

國立清華大學 101 學年度碩士班入學考試試題

系所班組別: 生命科學院丙組 科目: 微積分 科目代碼: 0601 共 1 頁 第 1 頁 *請在[答案卷]內作答

1. (10% each) Evaluate the following integrals

(1) $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x} dx$

(2) $\int_0^{\infty} e^{-ax} \sin bx dx$, where $a > 0$

(3) $\iint_D |\cos(x + y)| dx dy$, where $D = \{(x, y) \mid 0 \leq x \leq \pi \text{ and } 0 \leq y \leq \pi\}$

(4) $\oint_C xy^2 dx - x^2y dy$, where $C = \{(x, y) \mid x^2 + y^2 = a^2\}$, $a > 0$

2. (10% each) Evaluate the following limits

(5) $\lim_{x \rightarrow 0} \int_x^1 \frac{\cos t}{t^2} dt$

(6) $a_n = \sin(\pi\sqrt{n^2 + \pi})$, $\lim_{n \rightarrow \infty} a_n$

3. (7) (10%) Show by implicit differentiation that the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_0, y_0) is $\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1$.

(8) (10%) Expand $f(x) = \cos^3 x$ as a power series.

(9) (10%) Find the surface area of the part of the surface $z = x^2 + 2y$ that lies above the triangular region T in the xy -plane with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.

(10) (10%) Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.