

國立清華大學 101 學年度碩士班考試入學試題

系所班組別：工業工程與工程管理學系

考試科目（代碼）：統計學(1501 1601 1701)

共 2 頁，第 1 頁 \*請在【答案卷】作答

1. (10 pts.) Prove that Geometric distribution has memoryless property.
2. (15 pts) Prove that if  $X \sim \text{Bin}(n, p)$ , then as  $p \rightarrow 0$  with  $np = \mu$  constant,  $X$  converges in distribution to Poisson distribution.
3. (15 pts) Consider the simple linear regression model with no intercept term  $Y_i = \beta X_i + \epsilon_i$ . Derive the least square formula for  $\hat{\beta}$ . Note that the formula for  $\hat{\beta}$  when there is no intercept term is not the same as when there is an intercept term in the equation.
4. (10 pts) Let  $X$  and  $Y$  be independent random variables with  $E(X) = 1$ ,  $E(Y) = 2$ ,  $\text{Var}(X) = \text{Var}(Y) = \sigma^2$ , for what value of  $K$  is  $K(X^2 - Y^2) + Y^2$  an unbiased estimator of  $\sigma^2$ .
5. (12 pts.) Name the random variable  $Y$  and also determine the corresponding value of parameter(s).
  - (a)  $Y = \sum_{i=1}^{10} X_i/10$ , where  $X_1, \dots, X_{10}$  are independent normal random variables, each with mean 1 and variance 1.
  - (b)  $Y = \sum_{i=1}^{10} X_i$ , where  $X_1, \dots, X_{10}$  are independent chi-squared variables, each with parameter  $\nu = 1$ .
  - (c)  $Y = \sum_{i=1}^{10} X_i$ , where  $X_1, \dots, X_{10}$  are independent Exponential variables, each with expected value 1.
  - (d)  $Y = \sum_{i=1}^n (\frac{X_i - 10}{2})^2$ , where  $X_1, \dots, X_n$  are independent normal random variables, each with mean 10 and variance 4.
  - (e)  $Y = X^2$ , where  $X \sim$  student t distribution with degrees of freedom  $\nu$ .
  - (f)  $Y = \sum_{i=1}^{10} X_i$ , where  $X_1, \dots, X_{10}$  are independent Geometric distributions with parameter  $p$ .
6. (18 pts.)

$$f_X(x) = \begin{cases} 10e^{-10x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Draw the pdf of  $X$ .
- (b) What is the name of the random variable  $X$ ?
- (c) Compute  $P(1 \leq X \leq 2)$ .

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- (d) Compute the expected value of  $X$ ,  $\mu_X$ .
  - (e) Compute the standard deviation of  $X$ ,  $\sigma_X$ .
  - (f) Is the skewness  $E(X - \mu_X)^3$  positive, zero, or negative? Give your explanation.
7. (20 pts.) Suppose that  $X_1, X_2, \dots, X_n$  is a random sample of size  $n$  taken from a Bernoulli distribution with unknown parameter  $p = P(X_i = 1)$ . Regarding the sample mean  $\hat{P} = \sum_{i=1}^n X_i/n$ .
- (a) Prove or disprove that  $\hat{P}$  is a maximum likelihood estimator.
  - (b) Prove or disprove that  $\hat{P}$  is the estimator of  $p$  with the minimum mse (mean-squared error).