

國立清華大學 101 學年度碩士班考試入學試題

系所班組別：工業工程與工程管理學系

考試科目（代碼）：微積分(1801)

共 1 頁，第 1 頁 *請在【答案卷】作答

1. Calculate the following equations: (30 pts)

(a) $\int 2x^3 \sec^2(x^4 + 1) dx$

(b) $\int \frac{\ln x}{x} dx$

(c) $\int \sin^2 x \cos^5 x dx$

(d) $\int \frac{x^5+2}{x^2-1} dx$

(e) $\int \frac{1}{x\sqrt{4x^2+9}} dx$

2. (Multiple answers) Which of the following statements are correct for $f(x) = e^{1/x}$, $g(x,y) = \ln(x^2y)$, and $P=(1,1)$. (10 pts)

(a) The graph of $f(x)$ has a horizontal asymptote $y = 1$

(b) The graph of $f(x)$ has no inflection point

(c) $f(x)$ has no relative maximum nor relative minimum

(d) The gradient of $g(x,y)$ at P is $(2,1)$

(e) The maximum rate of change of $g(x,y)$ at P is $\sqrt{5}$

3. Determine the area of the region bounded by the graph of $f(x) = x^2 - 2x$ and the x -axis between $x = -1$ and $x = 3$. (10 pts)

4. Find the distance from the origin to the plane $x + 2y + 2z = 3$. (10 pts)

(a) Using a geometric argument (no calculus)

(b) Using the method of Lagrange multipliers and find the closest point in the plane.

5. Find the partial differential $\frac{\partial f}{\partial x}$ for $f(x,y) = \frac{2y}{y+\cos(x)}$. (5 pts)

6. Find the following limits respectively.

(a) $\lim_{x \rightarrow -\infty} \frac{\pi\sqrt{3}}{x^2}$ (5 pts)

(b) $\lim_{x \rightarrow \infty} \frac{5x^2+8x-3}{3x^2+2}$ (5 pts)

7. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, please calculate the coordinates of their locations? (10 pts)

8. An open rectangular box with square base is to be made from 48 ft.^2 of material. What dimensions will result in a box with the largest possible volume? (15 pts)