

國立清華大學 102 學年度碩士班考試入學試題

系所班組別：經濟學系

考試科目（代碼）：微積分與統計（4003）

共 2 頁，第 1 頁 \*請在【答案卷、卡】作答

[Please answer all questions and show your work in details.]

1.a. (5%) Find  $dy/dx$  in terms of  $x$  if  $2y + x^2 + y^2 - 25 = 0$  and  $y > -1$ .

1.b. (5%) Evaluate the following limit:  $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{x}$ .

2. Evaluate each of the following indefinite integrals.

a. (5%)  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

b. (5%)  $\int \frac{x^3}{\sqrt{9+x^2}} dx$

3.a. (5%) Inverse the coefficient matrix to solve the following system of equations:

$$2x_1 + x_2 = 4$$

$$6x_1 + 2x_2 + 6x_3 = 20$$

$$-4x_1 - 3x_2 + 9x_3 = 3$$

3.b. (5%) Use the Taylor approximation of *order one* to approximate  $F(L, K) = L^{2/3}K^{1/3}$  at  $(L, K) = (8.1, 26.8)$ .

4. (10%) Utilize the Lagrangian function to find the minimum distance from the origin to the ellipse  $x^2 + xy + y^2 = 3$ , and the corresponding Lagrange multiplier.

5.a. (5%) Find  $y(t)$  for the equation  $\frac{dy}{dt} + ay = b$  where  $a(\neq 0)$  and  $b$  are constants.

5.b. (5%) Find  $y(t)$  for  $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 0$  if  $y(0) = 0$  and  $\frac{dy}{dt} = 3$ .

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[Instructions: Please do all **FIVE** questions and show your work in details.]

- [10 pts] Suppose  $f(x, y) = 1$  for  $0 < x < 1, 0 < y < 1$  and  $= 0$  otherwise. Obtain  $f(x|X < Y)$ .
- A particular drug was given to a group of 100 patients (Group 1), and no drug was given to another group of 100 patients (Group 2). Assume that 60 patients of Group 1 and 50 patients of Group 2 recovered. We are interested in the difference of the mean rates of recovery of the two groups ( $\mu_1 - \mu_2$ ).
  - [5 pts] Please write down the null and alternative hypotheses.
  - [5 pts] Please write down the variance of the the mean difference ( $\mu_1 - \mu_2$ ).
  - [5 pts] Please construct an 80% confidence interval on the difference of the mean rates of recovery of the two groups ( $\mu_1 - \mu_2$ ). [Note:  $Z_{\alpha/2} = 1.285$ ]
- [5 pts] Suppose the following linear regression model:
$$Y_i = \alpha + \varepsilon_i, \quad i = 1, \dots, n$$
where  $\alpha$  is the intercept and  $\varepsilon_i$  is the random error term with mean zero and variance  $\sigma_i^2$ . Please derive the variance of the ordinary least squares (OLS) estimator of  $\alpha$ .
- The probability that a person will watch a movie on TV is 0.6. If a person is watching, the probability that the show is taped is one-third. If a person is not watching, the probability that the show will be taped is 0.9.
  - [5 pts] What is the probability that the show will be taped?
  - [5 pts] What is the probability that a show is being watched given that it is being taped?
- A parallel system is one that functions as long as at least one component of it functions. A particular parallel system is composed of three independent components, each of which has a life length with an exponential( $\lambda$ )distribution,  $f(x) = \frac{1}{\lambda} \exp[-x/\lambda]$ . The lifetime of the system is the maximum of the individual life lengths.
  - [5 pts] Please write down the random variable of the lifetime of the system.
  - [5 pts] What is the distribution function of the lifetime of the system?