國立清華大學 101 學年度碩士班考試入學試題

系所班組別：計量財務金融系（所）乙組

考試科目（代碼）：統計學(4503)

1. A factory has 3 machines that produce the same item. The probability that an item selected is defective is 5%. From the past experience, machines 1 and 2 each produce 25% of the total output, while machine 3 produce 50% of the total output. It is also known that 7% of machine 1's output is defective, while machine 2 produces 5% defective items. Given that an item selected is defective, what is the probability that it was produced by machine 3? (10%)

2. Let $x_1, x_2, ..., x_n$ be a random sample from the following *p.d.f.*

$$f(x)=\begin{cases}\dfrac{x-a}{c}, & \text{if } a\le x\le\dfrac{a+b}{2}\\ \dfrac{b-x}{c}, & \text{if } \dfrac{a+b}{2}<x\le b\\ 0 & \text{otherwise}\end{cases}$$

For some $a<b$.

(a) Find the constant c in terms of a and b. (5%)

(b) Find $Var(\bar{x})$. (10%)

3. Let $x_1, x_2, ..., x_n$ be independent and identically distributed with mean μ and variance σ^2.

Find the correlation coefficient between x_i and $\bar{x}$, where $\bar{x}=\sum x_i/n$? (10%)

4. A hotel chain has identically sized resorts in 5 locations. The data that follow resulted from analyzing the hotel occupancies on randomly selected days in the 5 locations.

ROW	Caymen	Pennkamp	California	Mayaguez	Maui
1	28	40	21	37	22
2	33	35	21	47	19
3	41	33	27	45	25

Analysis of Variance

Source	*df*	SS	MS	*F*	*p*
Location	4	963.6			
Error	10	210.0			
Total					

(a) Referring to the above table, the value of the element in the ANOVA table that always provides an estimate of the population variance is _____. (5%)

(b) Referring to the above table, the test ratio involves the ratio of 2 elements of the ANOVA table. Of these elements, the value of the one that provides an estimate of the population variance only when the null hypothesis is true is _____.(5%)

(c) Write down your assumptions and test whether all 5 locations have different mean occupancy rates at 5% significant level.(5%)

(d) Using Levene's test for homogeneity of variances, what are the null hypothesis, the value of the test statistic, the critical value at a 5% level of significance?(5%)

(e) What is the *p*-value of the test statistic for Levene's test for homogeneity of variances?(5%)

5. A student claims that he can correctly identify whether a person is a business major or an engineering major by the way the person dresses. Suppose in actuality that if someone is a business major, he can correctly identify that person as a business major 88% of the time. When a person is an engineering major, the student will incorrectly identify that person as a business major 15% of the time. Presented with one person and asked to identify the major of this person (who is either a business or engineering major), he considers this to be a hypothesis test with the null hypothesis being that the person is a business major and the alternative that the person is an engineering major.

(a) What would be a Type I error and a Type II error? (4%)

(b) What is the "actual level of significance" of the test? What is the "actual confidence coefficient"? What is the power of the test? (6%)

6. The head librarian at the Library of Congress has asked her assistant for an interval estimate of the mean number of books checked out each day. The assistant provides the following interval estimate: from 740 to 920 books per day. If the head librarian knows that the population standard deviation is 150 books checked out per day, and

(a) she asked her assistant for a 95% confidence interval, approximately how large a sample did her assistant use to determine the interval estimate? (10%)

(b) she asked her assistant to use 25 days of data to construct the interval estimate, what confidence level can she attach to the interval estimate? (5%)

7. Let $x_1, x_2, ..., x_n$ be an independent random sample from Bernoulli with probability of success p. Find the maximum likelihood estimator for the mean and variance. (15%)

STATISTICAL TABLES

TABLE 3 Cumulative Normal Distribution. Table Entry is $\Phi(z) = \text{Prob}[Z \leq z]$

z	*.00*	*.01*	*.02*	*.03*	*.04*	*.05*	*.06*	*.07*	*.08*	*.09*
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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　　*請在【答案卷、卡】作答

TABLE G.4 95th Percentiles of the F Distribution: Table Entries f for $\text{Prob}[F_{n1,n2} \leq f] = .95$

	n_1 = Degrees of Freedom for the Numerator								
n_2	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

n_2	10	12	15	20	30	40	50	60	∞
1	241.88	243.91	245.95	248.01	250.10	251.14	252.20	252.20	254.19
2	19.40	19.41	19.43	19.45	19.46	19.47	19.48	19.48	19.49
3	8.79	8.74	8.70	8.66	8.62	8.59	8.57	8.57	8.53
4	5.96	5.91	5.86	5.80	5.75	5.72	5.69	5.69	5.63
5	4.74	4.68	4.62	4.56	4.50	4.46	4.43	4.43	4.37
6	4.06	4.00	3.94	3.87	3.81	3.77	3.74	3.74	3.67
7	3.64	3.57	3.51	3.44	3.38	3.34	3.30	3.30	3.23
8	3.35	3.28	3.22	3.15	3.08	3.04	3.01	3.01	2.93
9	3.14	3.07	3.01	2.94	2.86	2.83	2.79	2.79	2.71
10	2.98	2.91	2.85	2.77	2.70	2.66	2.62	2.62	2.54
15	2.54	2.48	2.40	2.33	2.25	2.20	2.16	2.16	2.07
20	2.35	2.28	2.20	2.12	2.04	1.99	1.95	1.95	1.85
25	2.24	2.16	2.09	2.01	1.92	1.87	1.82	1.82	1.72
30	2.16	2.09	2.01	1.93	1.84	1.79	1.74	1.74	1.63
40	2.08	2.00	1.92	1.84	1.74	1.69	1.64	1.64	1.52
50	2.03	1.95	1.87	1.78	1.69	1.63	1.58	1.58	1.45
70	1.97	1.89	1.81	1.72	1.62	1.57	1.50	1.50	1.36
100	1.93	1.85	1.77	1.68	1.57	1.52	1.45	1.45	1.30
∞	1.83	1.75	1.67	1.57	1.46	1.39	1.34	1.31	1.30