to the vague and inarticulate notion of knowledge which we do entertain, we can claim that we know this desk without knowing the universe in any similar sense.
Is our common belief a pious superstition? Or rather is the assumption herein discussed basically untenable?

h. Logic has no content; it need not have any; but if it is useful in any practical way at all, it should be applicable to natural events or entities. In other words, in order that logic may be useful, such statements as "A is A", "A is not-A", must not only be logically significant, but also practically so. They may be true or false, or they may be neither, but we have to recognize their validity in some way before we can speak. But if they are known to be invalid to start with, then we can not possibly speak. The only statements that can possibly claim to be true are those that are made at point-instant. At any finite time and in any finite place no proposition concerning matters of fact can be made with any practical significance.

Whether pure logic is affected or not need not be considered in the present connection, it is certain that applied logic evaporates into thin air. An ironic situation arises: those who accept the supposition of this section can not say a word in favour of it, while those who argue in favour of it cannot possibly accept it and escape contradiction.

Our supposition, let us recall, is firstly that all relations are internal, and secondly, that an internal relation is a $\beta$ relation. Thus far we have been arguing that under such a supposition knowledge is impossible in the various senses already discussed. In order to free ourselves from such a dismal consequence, it is only necessary to overthrow either the first or the second part of the supposition. There are quite a number of people who believe that all relations are internal, but if what is meant by an internal relation is not a $\beta$ relation, then the specific objections herein set forth are not applicable, whether or not there are other objections. But if an internal relation is conceived as a $\beta$ relation, then we have to show reasons why some relations are external, and a clear distinction between an external and an internal relation is required. If what is meant by an external relation is made clear, then the proposition that there are external relations may be established. In that case it only remains for us to decide as to whether the specific relation between
the knower and the known is an external or internal relation or something totally different from both. What we have been arguing thus far is that that specific relation cannot be conceived as being internal in the sense of a b relation.

III

In this section, we shall take up for discussion in the first place the position of Mr. F. H. Bradley, and secondly that of Professor Spaulding. Both may have the misfortune of being misrepresented.

A. a, Mr. Bradley seems to have thought that relations are impossible. If entities are in relation, then the relation between them does not “relate”, for somehow they are already related; if on the other hand they are not related, then no relation can possibly relate them. In other words, if entities are in relation, to relate them is superfluous; if entities are not in relation, to relate them is impossible. It is impossible to relate entities not related, because for such a purpose an infinity of relations is required, and an infinity of relations is by definition unattainable. Comparatively detailed criticism will come at a later stage, for the moment, we should like to point out the following:

1. Relations and relational complexes should not be confused with each other. Relations simply are, but relational complexes may begin or end. The question is never whether a certain relation is, but always whether certain entities bear that relation to each other.

2. Relations should not be regarded as being engaged in activities. There is no sense in which they may be said to succeed or fail in their effort to relate, simply because there is no sense in which they can be said to make any effort at all.

3. Relations should not be regarded as possible terms, so that there is no sense in which two terms in relation can be said to have become three terms necessitating a further relation.

b. Bradley seems to have entertained the idea that all relations modify their terms. If A and B are related, then A is not the original A, nor B the original B. Each, being in relation, is directly influenced in some way by the relation and
indirectly influenced by each other. But if A and B were not in relation with each other, then the influence just mentioned would not have been there to affect them. That is to say, relations modify their terms.

c. Since relations modify their terms, some difference is made to them when they are in relation. Terms in relation must be somehow different from those very terms when out of it. If no difference is made to the terms, in what sense could we say that they are modified by their relation? Some differences therefore there must be between terms that are relata and terms that are mere terms. Of what that difference is, very few of us seem to have any clear notion. Bradley has the idea that qualities and relations imply each other, but in so far as I am aware, there is no attempt to regard them in any consistent way as tautological expressions. Later on we shall attempt to show that difference in quality implies difference in relation, but we are not justified in asserting that difference in relation implies difference in quality.

d. Entities in relation form a relational complex which whether or not further analyzable is at any rate not equivalent to the sum total of its parts. Thus, if we are dealing with such a relational complex as "Japan is to the east of China," we can not possibly say that this is equivalent to "Japan," "to the east of," and "China"; for obviously there are a number of possible combinations each of which is distinct from any other. There is therefore something unique about a relational complex, and this uniqueness which is essentially the uniqueness of the whole has been in some obscure way transformed into a uniqueness of the composing parts.

e. For reasons similar to the ones herein stated, some philosophers have come to the conclusion that all relations are internal, and an internal relation is such that the terms in relation are different from those very terms out of relation. Whether the reasoning is correct or not, we shall not examine for the moment, nor shall we concern ourselves with the conclusion as to whether it follows from the different stages of the argument. But if we accept the conclusion and interpret "modification of terms" as qualitative differences being made to them, then the objection
raised in the previous section would apply and would thus make knowledge something of a game of blind man’s bluff.

B, It is possibly for this reason that quite a few students of philosophy have come to the conclusion that the proposition that all relations are internal can not be accepted. Perhaps “hasty hands catches frogs for fishes”, and in an eager attempt to demolish that proposition, ideas have been put forth that could not stand any close examination. Professor Spaulding might be cited as an example of those whose valour has unfortunately become the better part of discretion.

It should however be pointed out that those who are opposed to the internal-relationists are not opposed to the idea of internal relation; they are merely opposed to the idea that all relations are internal. Hence what the external-relationists want is not to deny that there are internal relations, but merely to affirm that some relations are external.

a, Some relations, according to Spaulding, do not modify their terms. “Above and below,” “before and after” are such relations. If I walk around the table, I am not modified by the various positions which I hold in relation to the table. I may be taller or shorter than my friend, but I cannot be said to be thereby either tall or short. Terms related by this kind of relation are such that to say anything beyond their relation is unwarranted and consequently they are related in a way quite different from those that are related for instance by an organic relation.

b, If there are relations that do not modify their terms, it is believed to follow that there are relations by which no difference whatever is made to the terms. Entities that are relata in these relations are the same as those very entities that are not relata in these relations. It is believed, though I have no assurance in saying so, not only that terms before being related by an external relation are the same as they are after it, but also that terms in such a relation are the same as they are out of it. This may seem to be a distinction without a difference, but a difference there is, and in some cases, a very important one.

c, Since terms in any external relation are the same as they are out of it, it
follows that they are each independent of the other, and both independent of the relation. What is meant by independence is merely a lack of mutual modification. Independence is therefore not something which entities possess when they bear no relation to each other, it is rather something that represents the state of affairs in which they bear to each other an external relation. Since entities though related are yet independent of each other, it is thought that the whole in which they are the component parts is not such as to give rise to the idea of uniqueness. Thus, while the internal-relationists seemed to have confounded the uniqueness of the whole with that of the parts, the external-relationists seem to have mistaken a lack of uniqueness in the component parts for the lack of uniqueness in the whole.

The conclusion that Professor Spaulding arrived at is not merely that there are external relations, but also that the specific relation between the knower and the known is an example of such a relation. That is to say, though related, the knower and the known are yet independent of each other. The known is not merely relatum. It is that, but it is also something more; it is the original object. The objections raised in the second section of this article are therefore automatically removed.

There are people who are in sympathy with Spaulding's conclusion, but not with his reasoning; and there are perhaps others who are in sympathy to some extent with Bradley's reasoning, but find it by no means easy to swallow his conclusion. If external relation is to be justified, it must be on some basis and in some way other than those adopted by Professor Spaulding. Bertrand Russell has something to say on the subject, but what he has said does not amount to very much. Mr. G. Moore to my knowledge has probably the most adequate appreciation of the problem; in my opinion at any rate, he has certainly made the keenest remarks about it. I do not pretend to understand exactly what his theory is, but I have no doubt that I have been greatly influenced by him. At the same time, I must admit differences of opinion which it would be out of place here for me to discuss.
In this section, two views almost diametrically opposed to each other are presented. Which of the two is to be preferred? Before this question can be answered, there should be a general clarification of terms such as "difference", "modification", "influence", etc., etc. This clarification, I hope, will emerge as we proceed.

IV

We return to the three different relational complexes mentioned in section II. What we want to emphasize is the difference between \( a \) and \( b \) relations, and to decide whether the \( c \) relation belongs to either of them, and if so, which.

A. Points common to all relations.

a. There must be relations such as \( R \) and there must be a plurality of entities such as \( A, B, \) etc., etc.

b. There must be a possibility of such a relational complex as \( ARB \). Given relations and entities, we need not have relational complexes. In the actual world in which we live we are possibly never confronted with relations that are not involved in relational complexes. But from the point of view of analysis, relations are more primitive than relational complexes, and may be studied prior to and apart from a study of relational complexes.

c. There is something unique about such a relational complex as \( ARB \). It is some kind of whole that is not easily reducible to its parts, since these same parts combined in different ways may result in other and quite different wholes. Is is not the uniqueness of wholes that can be or should be denied, it is rather the attempt to assign a single pattern to the uniqueness that we are unable to follow.

d. Entities in relation are affected by the relation. To use Moore's phrase, they acquire a "relational property" which they would not have received if they had not been related.

Such being the case, there is a definite sense in which entities in relation can be said to be different from those very entities out of relation. \( A \) and \( B \) are entities out of relation, but in relation, they are also relata. The specific difference
in the case is the relational property each acquires from the other, and both from
the relation.

These points conceded to be common to all relations seem to substantiate the
view of the internal-relationists. If what is meant by an internal relation is one
that is analyzable into the state of affairs herein mentioned, then all relations are
internal. But this state of affairs has been by some at any rate perhaps unwill-
tingly confounded with the specific situation of a $b$ relation, and an internal
relation has come to mean much more than is conceded to all relations in the pre-
ceeding paragraphs. Before we take up the specific situation of a $b$ relation, we
should clarify a few terms that are needed for the moment.

B. There are different meanings to the term "identity" just as there are
different meanings to the term "equality". We are not here concerned with the
possible meanings which these term may embody, for our present purpose, the term
identity may be used as a characteristic of relations, and the term equality as a
characteristic of quality. The first shall be symbolized as "$I$", and the second as "$=$",
hence "$I$" stands for difference in relation, and "$\neq$" stands for difference in quality
or qualities. Differences in relation are easily grasable. At any finite place and
during any finite time $l$ change my relations, and differences of relations may be
asserted irrespective of empirical evidence, because they can always be inferred
from our knowledge of mathematics, physics and astronomy, with a suitable
construction of time and space. Qualities however are quite a different matter.
Their difference or equality is sometimes operational and always empirical, but
whereas a judgement of qualitative difference is final with respect to certain
operations, a judgement of equality in our sense at any rate is always tentative.
The latter can not be final, because the equality asserted is a mere lack of empiric-
al or experimental difference with respect to certain operations which may be
replaced by others, and when they are so replaced, we may be confronted with
qualitative differences formerly not experienced by us.

We are assuming here that there is some definite sense in which qualities are
different and may be detached from relations. As to whether our assumption is justified or not we shall discuss at a later stage. Nor are we concerned here with the specific sense in which qualities and relations are supposed to differ from each other. I can no more define qualities than I can define relations. But common sense considers them different, and in the absence of clear definitions and sharp distinctions we may accept the common sense view for our present purpose.

C. Let us take first the a relation in the a relational complex, namely, “This book is on the table”.

a. The book in this particular relational complex receives a relational property, namely, “on the table”. If there were no such relational complex, this specific property would not have been ascribable to the book. We are of course not asking whether or not there is a relation “on”; there always is. Our question is rather whether or not there is a relational complex, namely, “this book is on the table”. If there is, “this book” receives the relational property already mentioned.

b. Though this book is on the table, it can not be said that because it is on the table, it has changed its colour or increased its weight. It may change its colour, and it may increase its weight, but if it does either or both of these things, it does so independently of the relational property it has just required. There is a question of inference involved in this connection, but the problem of inference will be dealt with in a separate section.

c. Such being the case, we are justified in saying that the book in such a relation to the table is different from any book not in such a relation. The difference asserted is one of relation. We are also justified in saying that the book in such a relation is equal to this very book out of such a relation. The equality affirmed is one of quality.

The relational complex “this book is on the table” is a kind of whole, but it is difficult to say what kind of whole it is. The relation between the whole and its parts is not symmetrical, there is uniqueness of the whole which is qualitatively different from other wholes composed of the same parts, but there is no uniqueness
of parts which may enter different wholes without becoming themselves qualitatively different. One thing is certain, this relational complex is neither an organic whole, nor such as involves chemical action between its parts. And if "x" stands for either of these kinds of wholes, we can say definitely that the relational complex "this book is on the table" is not "x".

e. We shall employ "R" to symbolize a relation; "RP", the relational property of such a relation; and "x", the two kinds of wholes mentioned. The following expression will embody the points herein discussed.

\[ \mathcal{R}B, \mathcal{R}P, \mathcal{R}P\mathcal{A}, \mathcal{R}P=\mathcal{A}, \mathcal{R}B=\neg x. \]

D. Let us take up the b relation in the b relational complex, namely, "H₂O".

a. This is a relational complex in which the entities related are hydrogen and oxygen. Each acquires a relational property which it would not have, were it not so related to the other.

b. But in this case the hydrogen that is a relatum in this relational complex has changed its qualities by reason of its being a relatum. What can be said about it as an entity no longer applies when it has become a relatum.

c. From the above we are justified in saying that entities related in the b relational complex are different in relation and also unequal in quality to those very or similar entities not so related.

d. This relational complex is a sort of whole with emergent qualities describable in terms that are not applicable to the component parts separately. It is a kind of a whole that is equivalent to some kind or form of "x".

e. We shall employ "\mathcal{R}B" to symbolize this kind of relation, and "\mathcal{R}P", this kind of relational property. The following expression will embody the points herein discussed.

\[ \mathcal{R}B, \mathcal{R}P, \mathcal{R}P\mathcal{A}, \mathcal{R}P=\mathcal{A}, \mathcal{R}B=\neg x. \]

V

The topic for discussion in this section is not an easy one. To start with a few prefatory remarks are needed. Both a and b relations involve A\mathcal{R}P\mathcal{A}, but where-
as a relation embodies $A\overline{RP}=A$, $b$ relation contains just the opposite, $A\overline{RP}\neq A$. This seems to be the most important point of difference compared to which the other points mentioned may be ignored. The question is: What sort of relation, if any, subsists between $A\overline{RP}A$ and $A\overline{RP}=A$, and what relation, if any, between $A\overline{RP}A$ and $A\overline{RP}\neq A$? If the second part can be inferred from the first in each case, then there is contradiction somewhere. What we want to point out is that the essential difference lies in the nature of $\overline{RP}$ and $\overline{RP}$, namely, the two relational properties; that $A\overline{RP}A$ and $A\overline{RP}=A$ are separately inferable from $\overline{RP}$, and $A\overline{RP}A$ and $A\overline{RP}\neq A$ are separately inferable from $\overline{RP}$; but that in no way is $A\overline{RP}=A$ inferable from $A\overline{RP}A$, or $A\overline{RP}\neq A$ from $A\overline{RP}A$. It is therefore advisable to take up the problem of implication and inference in so far as it affects our discussion. We shall take up three different kinds of implications to be symbolized as “$\therefore$”, “$\triangle$” and “$\because$”.

A. A few words on propositions in general are not out of place here. There are propositions which have no existential import and which may be called purely logical propositions. There are others which have existential significance and which would be meaningless if the existence of the subject terms is denied. The relations of opposition between propositions depend upon the interpretation of the existence of their subject terms. Take for instance the A and E forms of propositions. These have no relations of opposition if their subject term is not assumed to exist. They are contrary if their subject term is either assumed or asserted to exist. Where the subject term is exist, its non-existence renders the proposition insignificant, but where the subject term is asserted to exist, its non-existence would mean that the proposition is false.

Aristotelian logic as revealed through text books seems to involve itself in self-contradiction. A, E, I, O are said to be categorical propositions implying an assertion of the existence of their subject terms. If I and O are categorical in the above sense, then they are not sub-contrary, and if they are sub-contrary, as they are declared to be, then they are not categorical in the above sense.

Take for instance the familiar proposition that “All men are mortals”, (terms
interpreted as classes). It might be interpreted as a proposition asserting an abstract and unique relation between "Humanity" and "Mortality" conceptualized and defined with such rigidity that the proposition may be translated into the following: "Whether there are men or not, if anything is human, he is mortal", just as one might say of an Euclidean straight line that whether it exists or not, if anything is a straight line, it is the shortest distance between two points. The familiar interpretation however of the above mentioned proposition is that all men are as a matter of fact mortals, or that all men are mortals and there are men. With either the familiar or the unfamiliar interpretation, being a man implies being a mortal, but the implication involved in the one case is quite different from that involved in the other.

B. The kind of implication here symbolized as "*" is the kind involved in the first and non-existential interpretation of propositions. It need not have anything to do with matters of fact. From certain postulates and certain definitions, both of which may be quite arbitrary, a number of propositions are said to follow in the sense that they are implied by the given postulates and definitions. To such a process matters of fact are quite irrelevant. Thus, as we have seen, with suitable definitions of humanity and mortality to which the existence of mankind may be considered as irrelevant, a proposition of the form "Whether there are men or not, if anything is human, he is mortal" may be so interpreted that being human "*=" being mortal. "p*q" means that the proposition "p" implies the proposition "q", whatever the facts may be. This is still "strict" implication, but it need not involve any empirical evidence.

C. But when the proposition "All men are mortals" is interpreted as asserting at the same time the existence of men, then the implication involved in it is quite different from that of the above. The proposition so interpreted becomes an inductive generalization and can claim no validity apart from experience. If there were no men to start with, or if there were, and they were all or some of them immortals, the proposition would be false, and being a man would not imply
being a mortal. Thus, the implication involved in this case cannot be divorced from matters of fact. It is possible for us to say at the present moment at any rate that all the citizens of New York are citizens of The United States of America; hence being a citizen of the former implies being a citizen of the latter. But this need not be so, and in the same way men need not be mortals. However, as a matter of fact, the generalization about men is valid; hence as a matter of fact, being a man does imply being a mortal. "pΔq" means that facts are such that the proposition p implies the proposition q.

D, a. These two implications are often confused. The way in which their distinction can be made clear leads me to think that the views presented here are somewhat akin to those of Mr. Moore. Take for instance; "A>B, B>C, therefore A>C", where ">" represents "greater than". We may have the expression:

\[ A > B \triangle (B > C \triangle A > C) \]

That is to say, if A is in fact greater then B, then the proposition "B is greater then C" implies the proposition "A is greater than C," whether or not there is any C at all. But we can not have the expression:

\[ A > B \neq (B > C \triangle A > C) \]

That is to say, the proposition "A is greater than B" does not imply the fact that B being greater than C, A must have been greater than C also.

b. The relation between "\( \neq \)" and "\( \triangle \)" should next engage our attention. Whenever there is a possibility of "\( \neq \)", there is a possibility of "\( \triangle \)", provided a certain condition is fulfilled. If for instance we have the implication "p\#q" and A comes within the definition of p, and B, q we may have the implication AΔB. Hence given A\( \neq \)0, and B\( \neq \)0, we are justified in saying (p\#q) \( \triangle \) (AΔB). The question as to whether facts will be so good-humoured as to work for our convenience and substantiate our theories is irrelevant, for, in the first place, we do not claim that (p\#q) \( \neq \) (AΔB), that is, we do not say that there will always be A and B bearing that relation; and secondly, if there are no A and B, or if there are and they can not be subsumed under p and q, the condition is not fulfilled, and the question of implication does not arise.
c. We have seen then that under certain conditions it is possible for us to have 
\[(p \ast q) \triangle (A \Delta B), \text{ though never } (p \ast q) \nless (A \Delta B)\]. It remains for us to see whether given \(A \Delta B\), we are ever justified in arriving at \(p \ast q\), even if the condition \(A \not p = 0, B \not q = 0\) is fulfilled. Personally, I see no way of arriving at any such conclusion. This is probably one sense in which empirical generalizations are said to be contingent. So far as we can see, there is no way in which \(A \Delta B\) can be transformed into \(p \ast q\). All that we can claim is that given \(A \Delta B\) there is a probability in favour of \(p \ast q\). It seems impossible to have either \((A \Delta B) \triangle (p \ast q)\), or \((A \Delta B) \nless (p \ast q)\); the uttermost we can assert is that \((A \Delta B) \therefore (p \ast q)\), and this, only when the condition already mentioned is fulfilled.

E. Let us not take up \(\therefore\). To start with, we may point out that this is not an implication in any strict sense; it might be called a sort of practical inference, an inference which was originally probably only physiological, but which has since become highly trained through the influence of civilization. All empirical knowledge when applied to things not yet experienced involves this kind of inference; but from the point of view of logical validity, it is highly problematical. There is in the situation no evidence of any necessity in any strict sense, all that can be claimed for it is a high or low degree of probability. It is something which in human psychology represents expectation, but what is expected need not be realized.

There is generally no confusion between \("\ast\"\) and \("\therefore\"\), but there is confusion between \("\triangle\"\) and \("\therefore\"\), arising chiefly from the view that with regard to matters of fact, there is no necessity. \("\triangle\"\) is an implication that is partly logical and partly factual. Empirical knowledge has enabled us to classify men as animals, hence being a man implies being an animal. If we are actually confronted with an ethereal man we are quite justified in calling him either a devil or a god. Empirically so far as we know, it is just as necessary for a man to be an animal as it is for a green book to be coloured. But a man need not remain a man, nor a green book either green or a book. There is however a probability that the status quo
will be maintained, and that probability is the source of the kind of inference which we have symbolized as "\( \therefore \)."

VI.

We shall now apply the above mentioned implications to the \( a \) and \( b \) relational complexes.

A. We have already said that entities in relation are affected by the relation, they receive a relational property which they would not have received, if they had not been related. But what is the relation between \( A \) and \( B \)'s being related and the acquiring by them of the relational property? With regard to this question, I confess that I am not sure of my ground. Sometimes I think in terms of a "\( \triangle \)" implication, but for the present I am inclined to think in terms of a "\( \ast \)" implication. Possibly I had in mind sometime ago the notion that the acquisition of a relational property is a characteristic of an existent entity; possibly I had felt that the ascription of a relational property to an entity that may itself be non-existent is meaningless. I am inclined to think at the present that whether or not there are "\( A' \)'s and "\( B' \)'s, the very assertion of a relation between them implies their acquiring a relational property of some kind. Our notion of a relation, though vague, contains at least this point: that if a relation is at all significant, it must give the terms related some kind of property which they would not have, had they not been so related. This notion of relation justifies us in saying that the proposition asserting or supposing a relation between \( A \) and \( B \) implies a proposition to the effect that \( A \) and \( B \) acquire a relational property of some kind. Hence we may have the following expression:

\[
a, \quad A\#B \#A\#P.
\]

\[
b, \quad A\#B \#A\#P.
\]

B. If \( A \) receives a relational property by virtue of its relation to \( B \), then it is relationally different from what it was when it had not had that property. There is again an implication involved, and like the one discussed in the preceding paragraph, it seems to be one that holds between propositions. Only in this case,
it seems to be more obvious. The assertion with regard to any entity that it possesses any kind of property whatever is significant only when some difference is made by the property mentioned to the entity that is asserted to possess it. If the assertion is at all significant as it is meant to be, then some kind of difference is implied; the only question is whether the implied difference is relational. It cannot be argued that the difference implied is not relational, for a relational difference in this connection is the minimum implied. And while it may be argued that more than a minimum may be implied, it can not be argued that a minimum is not implied; for the proposition asserting this minimal difference is, when the significance of propositions is borne in mind, tautological with the original proposition asserting a relation between A and B. Hence we have the expression:

\[ a, \; A \neq A \land A \neq \neg \neg A \]
\[ b, \; A \neq A \land A \neq \neg \neg A \]

C. With regard to the qualitative difference, the question is quite other than the above. We have noted qualitative equality in a relational complex, and qualitative difference in b relational complex, namely, 'A ≠ A' in the former, and \( A \neq A \) in the latter. These differences may safely be attributed to the inherent differences in the relational properties. But before we take up the question of implication, we have in passing to indulge in a few remarks on qualitative equality and difference. An assertion of qualitative equality as has already been pointed out is tentative; like an assertion of qualitative difference, it is based on empirical knowledge, but unlike the former, it only holds at a certain stage and with certain kinds of experiences or experiments and may therefore be modified, or supplemented, or overthrown by subsequent and more delicate and detailed experiments. If we divide experiments into varying degrees of crudeness and refinement, we may say that an assertion of qualitative equality is final with respect to all the experiments cruder than the one upon which the assertion is based, but tentative with respect to all the experiments that are more refined. A judgement of qualitative difference on the other hand is just the reverse; it is tentative with respect to the
crude, and final with respect to the more refined experiments. Strictly speaking, then, judgements of quality, in so far as their equality and difference are concerned, are empirically both final and tentative in opposite directions, and strictly speaking, we have no ground upon which we can justify our preference for either of them. But we have prejudices; somehow or other we have come to believe that the more refined an experiment is, the better it reveals reality. That being the case, we have greater faith in a judgement of qualitative difference, since it is final with respect to all the experiments more refined than the one upon which the judgement is based.

In the case of the a relational complex, we have a judgement of qualitative equality. With the above prejudice in mind, we can not say that our judgement is final. All that can be claimed is that there is a great probability in its favour, especially when repeated experiments confirm it. Psychologically we expect A1a to be qualitatively the same as A. They may be different, and their differences can be and often are attributed to other factors than the mere fact of their being related to each other. But we are not sure that the relation does not bring about some kind of qualitative difference not revealed or not yet revealed through further experience or experiment. We may of course define an a relation as one in which no qualitative difference is made to the entities related; hence if any qualitative difference is revealed through further experiment, then the relation involved is simply not an a relation. Such a definition is open to the following objections. In the first place, does the phrase "no qualitative difference being made" refer to an experience or experiment of a specific degree of refinement? If so, a more refined experiment may result in an a relation ceasing to be such. Does the phrase refer to any kind or degree of experience or experiment? If it does, the definition would be entirely abstract, since it does not refer to a particular kind of experience, and qualitative judgements would cease to be experiential. We may in either case claim that A1a and A should be qualitatively the same, and if as a matter of fact they are not, we may always claim that some factor other than the relation is
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responsible for the difference. In such a case we are merely substituting an abstract qualitative equality for an experienced qualitative difference, and while the substitution is justified when such a determining factor is found, it is not justified if such a factor is not ascertained to be there. From all that has been said above, it seems safe to conclude that the qualitative equality between $A \tilde{=} P$ and $A$ is not certain but merely probable. That is to say, given $A \tilde{=} P$, there is very high probability $A \tilde{=} P$ and $A$ are equal. Hence we have the expression:

$$a, \ A \tilde{=} P \vdash A \tilde{=} P = A.$$  

We may of course state the case negatively. Instead of speaking in terms of equality, we may speak in terms of inequality. Instead of saying that we expect qualitative equality, we may say that we do not expect qualitative inequality. The negative statement is probably stronger than the positive one from the point of view of logic. We may say that if $A$ and $B$ are related by an $a$ relation, it does not follow that $A \tilde{=} P$ and $A$ are qualitatively different. We may have the expression:

$$a, \ A \tilde{=} P \not\equiv A \tilde{=} P = A.$$  

In the $b$ relational complex we have a judgment of inequality, that is, of qualitative difference. Again with our prejudice in mind, we can say that a judgement of qualitative difference is final. This does not mean that qualitative difference in reality is always revealed in experience, it simply means that whenever it is revealed in experience, it is also there in reality. In the $b$ relational complex the inequality of $A \tilde{=} P$ and $A$ is invariably found in experience so that a part of the definition of such a relational complex is that entities that are its relata are qualitatively different from those that are not. Empirically a $b$ relation, since it involves a judgement of inequality that is regarded as final, can always be established; whereas an $a$ relation, since it involves a judgement of equality that is tentative, can not be empirically established though the probability in its favour may be such as to make it practically certain. Since this is the case, a $b$ relation can be defined without reference to any tentative element in our judgement, and
the implication in a \( b \) relational complex may be considered as one of factual necessity. We may therefore have the expression:

\[
b, \ A\neg P \triangle A\neg P \neg \forall A.
\]

E, We may summarize the above as follows:

\[
a, \ A\neg R\neg B \neg A\neg R \neg P \neg A \neg P \neg \forall A, A\neg P \vdash A \neg P = A;
b, \ A\neg R \neg B \neg A\neg R \neg P \neg A \neg P \neg \forall A, A\neg P \triangle A\neg R \neg P \neg \forall A.
\]

So far we have been considering essentially individual relational complexes, at least in so far as \( a \) relational complex is concerned. We may have the following modifications:

a, A and B may be considered as classes, in which case the relational complex having them as relata would be a class relational complex. The above formulae still hold.

b, So far also we have been comparing \( A \neg P \) with A, either as different stages of one individual, or different individuals of one class. We may compare \( A \neg P \) with anything whatever that has not the relational property which A has. If \( X \) represents such a thing or a class of such things, we may transform the above formulae into the following:

\[
a, \ A\neg P \neg A\neg P \neg A \neg R \neg P \neg X, A\neg P \neg A \neg R \neg P \neg \forall X;
b, \ A\neg R \neg B \neg A\neg P \neg A \neg R \neg P \neg X, A\neg P \triangle A\neg R \neg P \neg \forall X.
\]

F, The kind of relation that \( a \) is is what we call an external relation, and the kind of relation that \( b \) is is what we call an internal relation. The clear distinction between them lies in the different implications or inferences concerning qualitative equality and difference.

Those who maintain that all relations are internal in the above sense seem to have been confused about implications as well as about relational and qualitative differences. With regard to the \( a \) relation, the internal relationists, without taking notice of the empirical fact \( A \neg P = A \), seem to have thought that since \( A \neg R \neg P \neg A \) is the case, \( A \neg R \neg P \neg \forall A \) must be true. Their confusion is two-fold. They may have inferred in the first place a "\( \wedge \)" implication from a "\( \neg \)" implication. But we have seen
that that is possible only under a specific condition which must be satisfied before the inference is valid. In the second place, they may have thought that "I" implies "x" since "x" does as a matter of fact imply "I". We will see in the next section that although "x" implies "I", there is no implication the other way around.

a, Since the internal-relationists seem to have made the above two-fold mistake, their view on internal relation (viz, our b relation) also becomes fallacious. It is true that both relational and qualitative differences are involved in a b relation; it is also true that both are implied by the internal relational property, but it is not true that the former implies the latter. Qualitative difference between $A \bar{P} \bar{x} A$ and $A$ is an empirical fact, it is not inferred from the relational difference between them.

The internal-relationists seem to have been guilty of the following confusions.

1. "I" and "x" are not clearly distinguished.
2. Since "x" implies "I", it is thought that "I" implies "x".
3. $A \bar{P} \bar{x} A$ is considered as being inferred from $A \bar{P} \bar{I} A$ and not as an empirical fact.
4. Since under a certain condition, a "\( \Delta \)" can be inferred from a "\( * \)" the internal relationists perhaps thought that $A \bar{P} \Delta A \bar{P} \bar{x} A$ can always be inferred from $A \bar{P} * A \bar{P} \bar{I} A$.
5. Since the latter is true of all relations, the former must also be; hence all relations are internal in the sense defined.

b, Some of the external-relationists seem to have made the following mistakes:

1. "I" and "x" are also confused.
2. Since "x" implies "I", some external relationists probably draw the conclusion that "x" implies "I".
3. Since they recognize the qualitative equality between $A \bar{P}$ and $A$, they probably think that they can not help concluding that $A \bar{P}$ and $A$ are relationally also the same.

4. Since the above is the case, the somewhat careless conclusion is drawn
that external relations do not result in any difference whatsoever being made to their terms.

VII

From what has been said above, we can easily see that the two sources of confusion are the different implications and the lack of a clear distinction between qualitative and relational differences. The former has already been dealt with at some length, but the latter needs a little more detailed consideration than has been given in the preceding paragraphs. The whole problem hinges on our view of qualities and relations, and it is to these that we are now again turning our attention.

Bradley said in the third chapter of his "Appearance and Reality". "Qualities are nothing without relations" and further down "relation without quality is nothing." Bradley has always seemed to me to be more literary than lucid, and I cannot claim that I understand what he means by these sentences. It seems that the term relation has been used both as relational complexes and as relations pure and simple, and the term qualities has been used both as entities and as qualities pure and simple. It may be that relational complexes are nothing without entities, that Bradley thought that relations are also nothing without qualities; and that, since as a matter of fact entities are related in relational complexes, qualities are always related by relations. It is probably easy to argue from a synthetical dependence of relational complexes upon both relations and entities to the impossibility of an analytical independence between relations and qualities; for it is easy to forget that the whole is related to its parts in a way quite different from that in which parts are related to each other. The whole depends upon the parts for the kind of whole it is, but the parts do not depend upon each other for the kind of parts they are.

Furthermore, even if relations and qualities were mutually dependent upon each other, it does not mean that they are thus identified with each other. However dependent each may be upon the other for the relation that subsists between both,
a husband is not his wife, nor an effect its cause, nor yet a son his father. It is the identification that forms the subject of our discussion, and nothing short of such identification would make any difference to the argument herein set forth. If relations and qualities are identified with each other, then relational equality would mean qualitative equality, and relational difference qualitative difference; but if they are not so identified with each other, no such inference is possible.

A, To start with, we have to admit that qualitative differences involve relational differences. Whenever we experience qualitative difference, we can always experience relational difference of some kind at the same time. This is obvious, since qualitative differences can only be experienced in finite time; and in finite time, a change of temporal relations has already taken place. We may use Venn’s notation and say \( \text{D} \) \( \text{D} = 0 \) is true. Hence the proposition “\( \text{D} \triangle \text{D} \)” is also true. Does it follow then that our a relation, namely, external relation, is impossible in the sense that it is contradictory? The answer to this question depends on whether or not qualitative equality implies relational equality, when we grant that qualitative difference implies relational difference. If qualitative equality implies relational equality, then \( \text{A} \triangle \text{A} \) as an element in our a relational complex would imply \( \text{A} \triangle \text{A} \). But a relational complex also involves \( \text{A} \triangle \text{A} \); hence a relation would be contradictory and thus impossible. The problem then is whether “\( \text{D} \triangle \text{D} \)” implies “\( \text{D} = \text{D} \)”.

B, In order to tackle this problem, we may make use of the following eight propositions employing Venn’s notation for the sake of convenience.

- \( \text{A}, \ (\overline{\text{D}}) (\overline{\text{D}}) = 0 \)
- \( \text{A}', \ (\overline{\text{D}}) (\overline{\text{D}}) = 0 \)
- \( \text{E}, \ (\overline{\text{D}}) (\text{D}) = 0 \)
- \( \text{E}', \ (\overline{\text{D}}) (\text{D}) = 0 \)
- \( \text{I}, \ (\overline{\text{D}}) (\text{D}) > 0 \)
- \( \text{I}', \ (\overline{\text{D}}) (\text{D}) > 0 \)
- \( \text{O}, \ (\overline{\text{D}}) (\text{D}) = 0 \)
- \( \text{O}', \ (\overline{\text{D}}) (\text{D}) > 0 \)

a, It is possible that both \( \text{A} \) and \( \text{A}' \) are true propositions. If so, qualitative differences and relational differences are identical. Not only does “\( \text{D} \triangle \text{D} \)” imply “\( \text{D} = \text{D} \)”,
but "I" also implies "=": not only does "=" imply "I", but "I" also implies "=". And external relations in the sense defined would be impossible.

b. It is also possible that both E and E' are true. If so, there is no implication between relational and qualitative differences, and our definition of external and internal relations hold.

C. It is possible that qualitative and relational differences are neither identical with, nor non-implicative of each other. If so, it is possible to have the following possibilities:

a. All relational differences are qualitative differences. The following propositions would then be true: 1, (I) (¬E)=0, 2, (E) (¬E')=0, and 3, (E) (→E)=0. Since (I) (¬E)=0 is true, it follows that "I' implies '¬E'" is also true. Since "I' implies '¬E'" is true, it may happen that with a certain condition fulfilled, we can infer from AkπA, Akπ¬A, and our view of external relations is impossible, since it affirms the former and denies the latter.

b. Relational and qualitative differences overlap, viz, some of the one is some of the other, but neither is entirely the other. In this case the four particular propositions are true, and our definitions of internal and external relations remain unaffected by any of the propositions.

c. All qualitatative differences are relational differences. If so, the following propositions are true: 1, (¬E) (¬E')=0, 2, (I) (¬E')=0, and 3, (I) (¬E)=0. From (¬E) (¬E')=0 we can infer that "¬E" implies "I", but from (I) (¬E')=0 we can not infer that "I" implies "¬E". If "I" implies "¬E" is false, then AkπA does not imply Akπ¬A, and there is no contradiction in our view of external relation. At the same time, although "I" implies "=" is true, "=" implies "I" is false, so Akπ=A does not imply AkπA, and the contention of some extreme external relationists that external relations do not make any difference whatsoever to their terms can not be admitted.

D. That which is fatal to our view of external relations is the proposition that all relational differences either are or always involve qualitative differences.
We have seen that empirically all qualitative differences involve relational differences; hence if we accept the above assertion, we are either identifying quality with relation or asserting that they always involve each other. We are now confronted with the fundamental contention upon which the claim of the internal-relationists is based. In fact the proposition \((\overline{I}) \overline{(?)} = 0\) is tautological with the claim that all relations are internal. Have we then any reason for accepting this proposition?

a, We may try to answer this question from empirical evidence. Qualitative equality or difference is a question of experience. If we refuse to admit this, we may find that science and scientific procedure will be confronted with difficulties. In experience qualitative difference does involve relational difference, but in experience the latter does not involve the former. What experience can prove is the proposition that \((?!) (\overline{I}) = 0\), it can not prove the proposition \((\overline{I}) \overline{(?)} = 0\). If it is claimed that experience is too crude, we can enlarge the term to include scientific experiments, and the situation would remain the same as it was before. If it is urged that scientific experiments are themselves crude in the sense that infinitesimal differences are not revealed by them, it may be answered that the suggestion is valid only when qualitative equality or difference is abstracted from all its empirical bearings for the purpose of logical construction; but that if it is not so abstracted, if, that is to say, qualitative equality or difference is to be considered as anything that can be affirmed or denied by experience, then the proposition \((\overline{I}) \overline{(?)} = 0\) can never claim any empirical evidence whatsoever.

Thus far we have been saying that the above proposition can not be proved empirically. Can it be disproved by experience? This question depends for its answer upon whether or not relational differences are limited to those that are empirical. So far as our present stage of civilization is concerned, even our empirical relational differences are not accompanied by qualitative differences. Relatively therefore to our present scale of experience, the proposition \((\overline{I}) \overline{(?)} = 0\) can be disproved empirically. But as we have already pointed out, our present
judgements of qualitative equality are tentative with respect to experiments more delicate than those upon which the judgements are based. It is therefore possible to have a line of demarcation dividing the comparatively crude from the comparatively refined experiments such that our judgements of qualitative equality are final with respect to the former, and tentative with respect to the latter. It is possible for us to wake up some day to find our civilization so far advanced, or experiments so delicately adjusted and reliable, that for every empirical relational difference some qualitative difference can be experienced. Thus if we limit ourselves to relational differences that are empirical, our disproof of the proposition (I) \( \bar{\chi} = \chi \) can never be final. But fortunately or otherwise relational differences are never so limited; some of them are indeed empirical, others are merely inferred, and in so far as some of them are beyond finite experience, those that are so can never be experienced. At the same time qualitative differences are nothing if not empirical. Since experience is a series of finite events, and experiments, no matter how far advanced, must remain finite operations, not only can we never prove the proposition (i) \( \bar{\chi} = \chi \), but we can also disprove it conclusively from the point of view of our finite experience.

b, We have seen that so far as our experience is concerned there is no reason whatever for accepting the proposition that all relational differences involve qualitative differences. And what is more there are reasons for rejecting it. But empirical reasons need not be always applicable to the domain of logical contraction. What is empirically rejected need not be barred from entering into a theory or a system as a fundamental postulate to which facts may be considered irrelevant as long as it serves the function for which it is invoked. We hope to point out however that even in this sphere of logical construction, we have no reason for accepting the proposition (I) \( \bar{\chi} = \chi \).

r, In the first place, we have no reason for rejecting the proposition \( (\chi) \bar{\chi} =\chi \); hence if we accept (I) \( \bar{\chi} = \chi \), we are either identifying quality with relation, or else asserting that they involve each other to the extent that each is not without the other.
2. If qualities and relations are identified with each other, then, for reasons stated in section II of this article, we should not be able to know anything whatever. Nature would indeed be a passage, but it would be such a passage that no glimpse of it could ever be stolen. Hence pragmatically, from the point of view of convenience, the proposition \((\bar{I}) (\bar{\phi}) = 0\) should not be accepted.

3. 'Those who accept the proposition \((\bar{I}) (\bar{\phi}) = 0\) can hardly say anything in its favour unless the theory of types takes the arguments supporting it out of the universe of discourse of both relations and qualities. Whether such a defense is possible or not, I have not thought sufficiently to say anything definite. If it is open to the internal-relationists to invoke the aid of the theory of types, then their arguments may be consistent, but what they say can never be true. If, on the other hand, this defense is not open to them, then they can not argue at all, for their arguments, themselves in the universe of discourse of relations and qualities, would contradict the proposition for which these arguments are called forth.

Logically and pragmatically we see no reason for accepting the proposition \((\bar{I}) (\bar{\phi}) = 0\). Since this proposition is rejected, we have no possibility either of "\(\bar{I}\)" implying "\(\bar{\phi}\)" or of "\(=\)", implying "\(I\)", and our definitions of internal and external relation can be established. For the purpose of reminding our readers we repeat the formulae previously given.

\[ a, \overline{A \bar{R} B \ast A \bar{R} P \ast A \bar{R} P \bar{I} A}, A \bar{R} P \ast A \bar{R} P = A, A \bar{R} P \triangle A \bar{R} P \neq X. \]
\[ b, \overline{A \bar{R} B \ast A \bar{R} P \ast A \bar{R} P \bar{I} A}, A \bar{R} P \triangle A \bar{R} P \neq A, A \bar{R} P \triangle A \bar{R} P \neq X. \]

VIII

We shall now return to the problem of knowledge which may be made easier by the above discussion. Knowledge is a relational complex with terms that are themselves relational complexes thus involving a plurality of relations. Epistemology will have to determine how these entities and relations are to be construed; but for our present purpose, we shall consider the knower and the known as wholes united by a specific relation, namely, that of knowing. Various theories have
been put forth as to what kind of relation this specific one is, and some have concluded that it is an internal relation in the sense defined.

A. Is the knowledge-relation then internal or external? If it is internal in the sense of a $b$ relation, then all the arguments set forth in section II would render knowledge very much of a fabrication of our brain. Unfortunately we can not prove empirically what sort of relation our knowledge-relation is; we can not prove that it is either internal or external. At best we can only interpret it either as the one or the other or something quite different from both. The conclusion arrived at in this article is that it has to be interpreted as external.

a. But if all relations are internal, then the knowledge-relation must also be internal, since no other alternative is possible.

b. The purpose of this study is to disprove the proposition that all relations are internal (in the sense of a $b$ relation), and to establish at the same time the proposition that some relations are external. If we are successful in this attempt, we have at least two alternatives open to us for the interpretation of the knowledge-relation.

c. We have to stress the point that if we interpret the knowledge-relation as internal or external, we can not prove or disprove it to be either the one or the other.

B. Why do we interpret the knowledge-relation as external?

a. If it is regarded as internal, then we have no way of meeting the objections of section II.

b. If it is regarded as external, then these objections can not be urged against us.

c. We can not prove the knowledge-relation to be external for the simple reason that we can never compare any entity as knowledge-relatum with that entity as an external object.

C. What We want to make use of in an external relation is simply $A \vDash_p A$. If $A$ stands for any external object, $K$ stands for the knowing faculty, then $A \vDash A$
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stands for the knowledge relational complex and A&P stands for a knowledge-relatum. If the external object"happens to be this book on my desk, then the book in knowledge is relationally different from the book out of it, but that does not prevent us from inferring that the book as an external object and as a knowledge-relatum are qualitatively the same. In a word, this book can be known as an external object.