

科目：工程數學 C(3006)

校系所組：中央大學電機工程學系(電子組)

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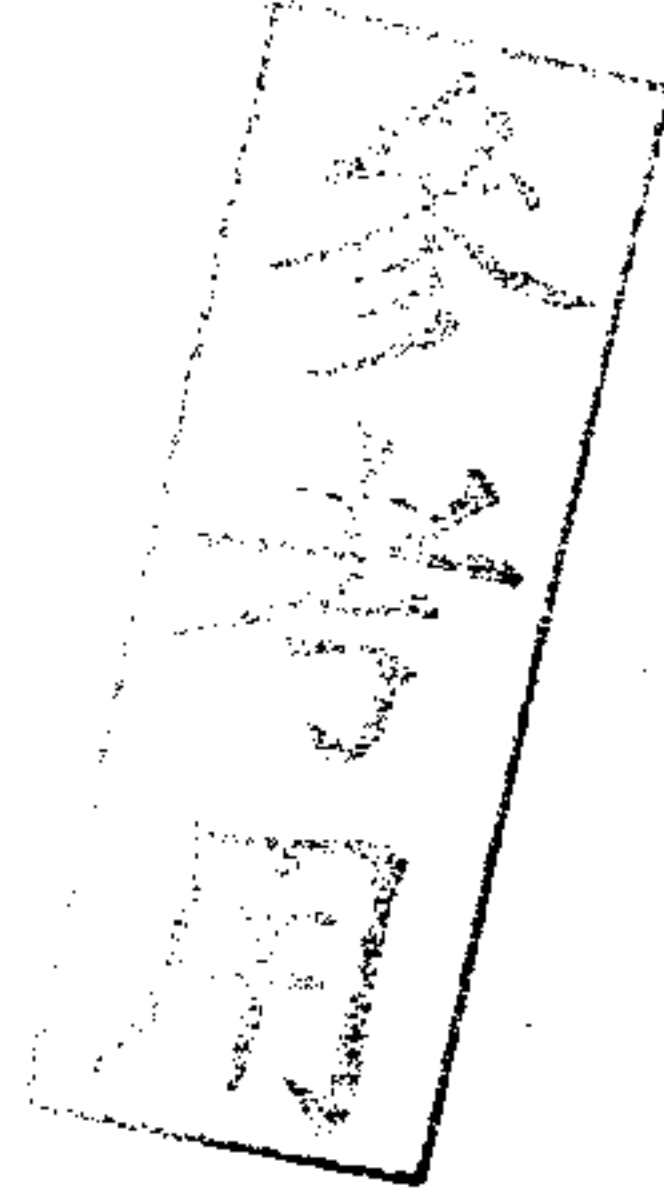
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- 本測驗試題為複選題(答案可能有一個或多個)，請選出所有正確或最適當的答案，並請用 2B 鉛筆作答於答案卡。
- 共二十題，每題完全答對得五分，答錯不倒扣。
- In the following questions, $\delta(t)$ is the Dirac delta function, $u(t)$ is unit-step function, \star is the convolution operator, $\mathcal{L} : f(t) \mapsto F(s)$ and $\mathcal{L}^{-1} : F(s) \mapsto f(t)$ denote the unilateral Laplace and inverse Laplace transforms for $t \geq 0$, respectively, boldface letters such as \mathbf{a}, \mathbf{b} , etc. denote vectors of proper length, A^T means the transpose of matrix A , and I is the identity matrix of proper size.

一、 Suppose that B_1 and B_2 are square invertible matrices. Let $\mathbf{0}$ denote an all-zero matrix of proper size. Define

$$B = \begin{bmatrix} B_1 & \mathbf{0} \\ \mathbf{0} & B_2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} B_1^{-1} & \mathbf{0} \\ \mathbf{0} & B_2^{-1} \end{bmatrix}.$$

Denote by P a permutation matrix of the same size as B and C . Which of the following statements are true?

- (A) $BC = P^T P$
 - (B) $P^T P = P P^T$.
 - (C) $B^T C B P C^{-1} B^{-1} P^T C^T = I$
 - (D) There exists an integer $k \neq 1$ such that $P^k = P$.
 - (E) None of the above are true.
- 二、 Using the forward elimination process with possible row exchanges to produce an upper triangular matrix U , which of the following statements are true?

- (A) When performing forward elimination on matrix $B = \begin{bmatrix} B_1 & \mathbf{0} \\ \mathbf{0} & B_2 \end{bmatrix}$ to produce an upper triangular matrix U , where B_1 and B_2 are square matrices and $\mathbf{0}$ denotes an all-zero matrix of proper size, U can be made to be equal to $U = \begin{bmatrix} U_1 & \mathbf{0} \\ \mathbf{0} & U_2 \end{bmatrix}$, where U_1 and U_2 are respectively the resulting forward elimination upper triangular outputs due to inputs B_1 and B_2 .
- (B) Suppose that $P_1 B_1 = L_1 U_1$ and $P_2 B_2 = L_2 U_2$, where P_1 and P_2 are permutation matrices, L_1 and L_2 are square lower triangular matrices, and U_1 and U_2 are square upper triangular matrices. Then,

$$\begin{bmatrix} P_1 & \mathbf{0} \\ \mathbf{0} & P_2 \end{bmatrix} \begin{bmatrix} B_1 & \mathbf{0} \\ \mathbf{0} & B_2 \end{bmatrix} = \begin{bmatrix} L_1 & \mathbf{0} \\ \mathbf{0} & L_2 \end{bmatrix} \begin{bmatrix} U_1 & \mathbf{0} \\ \mathbf{0} & U_2 \end{bmatrix}.$$

- (C) To perform forward elimination on a matrix $F = \begin{bmatrix} A & \mathbf{0} \\ C & D \end{bmatrix}$ to produce an upper triangular matrix U , where A , C and D are square invertible matrices, we can first do block-based forward elimination to obtain $G = \begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & D \end{bmatrix}$; then perform forward elimination respectively on A and D to obtain upper triangular U_A and U_D . The desired U is thus given by $\begin{bmatrix} U_A & \mathbf{0} \\ \mathbf{0} & U_D \end{bmatrix}$.

注意：背面有試題

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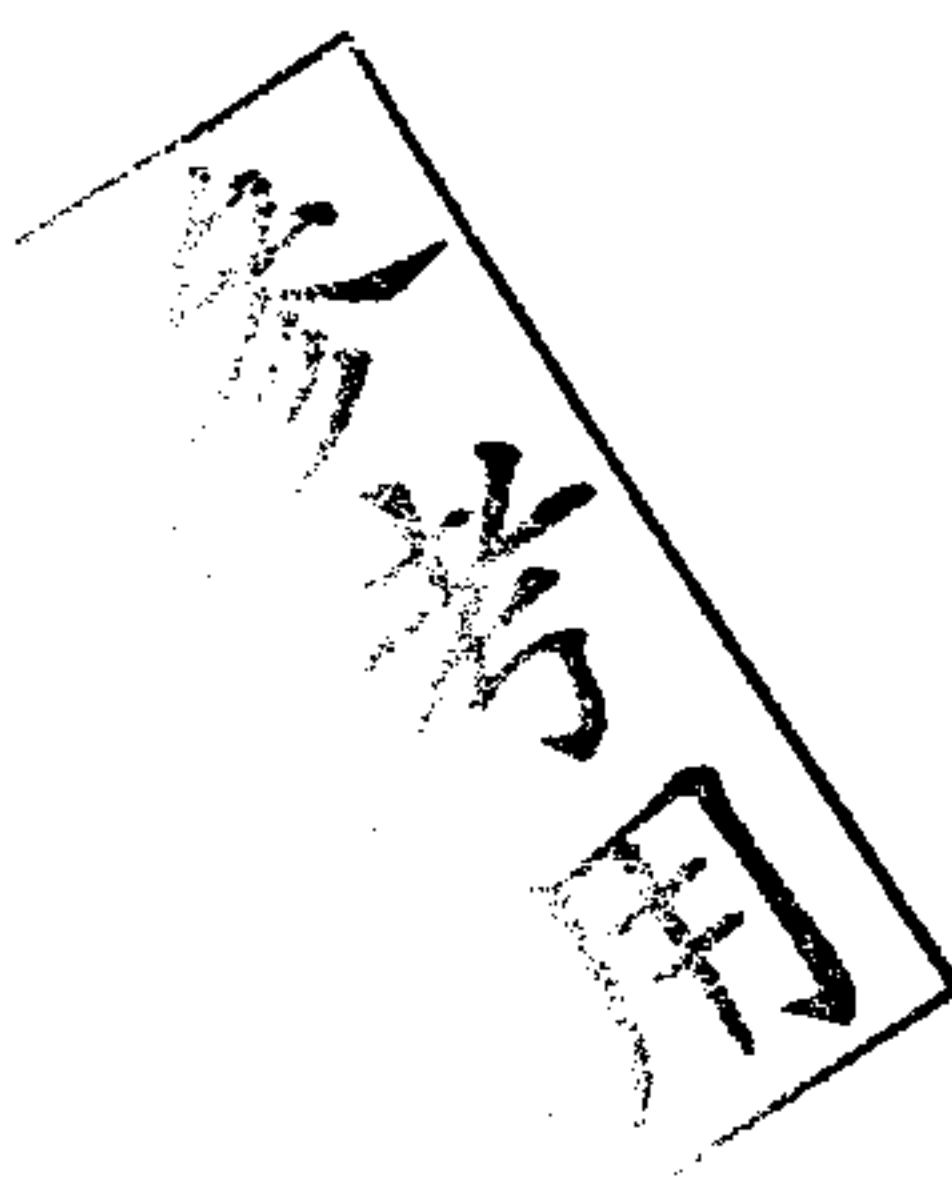
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- (D) To perform forward elimination on a matrix $F = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ to produce an upper triangular matrix U , where A, B, C and D are square invertible matrices, U can be made to be equal to $\begin{bmatrix} U_A & 0 \\ 0 & U_D \end{bmatrix}$, where U_A and U_D are respectively the forward elimination upper triangular outputs due to inputs A and D .
- (E) None of the above are true.

三、 Let $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ be nonzero matrix and nonzero vector, respectively. Which of the following sets are subspaces of \mathbb{R}^n ?

- (A) $\{x \in \mathbb{R}^n | Ax = b\}$.
- (B) $W_1 \cap W_2$, where W_1 and W_2 are two subspaces of \mathbb{R}^n .
- (C) $\{x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n | \sum_{i=1}^n x_i = 0\}$.
- (D) $\{x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T | x'(t) = Ax(t) + b\}$.
- (E) None of the above are true.

四、 Which of the following statements are true?

- (A) Let $u_1, \dots, u_k, k < n$, be linearly independent unit vectors in \mathbb{R}^n and $U = [u_1 \ \dots \ u_k] \in \mathbb{R}^{n \times k}$. Then, $I_n - UU^T$ is a projection matrix that projects a vector onto the column space of U , where I_n is an identity matrix of size $n \times n$.
- (B) Let $A \in \mathbb{R}^{n \times k}, k < n$ and $\text{rank}(A) = k$. Then $A(A^T A)^{-1} A^T$ is the projection matrix that projects a vector onto the column space of A .
- (C) Let $A \in \mathbb{R}^{n \times k}, n < k$ and $\text{rank}(A) = n$. Then $x = A^T(AA^T)^{-1}b$ is the solution of $Ax = b$ having minimum Euclidean norm.
- (D) All the orthogonal matrices of $\mathbb{R}^{2 \times 2}$ can be expressed either in the form of $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ or $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$.
- (E) None of the above are true.

五、 Let $A = QR$ be the QR factorization of A , where $A = [a_1 \ \dots \ a_n] \in \mathbb{R}^{n \times n}$, and $Q = [q_1 \ \dots \ q_n] \in \mathbb{R}^{n \times n}$ is an orthogonal matrix, and $R = [r_{i,j}] \in \mathbb{R}^{n \times n}$ is an upper triangular matrix. Which of the following statements are true?

- (A) a_k and q_k are linearly dependent for any $k \leq n$.
- (B) $\text{span}\{a_1, \dots, a_k\} = \text{span}\{q_1, \dots, q_k\}$ for any $k \leq n$.
- (C) $\{q_i | r_{i,i} \neq 0, i = 1, 2, \dots, n\}$ is an orthonormal basis of the column space of A .
- (D) If $r_{k,k} = 0$, then the vectors $\{a_1, \dots, a_k\}$ are linearly dependent.
- (E) None of the above are true.

六、 Consider a 4×4 real matrix A with three different eigenvalues 0, 1, 2. Which of the following statements are true?

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參考用

- (A) The determinant of A is 0.
 (B) There are three linearly independent eigenvectors.
 (C) The rank of A is 2.
 (D) The trace of A is 3.
 (E) None of the above are true.
- 七、 Consider two similar real matrices A and B . Which of the following statements are true?
- (A) A and B have the same set of eigenvalues.
 (B) A and B have the same set of eigenvectors.
 (C) A and B have the same null space.
 (D) A and B have the same rank.
 (E) None of the above are true.
- 八、 Consider an $m \times n$ real matrix A with linearly independent columns, and $m > n$. Which of the following statements are true?
- (A) $A^T A$ is positive definite.
 (B) AA^T is positive definite.
 (C) The column space of A is spanned by all the eigenvectors of AA^T .
 (D) The row space of A is spanned by all the eigenvectors of $A^T A$.
 (E) None of the above are true.
- 九、 Which of the following statements are true?
- (A) Let $A = \begin{bmatrix} 1 + \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \alpha_1 & 1 + \alpha_2 & \cdots & \alpha_n \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \alpha_2 & \cdots & 1 + \alpha_n \end{bmatrix}$. Then, $\det(A) = 1 + n \sum_{i=1}^n \alpha_i$.
- (B) Let A be a square matrix, and c and d be two column vectors. If $Ax = c$, then $\det(A + cd^T) = \det(A)(1 + d^T x)$.
- (C) Consider the 4×4 orthogonal projection matrix $Q = I - uu^T$, where $u \in \mathbb{R}^4$ and I is the 4×4 identity matrix. Then $\det(Q) + \text{rank}(Q) = 4$.
- (D) One can construct a 3×3 Hermitian matrix A with complex-valued entries such that $\det(A) = 1 + i$, where $i = \sqrt{-1}$.
- (E) None of the above are true.
- 十、 Which of the following statements are true?
- (A) Let T be a linear transformation from \mathbb{R}^n onto \mathbb{R}^m . Then T is one-to-one.
 (B) Let T be a linear transformation from \mathbb{R}^n to \mathbb{R}^m . Then $\{v_1, \dots, v_k\}$ can be linearly dependent even if $\{Tv_1, \dots, Tv_k\}$ is linearly independent.

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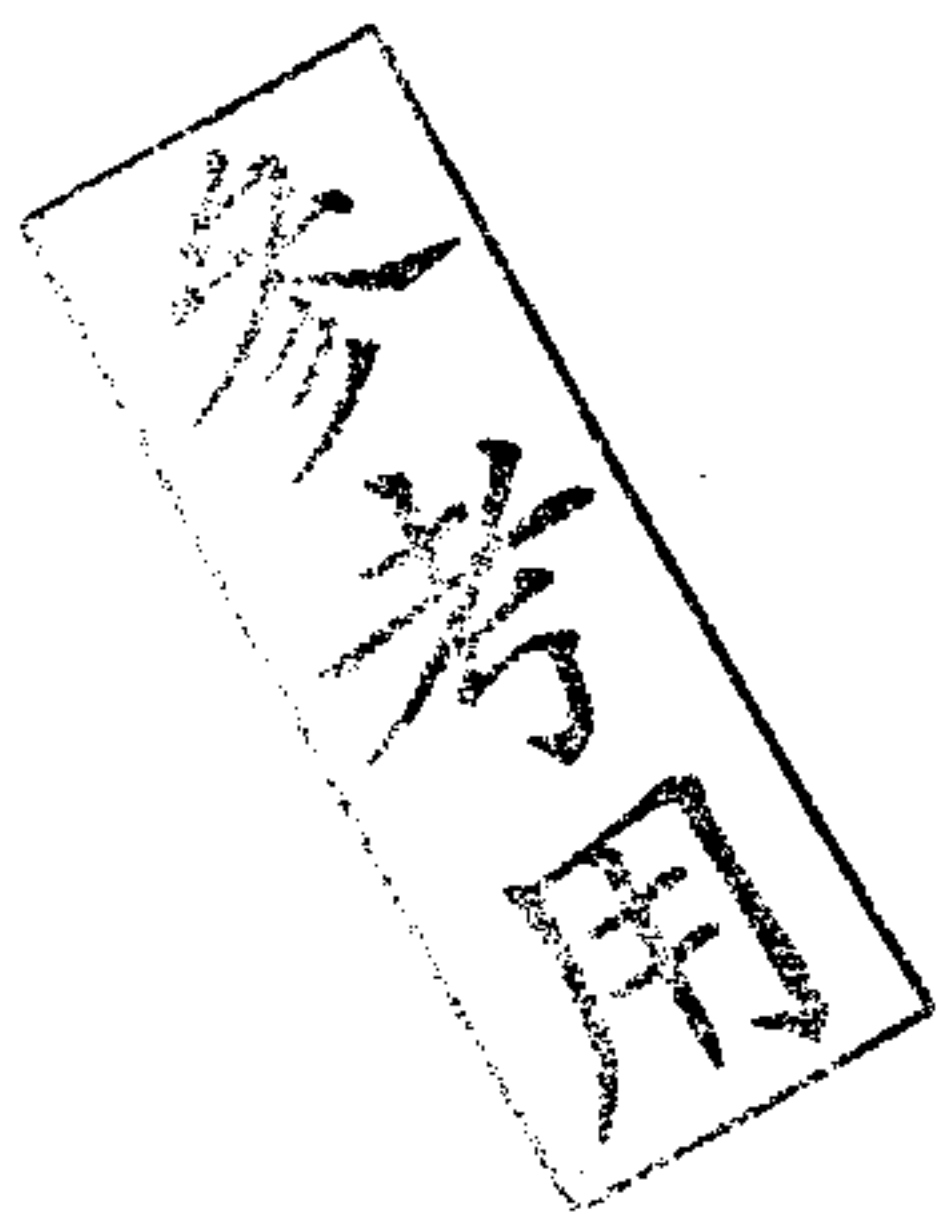
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(C) Let T be a linear transformation from \mathbb{R}^n to \mathbb{R}^n , and let $S \subset \mathbb{R}^n$ be a subspace such that $Ts \in S$ for all $s \in S$. Then $\dim(S) \in \{0, 1, n\}$, where $\dim(S)$ denotes the dimension of S .

(D) Let $A \in \mathbb{R}^{m \times n}$ with $m > n > 3$. Then $\text{rank}(AA^T) < \text{rank}(A)$.

(E) None of the above are true.

十一、 Consider the first order differential equation $(4xy + 1)dy + y^2dx = 0$. Which of the following statements are true?

(A) $x = y^{-1} + Cy^{-4}$ for some constant C is the general solution.

(B) $3xy^4 + y^3 = C$, where $y \neq 0$ and C is some constant, is an implicit solution.

(C) $y(x) = 0$ is a solution.

(D) If the solution curve passes through the point $(0, 3)$ in the $x - y$ plane, then it also passes through the point $(7, 1)$.

(E) None of the above are true.

十二、 Given one solution $y_1(x) = e^x$ to the homogeneous second order linear differential equation $(x + 1)y''(x) - (x + 2)y'(x) + y(x) = 0$ with $x > -1$, the second linearly independent solution $y_2(x)$ takes the form of $y_2(x) = v(x)y_1(x)$. Which of the following statements are true?

(A) The function $v(x)$ satisfies $(x + 1)v''(x) - (x + 2)v'(x) = 0$.

(B) The function $v(x)$ satisfies $(x + 1)v''(x) + xv'(x) = 0$.

(C) $v(x) = xe^x$.

(D) $v(x) = (2 + x)e^{-x}$.

(E) None of the above are true.

十三、 Solve the third order differential equation $y'''(x) - \frac{3}{x}y''(x) + \frac{6}{x^2}y'(x) - \frac{6}{x^3}y(x) = \frac{1}{x^4}$ with $x > 0$. Which of the following statements are true?

(A) $y(x) = -\frac{1}{24}x^{-1}$ is a solution.

(B) $y(x) = \frac{1}{24}x$ is a solution.

(C) $y(x) = \frac{1}{24}(x - x^{-1})$ is a solution.

(D) If the solution $y(x)$ satisfies $y(1) = 0$, $y'(1) = \frac{1}{6}$, and $y''(1) = \frac{1}{6}$, then $y(2) = \frac{1}{16}$.

(E) None of the above are true.

十四、 Consider the system of linear differential equations

$$\mathbf{x}'(t) = A\mathbf{x}(t), \quad \text{where } A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}.$$

Which of the following statements are true?

(A) $\mathbf{x}(t) = \begin{bmatrix} 2 \cos(t) - \sin(t) \\ \cos(t) \end{bmatrix}$ is a solution.

(B) $\mathbf{x}(t) = \begin{bmatrix} \cos(t) - 2 \sin(t) \\ \sin(t) \end{bmatrix}$ is a solution.

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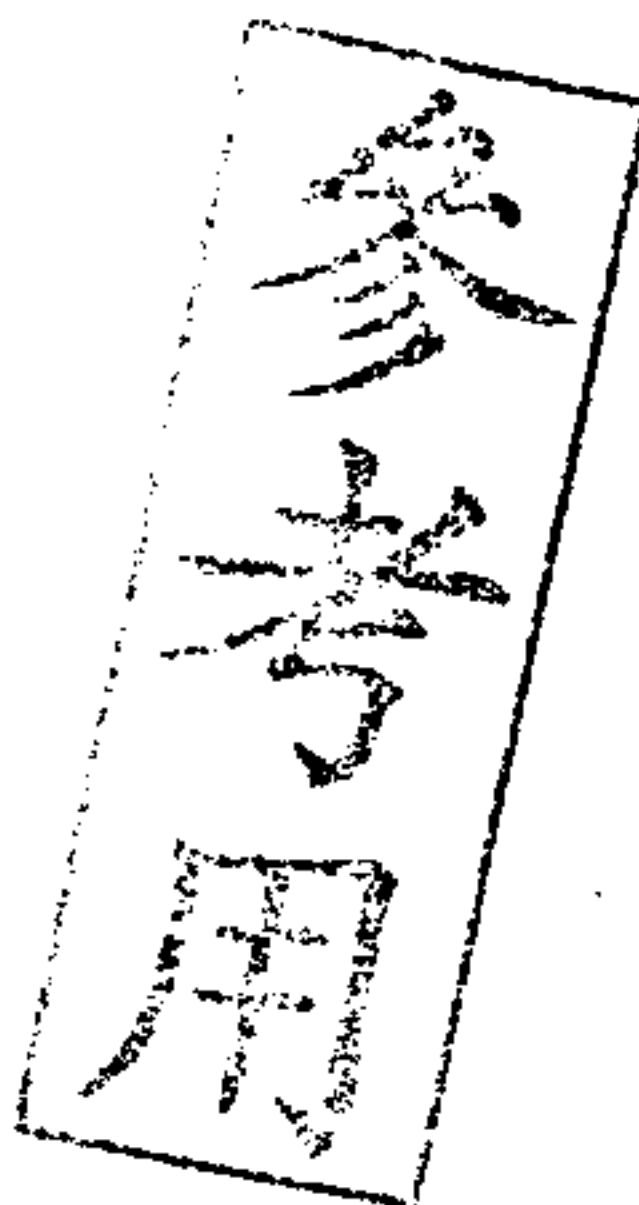
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(C) $e^{\pi A} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

(D) $e^{\frac{\pi}{2}A} = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$.

(E) None of the above are true.

十五、 Which of the following statements are true?

(A) $\mathcal{L} \left\{ \int_0^t \sin(a(t-\tau)) \sin(a\tau) d\tau \right\} = \frac{1}{(s^2+a^2)^2}$.

(B) $(\delta * f)(t) \neq f(t)$.

(C) $(u * f)(t) = f(t)$, where $u(t)$ is the unit-step function.

(D) If $y'(t) + 2y(t) + \int_0^t y(\tau) d\tau = u(t-1)$ and $y(0) = 0$, then $Y(s) = \mathcal{L}\{y(t)\} = \frac{e^{-s}}{(s+1)^2}$.

(E) None of the above are true.

十六、 Let $x(t)$ be the solution of the initial value problem $x''(t) + p_0x'(t) + q_0x(t) = f(t)$, $t \geq 0$, $x(0) = a$ and $x'(0) = b$. Let $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{x(t)\} = X(s)$. If $X(s) = \frac{2s+1}{s^2+2s+1} + \frac{F(s)}{s^2+2s+1}$, then which of the following statements are true?

(A) $p_0 + q_0 = -3$.

(B) $a + b = -1$.

(C) If $f(t) = 0$, then $x(t) = 2e^{-t} - te^{-t}$.

(D) If $f(t) = \delta(t-3)$, then $\mathcal{L}^{-1}\left\{\frac{F(s)}{s^2+2s+1}\right\} = u(t-3)te^{-t}$.

(E) None of the above are true.

十七、 Consider the following non-homogeneous linear system

$$x'(t) = Ax(t) + f(t), \text{ where } A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix} \text{ and } f(t) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} \quad (*)$$

Given that $\lambda_1 = 1$ and $\lambda_2 = 1 + 2i$ are eigenvalues for A and that $\begin{bmatrix} 0 \\ 1 \\ -i \end{bmatrix}$ is an eigenvector of A associated with the eigenvalue λ_2 , which of the following statements are true?

(A) $\begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} e^t$ is a solution to $x'(t) = Ax(t)$.

(B) $e^{At} = \begin{bmatrix} 2e^t & 0 & 0 \\ -3e^t & e^t \cos(2t) & e^t \sin(2t) \\ 2e^t & e^t \sin(2t) & -e^t \cos(2t) \end{bmatrix}$.

(C) The system (*) has a particular solution $x_p(t) = be^{2t}$ with $(A - 2I)b = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

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(D) $x_p(t) = \begin{bmatrix} -1 \\ \frac{4}{5} \\ \frac{7}{5} \\ -1 \end{bmatrix} e^{2t}$ is a particular solution to the system (*).

(E) None of the above are true.

十八、 Let $f(t) = \sum_{n=-\infty}^{\infty} g(t-n)$, where $g(t) = t^2[u(t) - u(t-2)]$. Represent $f(t)$ in terms of Fourier series as $\tilde{f}(t) = \sum_{n \geq 0} a_n \cos(n\pi t) + b_n \sin(n\pi t)$. Which of the following statements are true?

(A) $f(t)$ is a periodic function with minimum period 2.

(B) $b_n = -\frac{4(1+(-1)^n)}{n\pi}$ for $n \geq 1$.

(C) $b_n = -\frac{4(1-(-1)^n)}{n\pi}$ for $n \geq 1$.

(D) $\tilde{f}(t=1) = 3$.

(E) None of the above are true.

十九、 Consider the following boundary value problem for the bivariate function $g(x, t)$ with $10 < x < 20$

$$\frac{\partial g(x, t)}{\partial t} = 5 \frac{\partial^2 g(x, t)}{\partial x^2} \quad \text{for } 10 < x < 20 \text{ and } t > 0$$

$$g(10, t) = g(20, t) = 0 \quad \text{for } t > 0$$

Assuming $g(x, t) = X(x)T(t)$ is separable, which of the following statements are true?

(A) The function $X(x)$ satisfies $X''(x) + X(x) = 0$ with $X(10) = X(20) = 0$.

(B) Solutions to $X(x)$ take the form of $(-1)^n \sin\left(\frac{n\pi}{10}x\right)$ for $n = 1, 2, \dots$

(C) Assuming $X(x) = \sin\left(\frac{n\pi}{20}x\right)$ for some positive integer n , the corresponding $T(t)$ satisfies $T'(t) + \frac{n^2\pi^2}{80}T(t) = 0$.

(D) Assuming $X(x) = \sin\left(\frac{n^2\pi}{10}x\right)$ for some positive integer n , the corresponding $T(t)$ satisfies $T'(t) + \frac{n\pi}{10}T(t) = 0$.

(E) None of the above are true.

二十、 Continued from Question 十九, we further assume that $g(x, 0) = x$ for $10 < x < 20$ and that the solution $g(x, t)$ takes the form of

$$g(x, t) = \sum_{n=1}^{\infty} c_n X_n(x) T_n(t)$$

for some constants c_n . Which of the following statements are true?

(A) $c_n = \frac{20((-1)^n - 2)}{n\pi}$, $X_n(x) = \sin\left(\frac{n\pi}{10}x\right)$ and $T_n(t) = \exp\left(-\frac{n^2\pi^2}{20}t\right)$.

(B) $c_n = \frac{20(1-2(-1)^n)}{n\pi}$, $X_n(x) = \sin\left(\frac{n\pi}{10}x\right)$ and $T_n(t) = \exp\left(-\frac{n^2\pi^2}{20}t\right)$.

(C) $\sum_{n=1}^{\infty} c_n X_n(20) T_n(0) = 20$.

(D) $\sum_{n=1}^{\infty} c_n X_n(10) T_n(0) = 0$.

(E) None of the above are true.