

國立清華大學 101 學年度碩士班入學考試試題

系所班組別：生醫工程與環境科學系(0522) 甲組(分子生醫工程組)
考試科目 (代碼)：應用數學(2203)、Z403 丙組(醫學物理與工程組)

共 2 頁，第 1 頁 *請在【答案卷、卡】作答

1. Solve the following ordinary differential equations.

(a) $x dx + (x^2 y + 4y) dy = 0$ (8 pts)

(b) $(x + 3y) dx - (3x + y) dy = 0$ (8 pts)

(c) $x^3 y''' - 6y = 0$ (8 pts)

2. Suppose a student carrying a flu virus return to an isolated college campus of 100 students. If assumed that the rate at which the virus spreads is proportional not only to the number of infected students but also to the number of students not infected (i.e. dx/dt is proportional to $x(100-x)$, x : infected student, t : time), determine the time that half students are infected. It is further observed that 20 students were infected after 2 days. (8 pts)

3. A free undamped spring/mass system oscillates with a period of 3 s. When 8 Kg is removed from the spring, the system then has a period of 2 s. What was the weight of the original mass on the spring?

[Hint: The system can be modeled by differential equation $m \frac{d^2 x}{dt^2} = -kx$.] (8 pts)

4. A differential equation system

$$\frac{d^2 x}{dt^2} + 3 \frac{dy}{dt} + 3y = 0$$

$$\frac{d^2 x}{dt^2} + 3y = t e^{-t} \quad x(0) = 0, x'(0) = 2, y(0) = 0 \quad (8 \text{ pts})$$

5. Let $x(t) = u(t - 3) - u(t - 5)$ and $h(t) = e^{-3t} u(t)$.

(a) Compute $y(t) = x(t) * h(t)$. (5pts)

(b) Compute $g(t) = (dx(t)/dt) * h(t)$. (5pts)

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6. Supply a first column so that the matrix is orthogonal.

$$\begin{pmatrix} \square & -1/\sqrt{2} & 1/\sqrt{3} \\ \square & 0 & 1/\sqrt{3} \\ \square & 1/\sqrt{2} & 1/\sqrt{3} \end{pmatrix} \quad (8 \text{ pts})$$

7. Solve $\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^t$ by variation of parameters. (a) Find the fundamental matrix $\Phi(t)$ (5pts) (b) Solve the general solution of $\mathbf{X}(t)$. (5pts)

8. $f(x) = \begin{cases} -1, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$, find the complex Fourier series. (8 pts)

9. Find the Fourier Transform of $x(t) = e^{-3|t|}$. (8 pts)

10. Find the temperature $u(x,t)$ in a rod of length 100 if the initial temperature is $f(x) = 100 - x$ and if the ends $x=0$ and $x=100$ are insulated. The thermal diffusivity constant k of this rod is 2. (8 pts)

[Hint: heat equation $k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.]