

國立清華大學 102 學年度碩士班考試入學試題

系所班組別：資訊工程學系

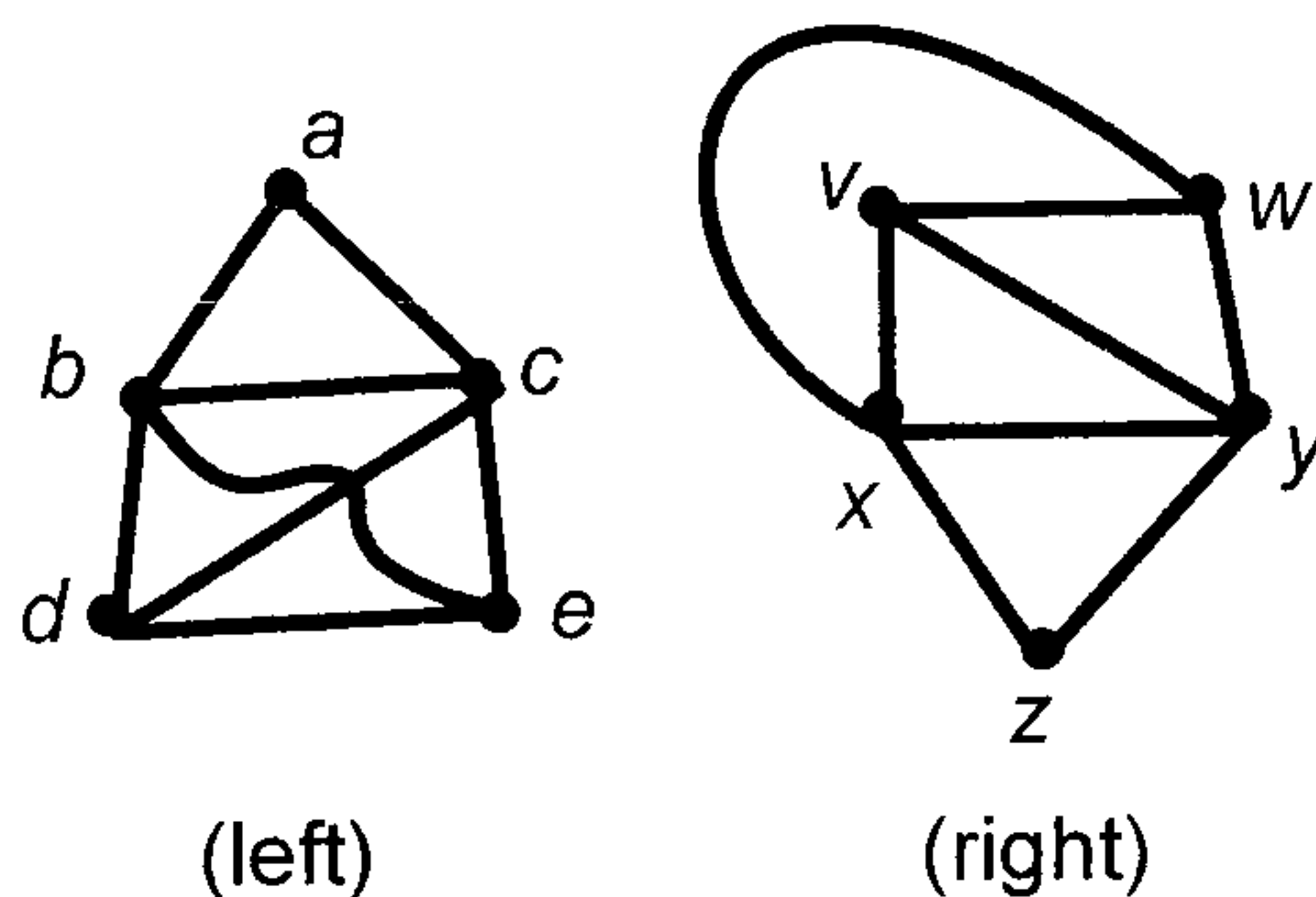
考試科目（代碼）：基礎計算機科學(2001)

共 8 頁，第 1 頁*請在【答案卷、卡】作答

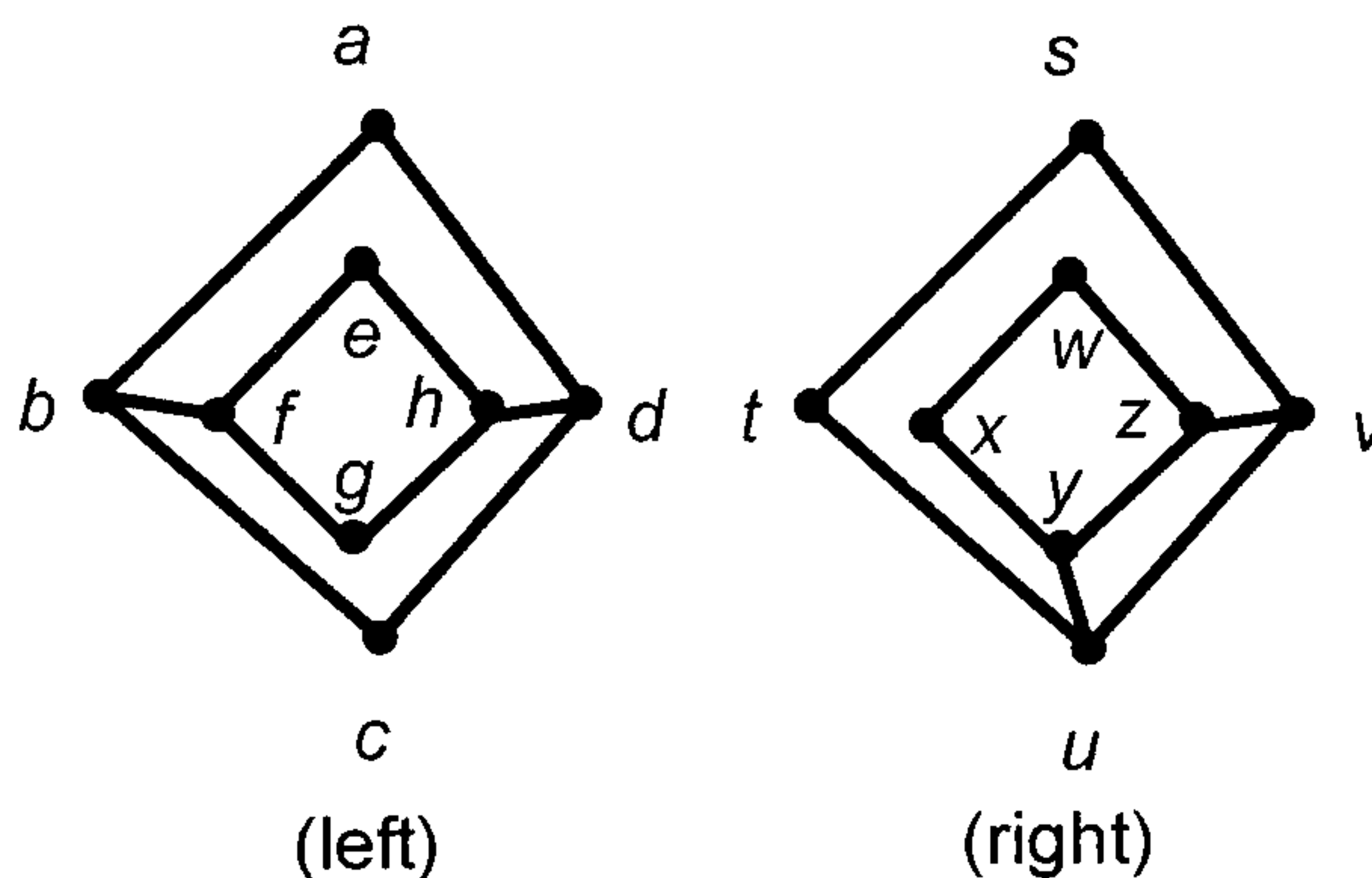
1. (17%) Answer the following questions.

(A) (4%) Prove or disprove whether the two graphs in each subproblem are isomorphic. No point will be given without a formal proof.

(a) (2%)



(b) (2%)



(B) (4%) Prove that if $n \in \mathbb{Z}^+$ and $n \geq 3$, then $\prod_{i=2}^n (1 - i^{-2}) = \frac{n+1}{2n}$.

(C) (4%) How many $n < 801, n \in \mathbb{Z}^+$ are not divisible by 2, 3, 4, 5, 6, 8, and 10?

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(D) (5%) Define $p(x)$, $q(x)$, and $r(x)$ as follows:

$$p(x): x^2 - 26x + 120 = 0$$

$$q(x): x \text{ is even}$$

$$r(x): x < 0$$

Determine the truth or falsity of the following statements, for the universe of integers. For a false statement, give a counter example.

(a) (1%) $\forall x [q(x) \rightarrow p(x)]$

(b) (1%) $\exists x [q(x) \rightarrow p(x)]$

(c) (1%) $\exists x [p(x) \rightarrow (q(x) \wedge r(x))]$

(d) (1%) $\forall x [\neg q(x) \rightarrow \neg p(x)]$

(e) (1%) $\forall x [(p(x) \vee q(x)) \rightarrow r(x)]$

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2. (17%) Answer the following questions.

(A) (5%) Solve the following recurrence:

$$G_0 = 1, G_1 = 2, G_k = 5G_{k-1} + 6G_{k-2} \text{ for } k \geq 2.$$

(B) (4%) In a 10-sided polygon,

(i) How many diagonals are there?

(ii) Suppose that no three diagonals meet at one point.
How many intersections will the diagonals form?

(C) (4%) How many positive integral solutions (x, y, z) for the following inequality?

$$x + y + z \leq 100$$

(D) (4%) Find the greatest common divisor of 41537 and 248419.

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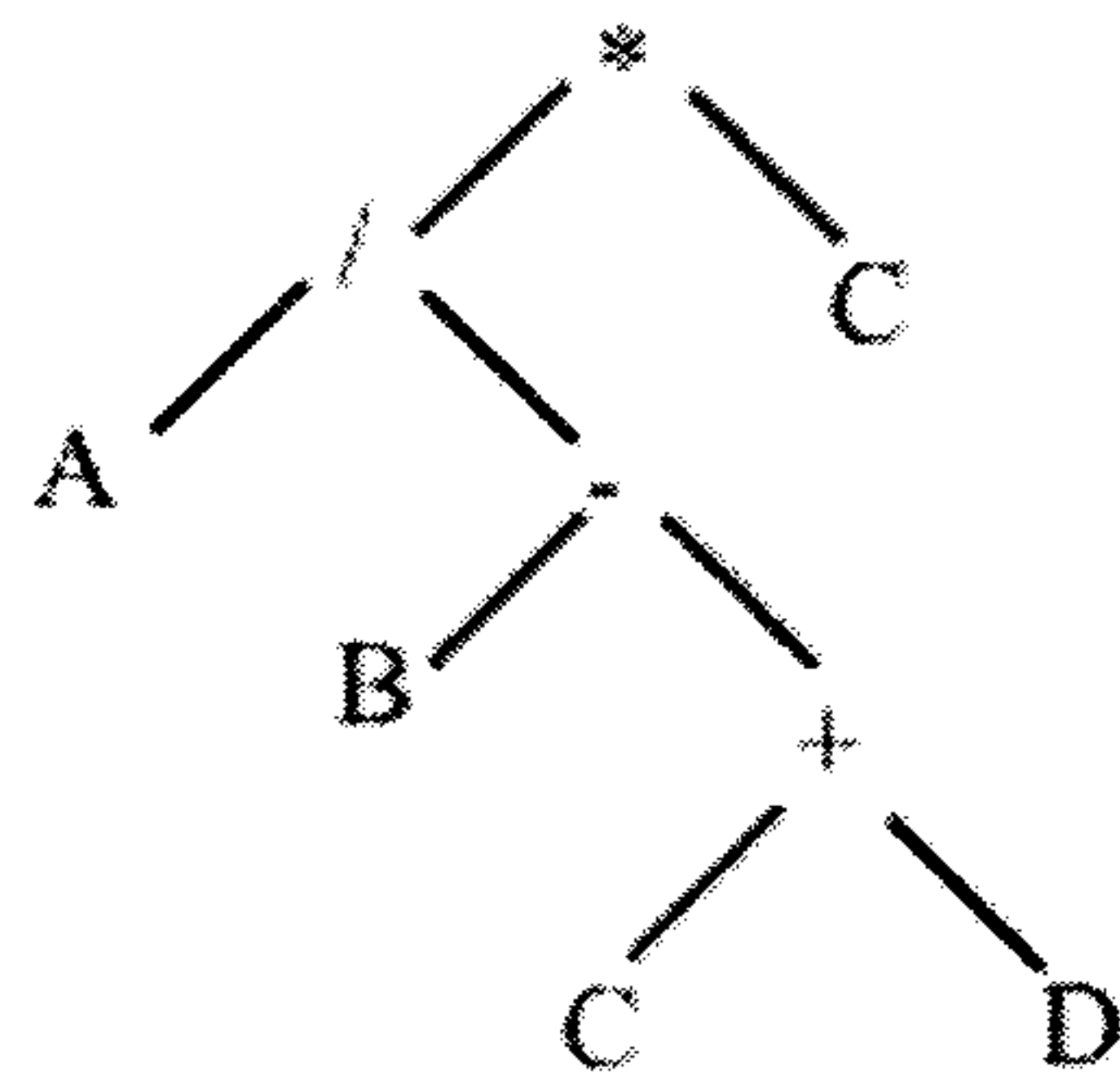
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3. (17%) Answer the following questions.

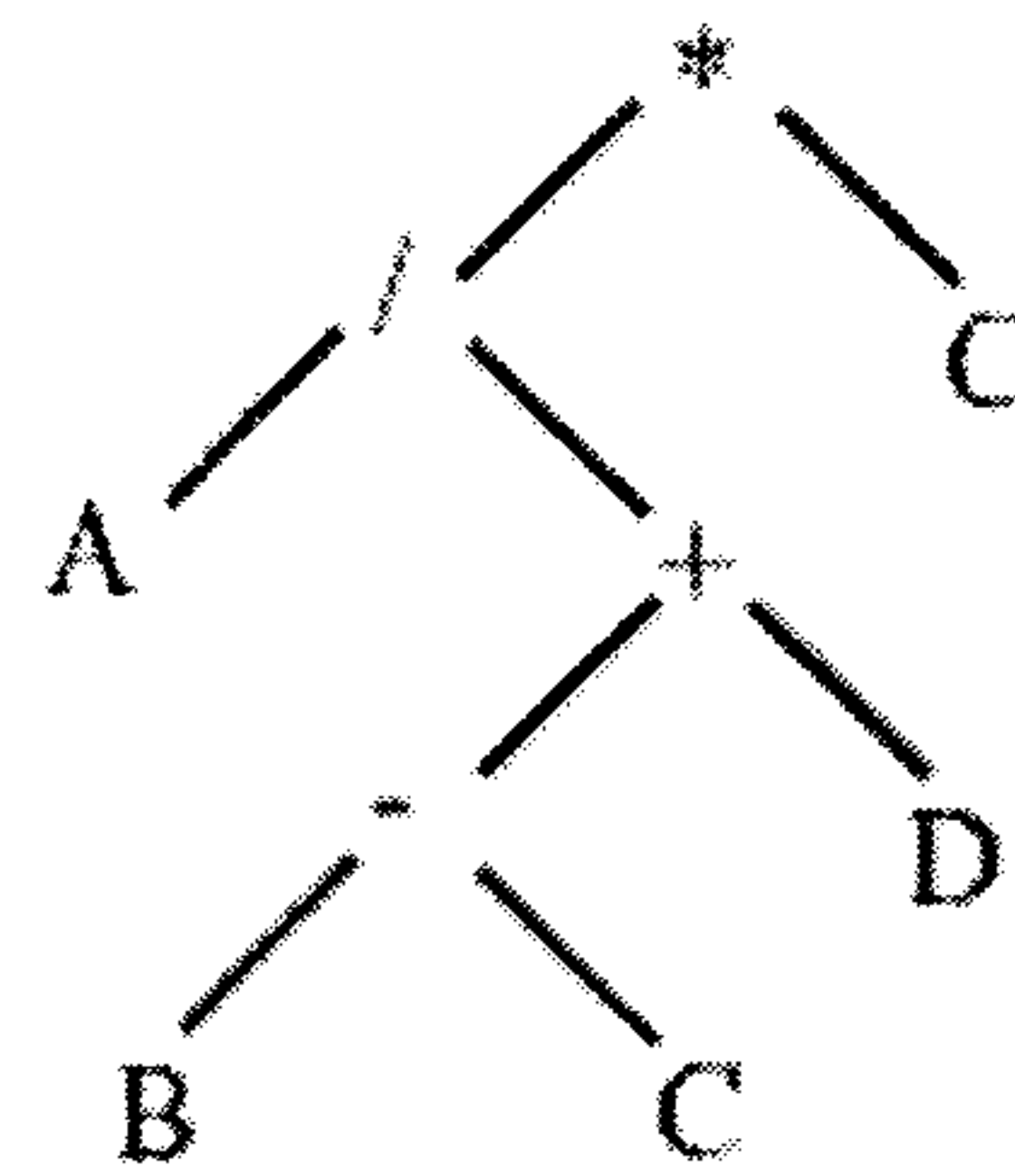
(A) Consider the infix expression $(A/(B-C+D))*C$ and answer the following two questions:

(a) (3%) Which of the following is the expression tree of the given expression?

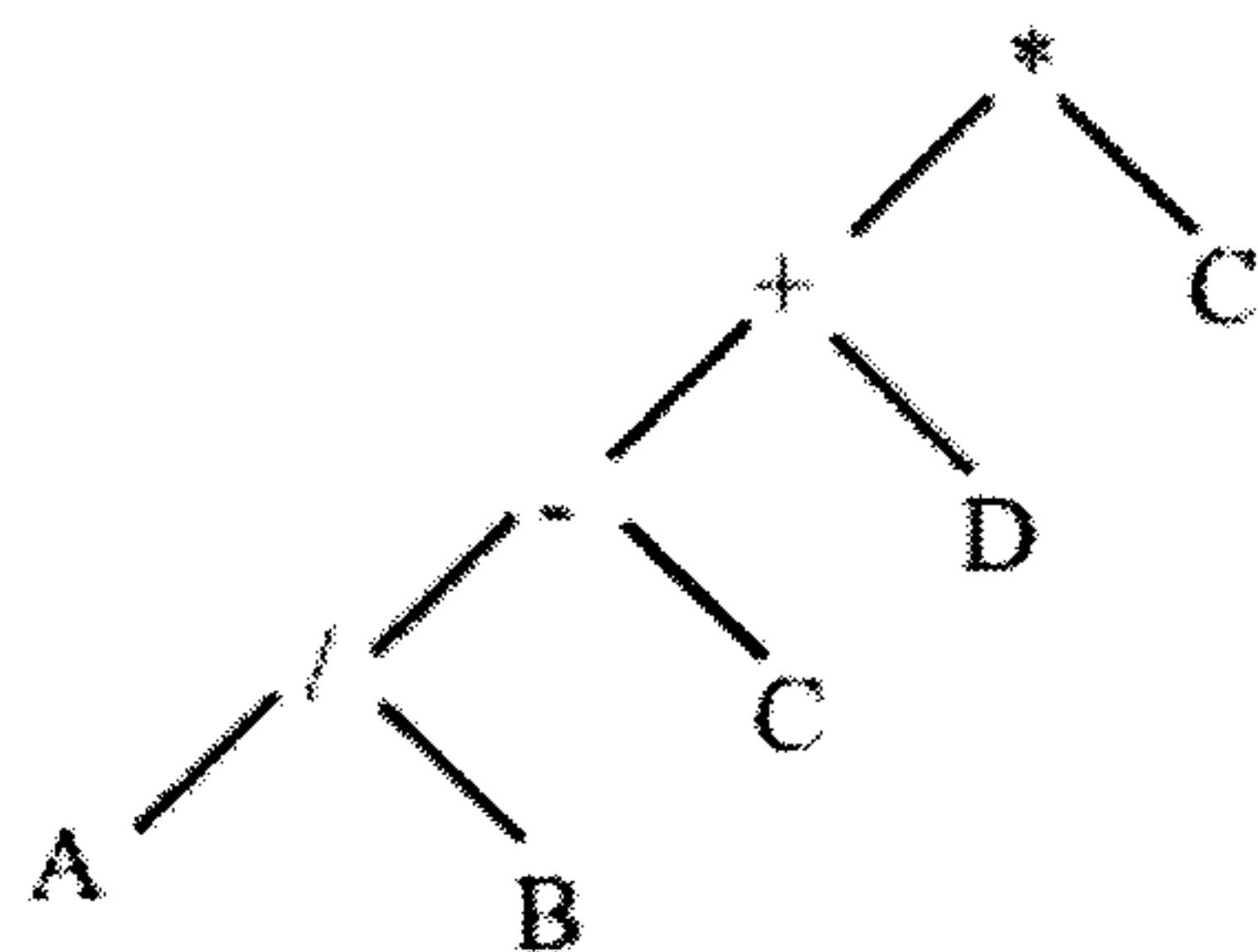
(1)



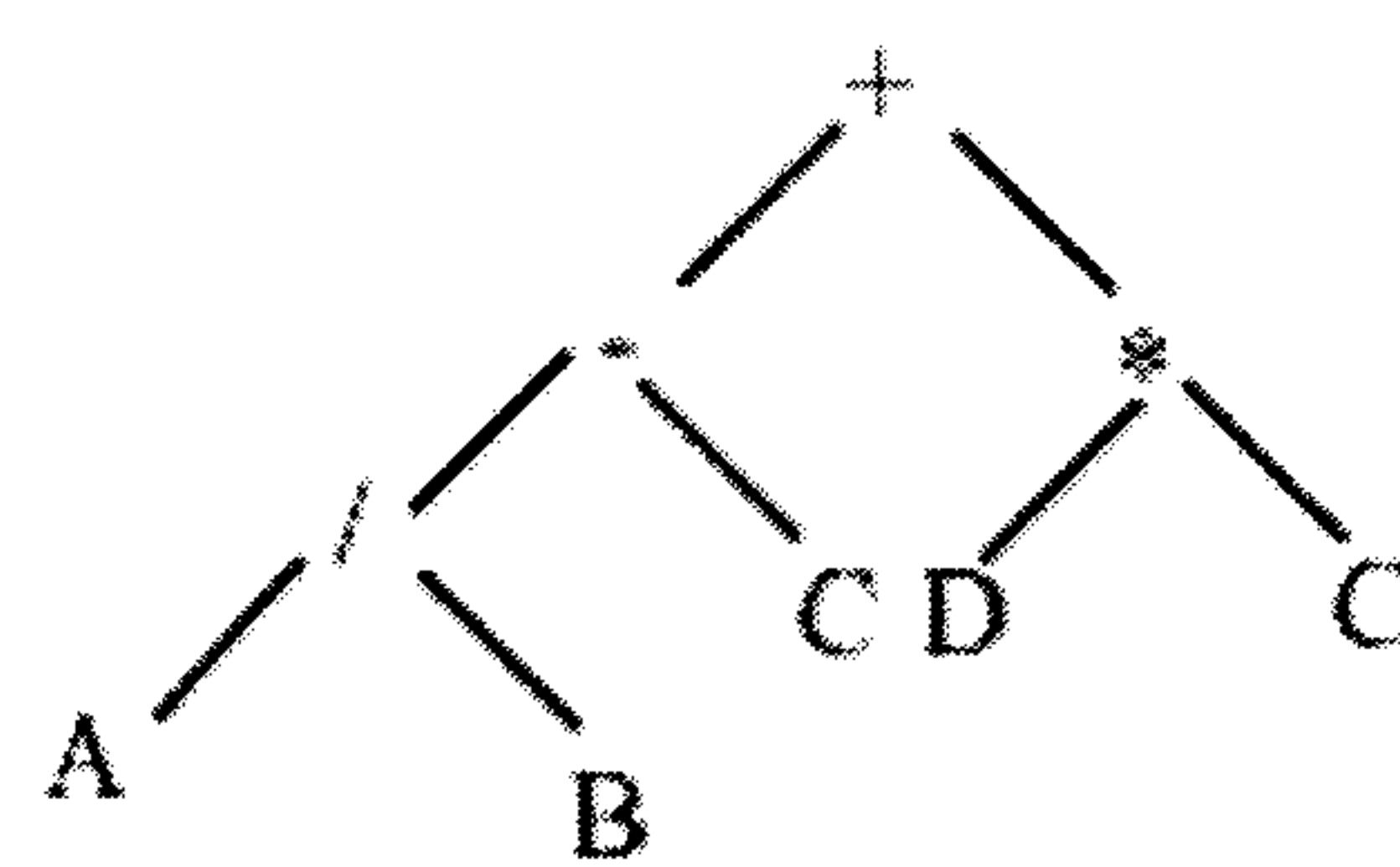
(2)



(3)



(4)



(b) (4%) Write down the equivalent postfix expression.

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共 8 頁，第 5 頁 *請在【答案卷、卡】作答

(B) Consider using arrays as the underlying representations of binary trees. Answer the following questions.

(a) (4%) Insert a set of keys 8, 29, 14, 16, 31, 19, 13 in this order into an initially empty max heap H . Show the content of the array that stores H .

(b) (3%) Insert a set of keys 27, 3, 15, 2, 30 into an initially empty binary search tree T . Show the content of the array that stores T .

(c) (3%) Treat H as a priority queue, pop two keys from it, and then insert the two keys into T . Show the content of the array that stores T again.

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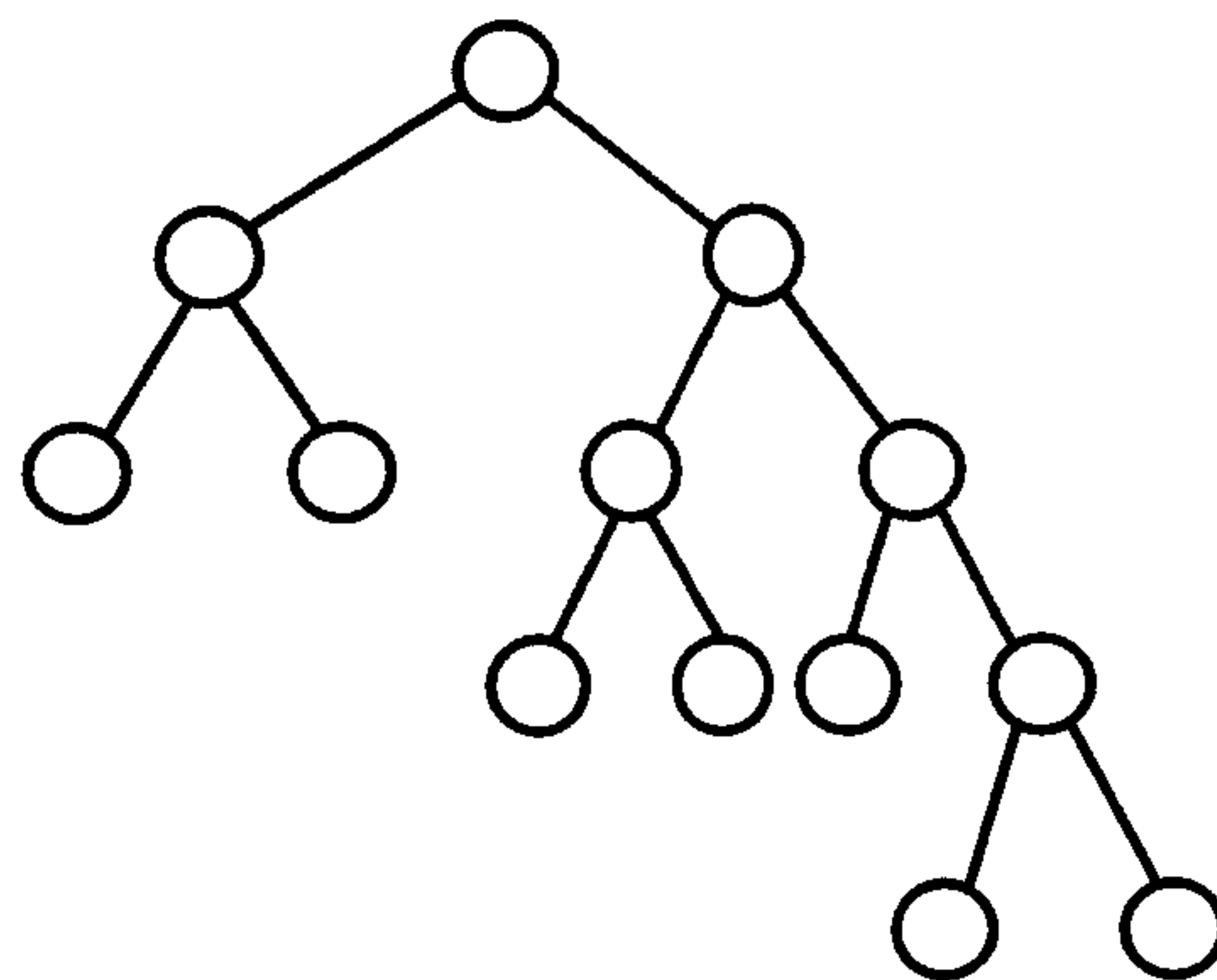
4. (17%) Answer the following questions.

(A) (2%) Please choose an answer that best completes the following sentence: The number of comparisons required to find an element in a hash table with N buckets, of which M are full,

- a. is approximately $\log_2 M$.
- b. is approximately $\log_2 N$.
- c. is usually only slightly less than N .
- d. may be large if M is only slightly less than N .

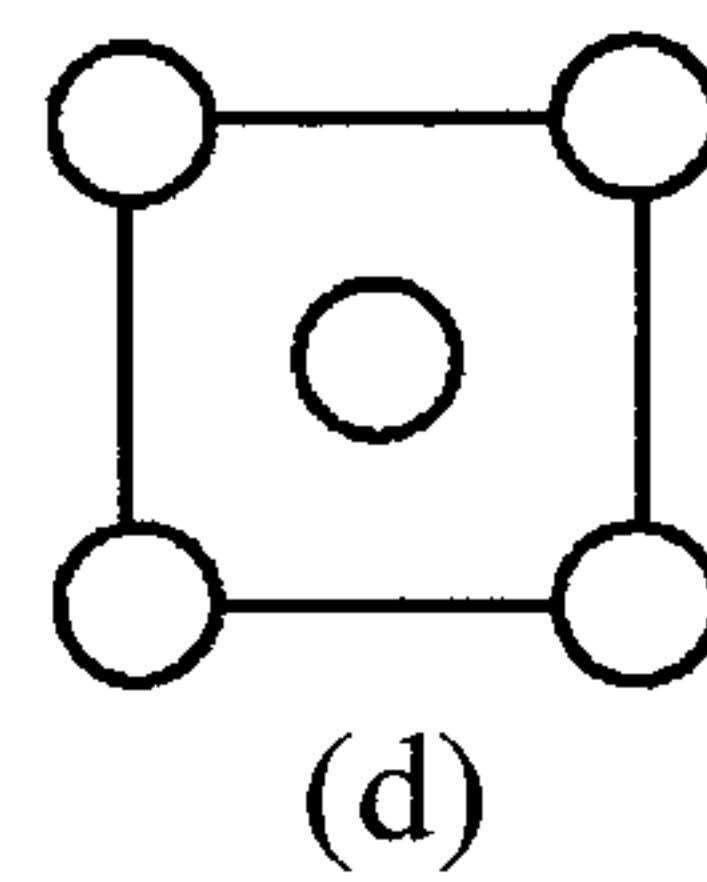
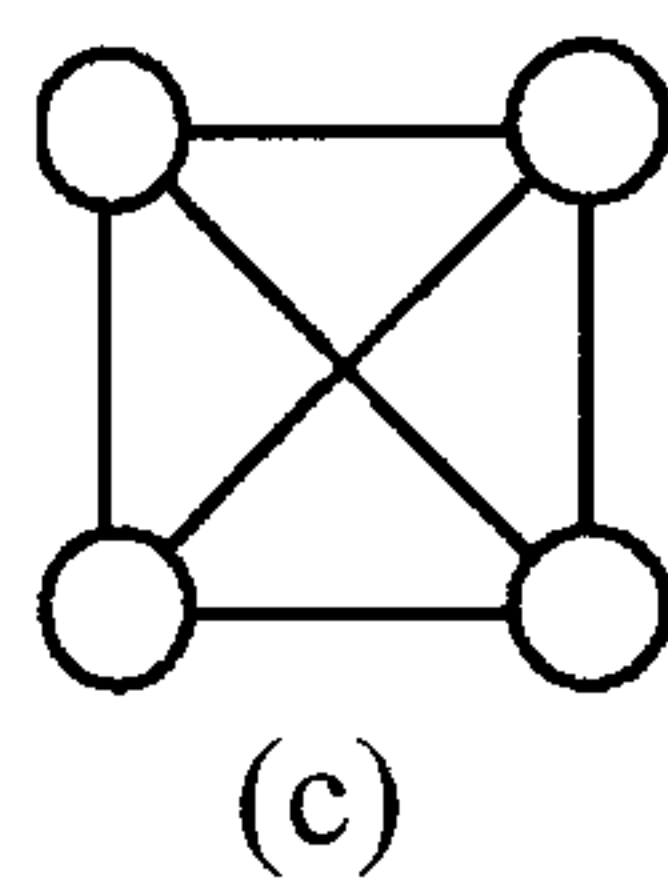
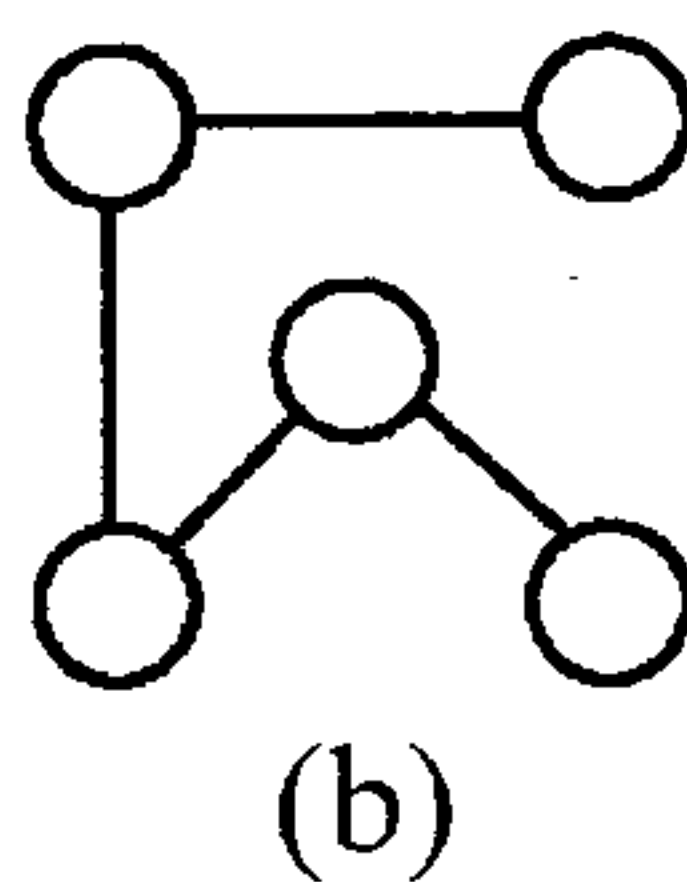
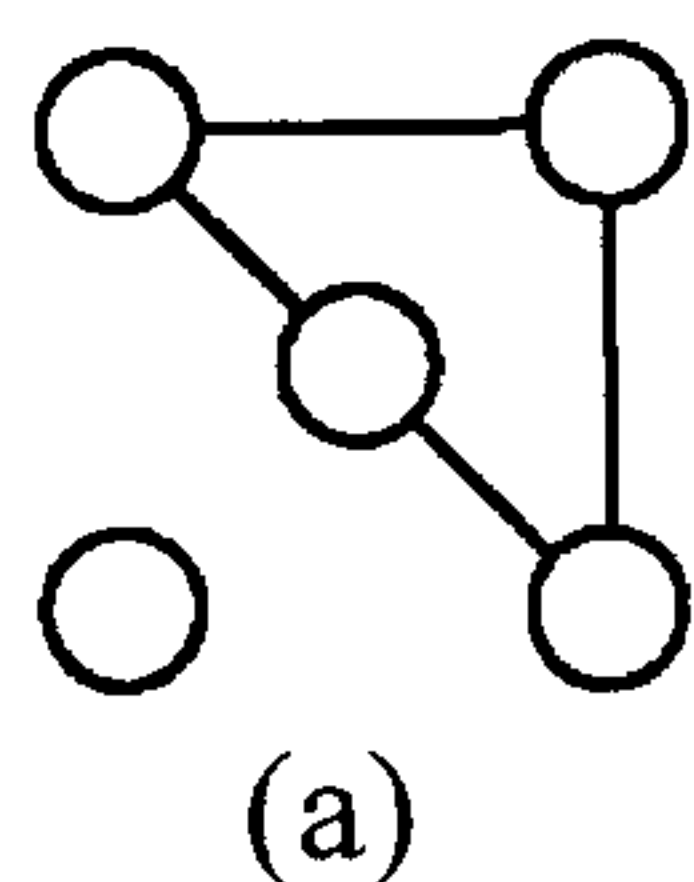
(B) (9%)

- a. How many different binary trees can be made from three nodes that contain the key values 1, 2, and 3? Explain why.
- b. How many different binary search trees can be made from three nodes that contain the key values 1, 2, and 3? Explain why.
- c. Color the nodes in the following tree so that it is a red-black tree.



(C) (6%)

- a. Which of the following graphs are bipartite?
- b. Which of the following graphs are biconnected?
- c. What are the real-world applications that would use the biconnected graph? Please give an example.



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5. (17%) Answer the following questions.

(A) (9%) Consider the following C program of a recursive function:

```
void f(int n)
{
    int i;
    if (n>1) {
        for (i=1; i<=n; i++) {
            printf("Computer Science.\n");
        }
        f(n/2);
        f(n/2);
    }
}
```

- (a) How many lines does the program print if we call `f(1024)`?
- (b) Write a recurrence relation for the time complexity.
- (c) How many lines, as a function of `n` in $\Theta(\cdot)$ form, does the program print? Assume `n` is a power of 2.

(B) (6%) The following code block is an implementation of Floyd's algorithm for finding all-pairs shortest paths. Initially, `dist[i][j]` is the length of the directed edge from vertex `i` and vertex `j` if it exists, and is `infinity` otherwise. Please write down the missing code for (a) and (b).

```
for (k=1; k<=N; k++)
    for (i=1; i<=N; i++)
        for (j=1; j<=N; j++)
            dist[i][j] = min( (a) , (b) );
```

(C) (2%) Please describe the maximum-flow problem.

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6. (15%) Answer the following questions.

(A) (10%) Let $A(1:n,1:n)$ be a positive definite matrix. The following algorithm takes a positive definite matrix A , as input and decomposes A into the product of L and L^t (the transpose of matrix L), where L is a lower triangular matrix stored in the diagonal and lower part of A .

```
for j=1,2, ...,n
{ t=A(j,j);
  for r=1,2, ...,j-1
  { t=t-A(j,r)*A(j,r); }
  A(j,j)=sqrt(t);
  for i=j+1, j+2, ...,n
  { s=A(i,j);
    for r=1,2, ..., j-1
    { s=s-A(i,r)*A(j,r); }
    A(i,j)=s/A(j,j);
  }
}
```

Given the following matrix $A(1:3, 1:3)$ as input

$$A = \begin{bmatrix} 9 & 3 & -3 \\ 3 & 17 & 3 \\ -3 & 3 & 27 \end{bmatrix}$$

What is the result of A after running the algorithm? (show only the integer part)

(B) (5%) Let a and n be positive integers with $1 < a < n$ and $\gcd(a,n)=1$, that is, they are relatively prime. The order of $a \pmod n$ is the smallest positive integer r such that $a^r \equiv 1 \pmod n$, we denote $r = \text{ord}_n(a)$. For example, $\text{ord}_4(3) = 2$.

(a)(2%) What is $\text{ord}_{11}(3)$?

(b)(3%) What is $\text{ord}_{31}(3)$?