

國立清華大學 101 學年度碩士班考試入學試題

系所班組別：工業工程與工程管理學系

考試科目（代碼）：統計學(1501 1601 1701)

共 2 頁，第 1 頁 *請在【答案卷】作答

1. (10 pts.) Prove that Geometric distribution has memoryless property.
2. (15 pts) Prove that if $X \sim \text{Bin}(n, p)$, then as $p \rightarrow 0$ with $np = \mu$ constant, X converges in distribution to Poisson distribution.
3. (15 pts) Consider the simple linear regression model with no intercept term $Y_i = \beta X_i + \epsilon_i$. Derive the least square formula for $\hat{\beta}$. Note that the formula for $\hat{\beta}$ when there is no intercept term is not the same as when there is an intercept term in the equation.
4. (10 pts) Let X and Y be independent random variables with $E(X) = 1$, $E(Y) = 2$, $\text{Var}(X) = \text{Var}(Y) = \sigma^2$, for what value of K is $K(X^2 - Y^2) + Y^2$ an unbiased estimator of σ^2 .
5. (12 pts.) Name the random variable Y and also determine the corresponding value of parameter(s).
 - (a) $Y = \sum_{i=1}^{10} X_i/10$, where X_1, \dots, X_{10} are independent normal random variables, each with mean 1 and variance 1.
 - (b) $Y = \sum_{i=1}^{10} X_i$, where X_1, \dots, X_{10} are independent chi-squared variables, each with parameter $\nu = 1$.
 - (c) $Y = \sum_{i=1}^{10} X_i$, where X_1, \dots, X_{10} are independent Exponential variables, each with expected value 1.
 - (d) $Y = \sum_{i=1}^n (\frac{X_i - 10}{2})^2$, where X_1, \dots, X_n are independent normal random variables, each with mean 10 and variance 4.
 - (e) $Y = X^2$, where $X \sim$ student t distribution with degrees of freedom ν .
 - (f) $Y = \sum_{i=1}^{10} X_i$, where X_1, \dots, X_{10} are independent Geometric distributions with parameter p .
6. (18 pts.)

$$f_X(x) = \begin{cases} 10e^{-10x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Draw the pdf of X .
- (b) What is the name of the random variable X ?
- (c) Compute $P(1 \leq X \leq 2)$.

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- (d) Compute the expected value of X , μ_X .
 - (e) Compute the standard deviation of X , σ_X .
 - (f) Is the skewness $E(X - \mu_X)^3$ positive, zero, or negative? Give your explanation.
7. (20 pts.) Suppose that X_1, X_2, \dots, X_n is a random sample of size n taken from a Bernoulli distribution with unknown parameter $p = P(X_i = 1)$. Regarding the sample mean $\hat{P} = \sum_{i=1}^n X_i/n$.
- (a) Prove or disprove that \hat{P} is a maximum likelihood estimator.
 - (b) Prove or disprove that \hat{P} is the estimator of p with the minimum mse (mean-squared error).