

國立清華大學 102 學年度碩士班考試入學試題

系所班組別：經濟學系

考試科目（代碼）：微積分與統計（4003）

共 2 頁，第 1 頁 *請在【答案卷、卡】作答

[Please answer all questions and show your work in details.]

1.a. (5%) Find dy/dx in terms of x if $2y + x^2 + y^2 - 25 = 0$ and $y > -1$.

1.b. (5%) Evaluate the following limit: $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{x}$.

2. Evaluate each of the following indefinite integrals.

a. (5%) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

b. (5%) $\int \frac{x^3}{\sqrt{9+x^2}} dx$

3.a. (5%) Inverse the coefficient matrix to solve the following system of equations:

$$2x_1 + x_2 = 4$$

$$6x_1 + 2x_2 + 6x_3 = 20$$

$$-4x_1 - 3x_2 + 9x_3 = 3$$

3.b. (5%) Use the Taylor approximation of *order one* to approximate $F(L, K) = L^{2/3}K^{1/3}$ at $(L, K) = (8.1, 26.8)$.

4. (10%) Utilize the Lagrangian function to find the minimum distance from the origin to the ellipse $x^2 + xy + y^2 = 3$, and the corresponding Lagrange multiplier.

5.a. (5%) Find $y(t)$ for the equation $\frac{dy}{dt} + ay = b$ where $a(\neq 0)$ and b are constants.

5.b. (5%) Find $y(t)$ for $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 0$ if $y(0) = 0$ and $\frac{dy}{dt} = 3$.

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[Instructions: Please do all **FIVE** questions and show your work in details.]

- [10 pts] Suppose $f(x, y) = 1$ for $0 < x < 1, 0 < y < 1$ and $= 0$ otherwise. Obtain $f(x|X < Y)$.
- A particular drug was given to a group of 100 patients (Group 1), and no drug was given to another group of 100 patients (Group 2). Assume that 60 patients of Group 1 and 50 patients of Group 2 recovered. We are interested in the difference of the mean rates of recovery of the two groups $(\mu_1 - \mu_2)$.
 - [5 pts] Please write down the null and alternative hypotheses.
 - [5 pts] Please write down the variance of the the mean difference $(\mu_1 - \mu_2)$.
 - [5 pts] Please construct an 80% confidence interval on the difference of the mean rates of recovery of the two groups $(\mu_1 - \mu_2)$. [Note: $Z_{\alpha/2} = 1.285$]
- [5 pts] Suppose the following linear regression model:
$$Y_i = \alpha + \varepsilon_i, \quad i = 1, \dots, n$$
where α is the intercept and ε_i is the random error term with mean zero and variance σ_i^2 . Please derive the variance of the ordinary least squares (OLS) estimator of α .
- The probability that a person will watch a movie on TV is 0.6. If a person is watching, the probability that the show is taped is one-third. If a person is not watching, the probability that the show will be taped is 0.9.
 - [5 pts] What is the probability that the show will be taped?
 - [5 pts] What is the probability that a show is being watched given that it is being taped?
- A parallel system is one that functions as long as at least one component of it functions. A particular parallel system is composed of three independent components, each of which has a life length with an exponential(λ)distribution, $f(x) = \frac{1}{\lambda} \exp[-x/\lambda]$. The lifetime of the system is the maximum of the individual life lengths.
 - [5 pts] Please write down the random variable of the lifetime of the system.
 - [5 pts] What is the distribution function of the lifetime of the system?