

科目：應用數學(2001)

校系所組：中央大學光電科學與工程學系照明與顯示科技碩士班

交通大學電子物理學系(丙組)

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參考用

- 一. (15%) Consider a linear mapping T that acts on the linear vector space V formed by all 2×2 real matrices. For any matrix A that belongs to V

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

one finds that $T[A]$ is given by

$$T[A] = \begin{pmatrix} 2c & a+c \\ b-2c & d \end{pmatrix}.$$

Find all independent λ and matrices A with a, b, c, d being relatively prime integers such that $T[A] = \lambda A$ is satisfied.

- 二. (8 %) Evaluate the integral $\iint_S \vec{r} \cdot \hat{n} ds$ over the surface (which is a triangle) in the first octant formed by the plane $2x + 3y + 5z = 30$ and x, y, z axes. Here \hat{n} is the unit normal vector to the surface, pointing away from the origin.

- 三. (12%) Consider a vector field $\vec{V}(x, y, z) = (2xy + y^2, 2xy + x^2, z)$.

(a) Find the line integral $\int_C \vec{V} \cdot d\vec{r}$ along the path C from $(0,0,0)$ to $(1,1,1)$ for C being the curve

intersecting by two surfaces: $y - x^2 = 0$ and $z^3 - x = 0$.

(b) Consider the same line integral from $(0,0,0)$ to $(1,1,1)$ but with C being from $(0,0,0)$ to $(1,0,0)$ and then from $(1,0,0)$ to $(1,1,0)$ and finally to $(1,1,1)$. All intermediate connections are straight lines. Is the value of the line integral the same? Why? Is \vec{V} conservative? If your answer is yes, construct its potential function.

- 四. (15%) Consider a matrix

$$A = \begin{pmatrix} -2 & -1 \\ 4 & 3 \end{pmatrix}.$$

Let $B = A + A^2 + A^3 + \dots + A^{10}$. Find explicit expression of B as a 2×2 matrix.

注意：背面有試題

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五. (15%) Find the Laurent series for the function $\frac{1}{(z-1)(z-2)}$ in each of the following domains:

(a) $|z| < 1$, (b) $1 < |z| < 2$, (c) $|z| > 2$.

六. (15%) Perform the integral $\int_0^{\infty} \frac{\sqrt{x}}{x^2+1} dx$ by contour integral method.

七. (20%) Solve the following PDE boundary problem.

$$\begin{cases} u_t = c^2 u_{xx} & (0 < x < l) \\ u(t, 0) = u(t, l) = 0 \\ u(0, x) = x(l-x) \end{cases}$$