国立清華大學 101 學年度碩士班考試入學試題

系所班組別：工程與系統科學系碩士班　丙組
　　　　　　先進光學科技碩士學位學程　工程與系統科學組

考試科目（代碼）：電磁學（9803）

　　*請在【答案卷、卡】作答

1. Consider a capacitor formed by two large perfect conducting plates and two dielectric slabs (dielectric constant $\varepsilon_r = 2$, thickness d, spacing t), all in parallel, as shown in Fig. 1 (cross sectional view). If two *surface charges* of densities, σ and $-\sigma$ are placed on one side of the dielectric surfaces (see Fig. 1, marked in dashed line) and the two conducting plates are held at the same electric potential, e.g, V_0,
   (a) find the electric fields (in terms of d, t, σ, V_0, etc) in each region between the conductors, (neglect the fringing field effect) (10 %)
   (b) find the *surface charge densities* on the two conductors. (5 %)

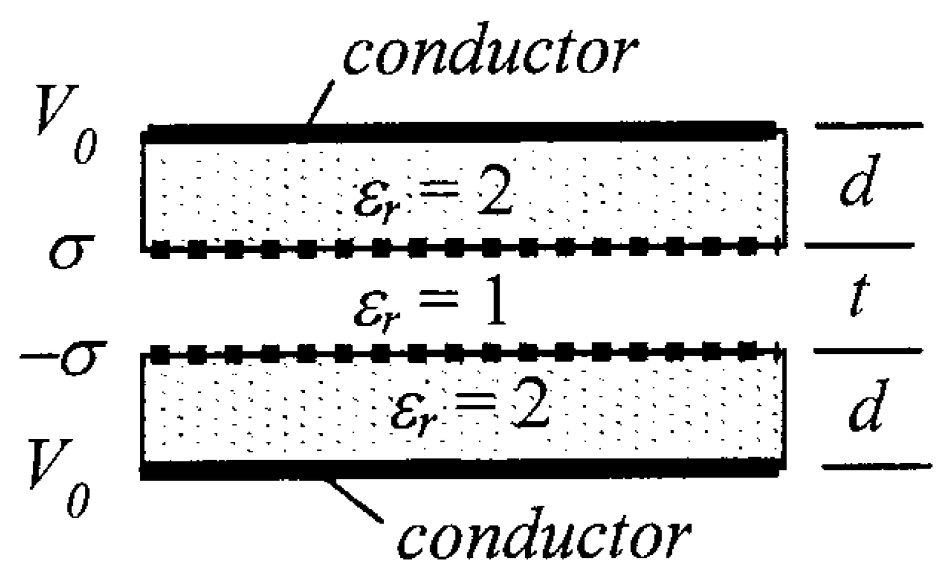
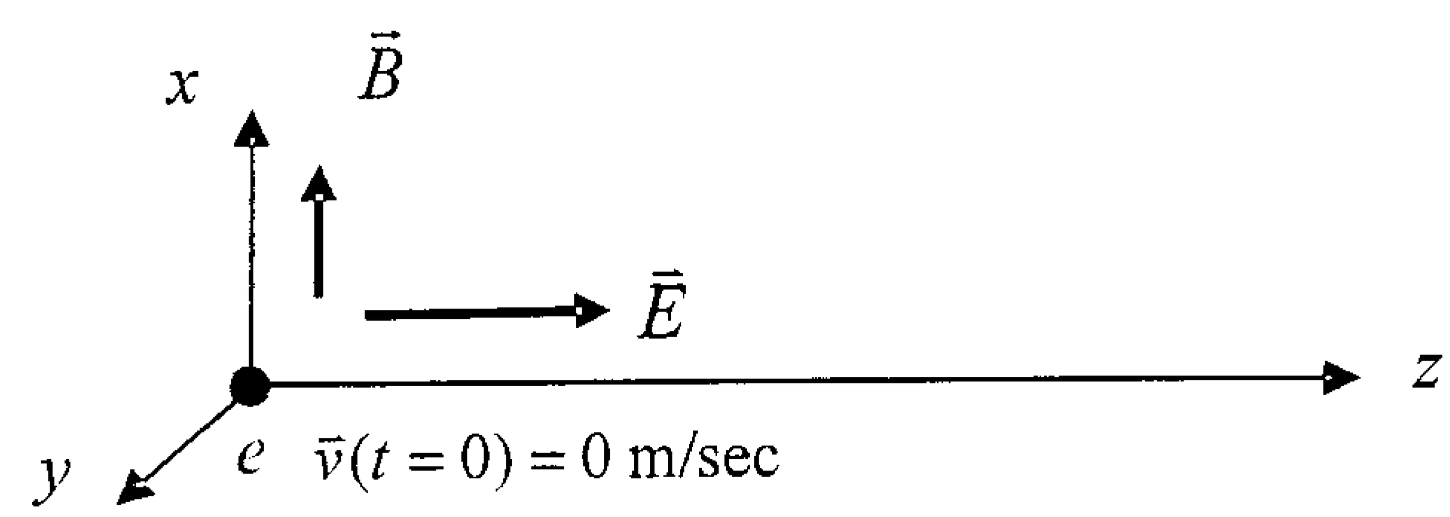


Fig. 1

2. Consider an *electron* with a velocity, at time $t = 0$ sec, $\vec{v}(t=0) = 1.0\hat{y} + 1.0\hat{z}$ m/sec .
   (a) If the electron is placed in a uniform magnetic field, $\vec{B} = 0.5\,\hat{z}$ T, find electron *trajectory* (mathematical expression and/ or description) for $t > 0$. (5 %)
   (b) If the electron is immersed in static *electric* and *magnetic* fields, $\vec{E} = 0.5\,\hat{y}$ V/m , $\vec{B} = 0.1\,\hat{y}$ T , respectively, find the *work done per unit time* on the electron by the fields (at $t = 0$ sec). (5 %)
   (c) If the electron is initially at *rest* ($\vec{v}(t=0) = 0$ m/sec) and it is placed in a region having an electric field $\vec{E} \,//\, \hat{z}$, and a magnetic field, $\vec{B} \,//\, \hat{x}$, both are uniform, as shown in Fig. 2, find electron *trajectory* (mathematical expression and/ or description) for $t > 0$. (derivation not needed, explain your answer) (5 %)

Fig. 2



3. (a) Consider a **long solenoid** (20 turns/m) of diameter 2 m, as shown in Fig. 3. Find the *energy per unit length* stored in the solenoid if a dc current of 1.0 A (per turn) is passed in the wire. (5 %)

(b) In Fig. 3, if a 2nd wire loop having a winding of 2 turns (diameter 1.0 m) is placed at the center of the solenoid and oriented at angle 30° with respect to the solenoid axis, find the *mutual inductance* between the solenoid and the 2nd wire loop. (5 %)

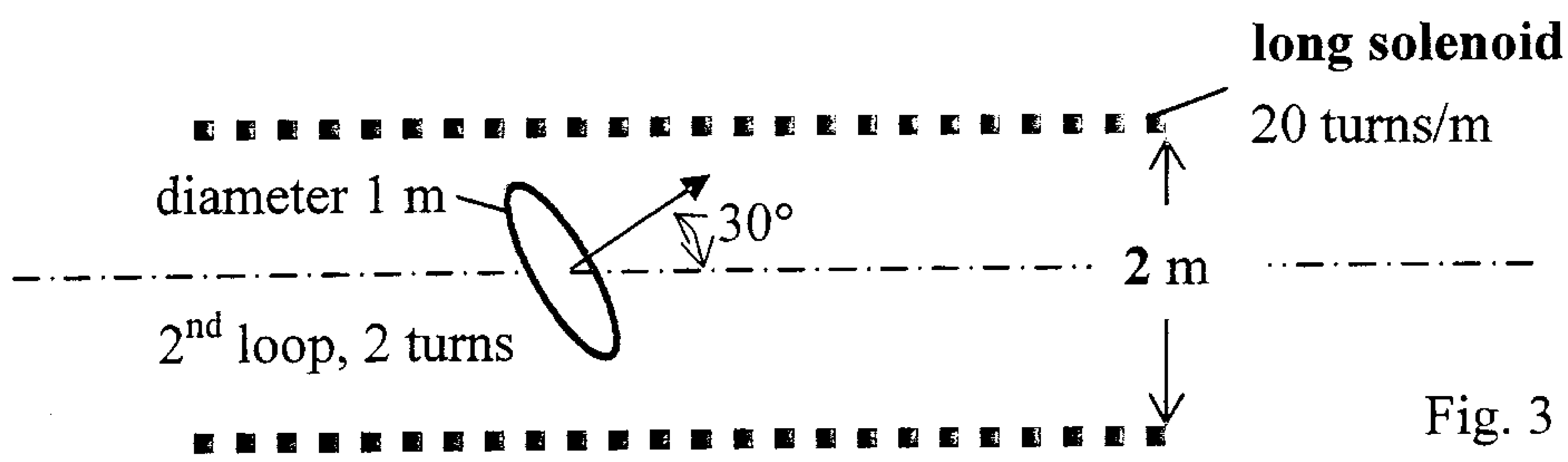


Fig. 3

4. (a) In a "good conductor" of *conductivity* $\sigma = 1.5 \times 10^{-4}$ S/m, what would be the *density of power dissipated* in the conductor if the electric field strength is $E = 2.0$ V/m. (5 %)

(b) Consider a *sinusoidal* plane electromagnetic wave propagating in a medium of refraction index $n = 1.5$ with a wave vector, $\vec{k} = 1.0\hat{x}$ m^{-1}. The corresponding *magnetic* field has an amplitude $B_0 = 1.0 \times 10^{-6}$ T and a direction $\vec{B} \parallel \hat{y}$, write down the mathematical (spatial and temporal dependence) of the corresponding electric field, $\vec{E}(\vec{r},t)$. (10 %)

5. A *dielectric sphere* of radius R and dielectric constant ε_d is placed in a medium of dielectric constant ε_p and immersed in an uniform external electric field, $\vec{E}_0 = E_0\hat{z}$, as shown in Fig. 4. By solving the Laplace equation (in spherical coordinates, see "useful information" below), find
   (a) the *electric field* distribution inside the sphere ($r < R$), (10 %)
   (b) the total induced *electric dipole moment* of the *dielectric sphere*. (5 %)

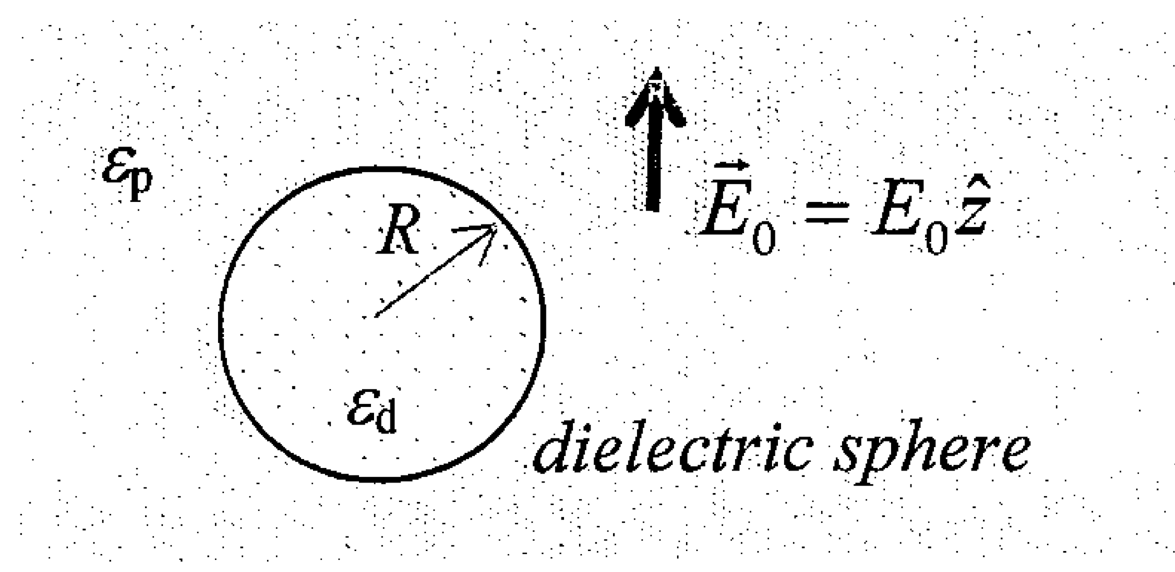


Fig. 4

Useful information: general solution for the Laplace equation in spherical coordinates (with azimuthal symmetry):

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta) \qquad (1)$$

$$\int_{-1}^{1} P_l(x)P_{l'}(x)dx = \int_0^{\pi} P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta d\theta = \begin{cases} 0, & \text{if } l' \neq l \\ \dfrac{2}{2l+1}, & \text{if } l' = l \end{cases} \qquad (2)$$

6. Consider electric *dipole radiations*, i.e., radiations generated by a sinusoidally oscillating *electric dipole* and under the "*approximations*", $d << \lambda << r$, where d, λ, and r being the *dipole size*, *wavelength* and *distance* from the dipole (to the observer),

(a) for the electromagnetic radiations in the ***far field zone*** from an oscillating *electric* dipole pointing along the z-direction, $\vec{p} = p_0 \cos \omega t\, \hat{z}$, and located at the origin (0, 0, 0), (in Cartesian coordinates), as shown in Fig. 5.

(*i*) *qualitatively* plot the *intensity patterns* (or radiation pattern, 2D polar plots of $I(\theta)$ in "E" and "H" planes,). Note that you would need to define your "E" and "H" planes in the answer. (5 %)

(*ii*) what are the "*directions*" of the *electric* and *magnetic fields*, as well as the *Poynting vector* of the radiation, (give your answer in spherical coordinates unit vectors, $\hat{r}, \hat{\theta}, \hat{\phi}$) (5 %)

(b) if the *frequency* of the time varying electric dipole in (a) is f = 500 MHz and the *radiation intensity* (time averaged power density) measured by an observer located at position $\vec{A}$ (0, 1 km, 0) is 1 mW/m^2, then what would be the *radiation intensity* at position $\vec{B}$ (0.5 km, 0, 0), if the *strength* of the oscillating dipole increases from p_0 to $2p_0$ while its *frequency* is changed from 500 MHz to 250 MHz? (5 %)

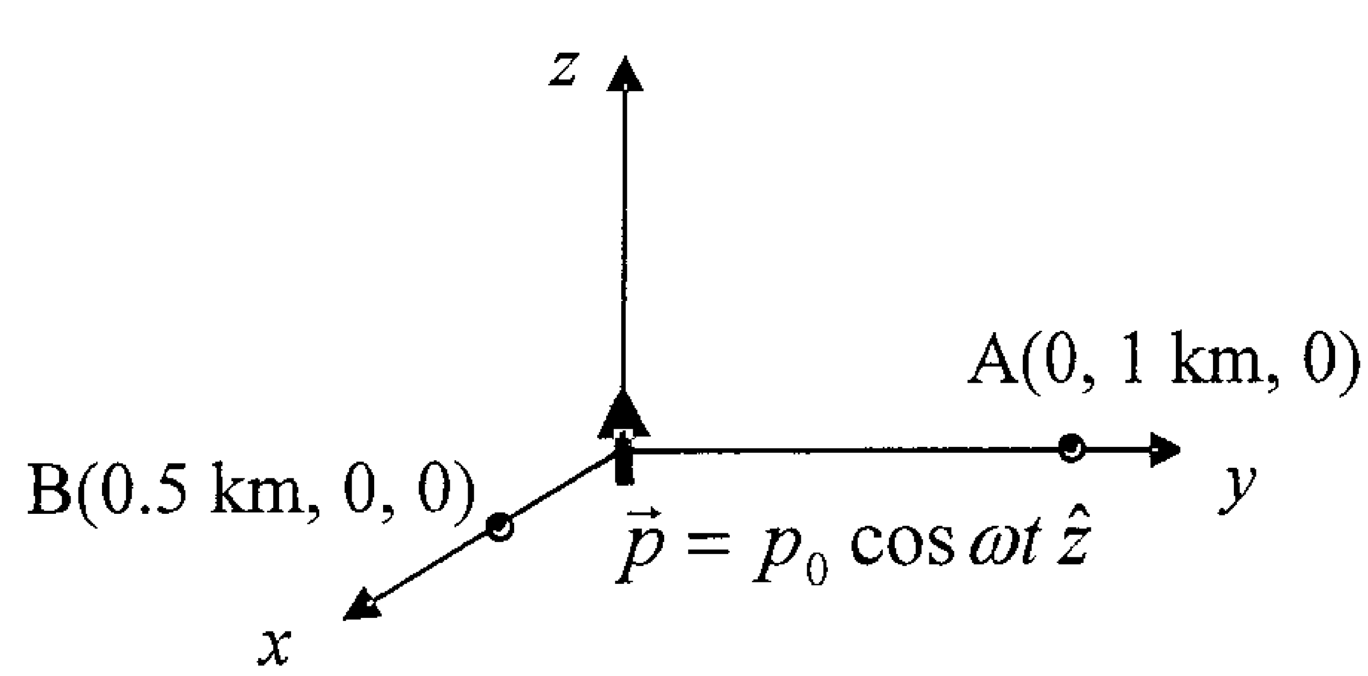


Fig. 5

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7. Consider electromagnetic waves propagating (along +z direction) in a waveguide with a uniform cross section.

(a) If the waveguide is formed by two parallel rods of *perfect conductors* as shown in Fig. 6(a), can TEM (transverse electromagnetic) wave propagate in this waveguide (in the space between the two conductors)? Explain your answer. Give an example with a different cross section structure where the TEM wave *can not* propagate. Please explain your answer. (5 %)

(b) For the **TM** modes in a *air*-filled *rectangular waveguide* formed by *perfect conductors* of width a and height b (waveguide aligned along the z-axis), as shown in Fig. 6(b), qualitatively plot the "*dispersion relation*", i.e., $k(\omega)$ or ω vs kc (c being speed of light in vacuum). Mark the *cutoff frequency*, ω_{cutoff}, on the plot and explain its physical meaning (what happen to the wave, if $\omega > \omega_{cutoff}$, and/or if $\omega < \omega_{cutoff}$). (5 %)

(c) From (b), which one, *phase velocity* (v_{ph}), or *group velocity* (v_g), that is higher than the *speed of light*? Explain your answer. Both phase and group velocities are frequency dependent, i.e., $v_{ph}(\omega)$ and $v_g(\omega)$, respectively. What is the minimum value of the *group velocity* (and occurring at what frequency, ω / ω_{cutoff} = ??)? (5 %)

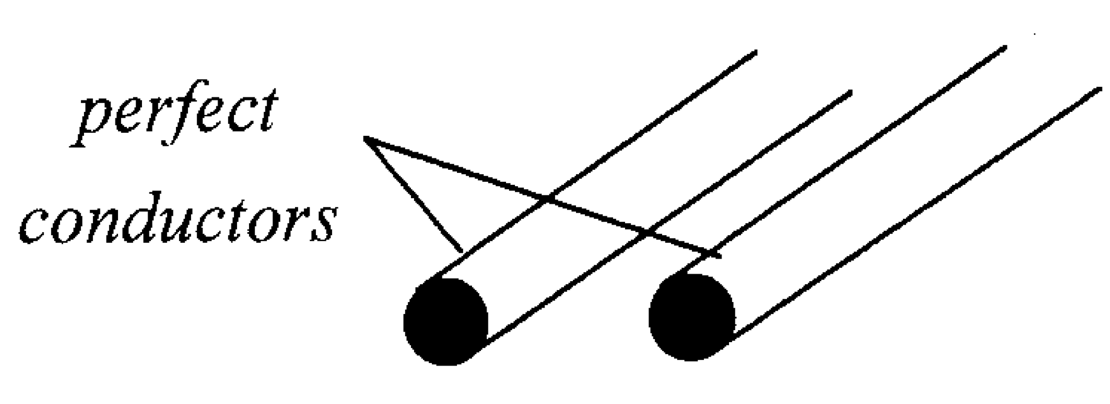

Fig. 6 (a)

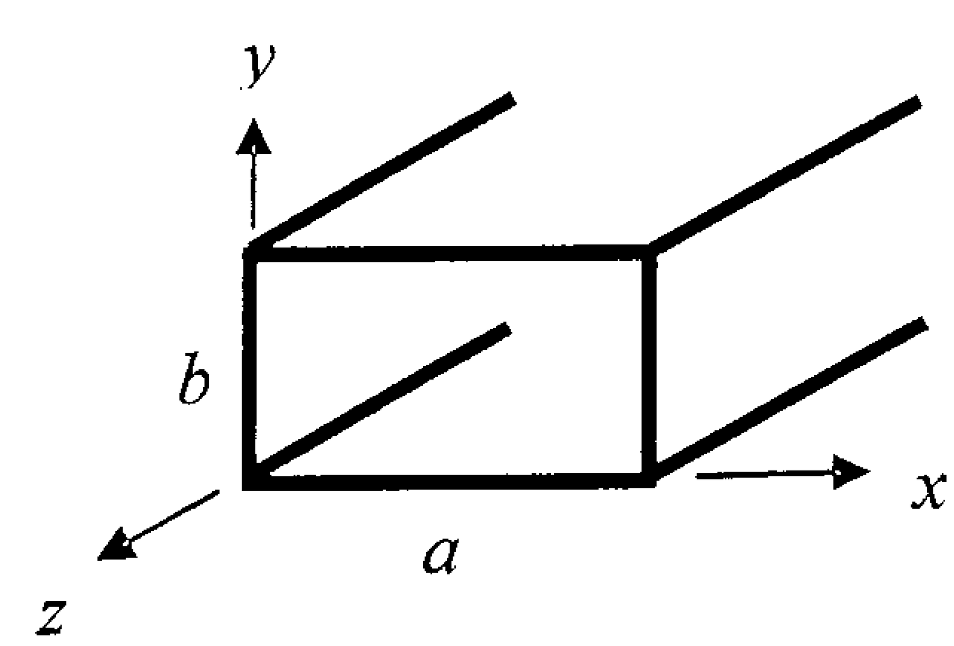

Fig. 6 (b)