科目：近代物理(300G)　　校系所組：交通大學電子研究所（甲組）
交通大學電信工程研究所（乙組）
清華大學光電工程研究所
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參考用

(一) 單選題 ： 1~6 題 每題5分 答錯倒扣1.25分
(二) 複選題 ： 7~12 題 每題5分 答錯倒扣1分
(三) 非選擇題: 13~16 題 每題10分

(一) 單選題：1~6 題 每題5分 答錯倒扣1.25分

1. The energy of 13.6eV is required to separate a hydrogen atom into a proton and an electron, i.e. the binding energy is E= -13.6 eV at this state. What are the potential energy $PE$, the orbital radius $r$, and velocity $\upsilon$ of the electron in a hydrogen atom corresponding to this state. (5%)
(A) $PE=-27.2eV,\ r=2.65\times10^{-11}m,\ \upsilon=3.097\times10^{6}m/s$
(B) $PE=-13.6eV,\ r=5.29\times10^{-11}m,\ \upsilon=3.1\times10^{6}m/s$
(C) $PE=-13.6eV,\ r=2.65\times10^{-11}m,\ \upsilon=3.097\times10^{6}m/s$
(D) $PE=-27.2eV,\ r=5.29\times10^{-11}m,\ \upsilon=2.19\times10^{6}m/s$
(E) $PE=-13.6eV,\ r=5.29\times10^{-11}m,\ \upsilon=2.19\times10^{6}m/s$

2. $\Psi_1$ and $\Psi_2$ represent two wave functions achieved by solving schrodinger wave equation. Which one is correct statement for these wave functions ? (5%)
(A) $|\Psi_1+\Psi_2|^2=|\Psi_1|^2+|\Psi_2|^2$
(B) $\int_{x_1}^{x_2}|\Psi_i|^2dx=1,\ (i=1,2)\ x_1\ and\ x_2\ are\ finite$
(C) $\frac{\partial\Psi}{\partial x}, \frac{\partial\Psi}{\partial y}, \frac{\partial\Psi}{\partial z},\ \Psi=\Psi_1\ or\ \Psi_2$ are continuous and single valued everywhere
(D) $\Psi_1(x\to\pm\infty)\neq0\ and\ \Psi_2(x\to\pm\infty)\neq0$
(E) $\frac{\partial\Psi_1}{\partial x}(x\to\pm\infty)=0\ and\ \frac{\partial\Psi_2}{\partial x}(x\to\pm\infty)=0$

3. In Boltzmann's theory, the distribution function ($f$) has arguments, time ($t$), velocity ($\vec{v}$), and position ($\vec{r}$). The first order of Taylor's expansion of this distribution function is equivalent to the velocity integral of the discrepancy between the backward scatterings (denoted by $f'$ and $\vec{v}'$) and the forward scatterings, as shown below:

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial\vec{r}}+\frac{\vec{F}}{m}\cdot\frac{\partial f}{\partial\vec{v}}=\left(\frac{\partial f}{\partial t}\right)_{collision}$$

$$\left(\frac{\partial f}{\partial t}\right)_{collision}:=\left(\int d^3\vec{v}_1\int d\omega\sigma|\vec{v}-\vec{v}_1|\{f'(\vec{r},\vec{v}',t)f'(\vec{r},\vec{v}'_1,t)-f(\vec{r},\vec{v},t)f(\vec{r},\vec{v}_1,t)\}\right)$$

This is called "Boltzmann's equation", where $(\partial f/\partial t)_{collision}$ is called "collision term". $\vec{F}$ is an external field, and m is the particle mass. It is regarded that the collisions occur at $\vec{r}$ with the spatial volume being $d^3\vec{r}$. We can call it "Detailed Balance Condition" when $f'(\vec{r},\vec{v}',t)f'(\vec{r},\vec{v}'_1,t)=f(\vec{r},\vec{v}_1,t)f(\vec{r},\vec{v}_1,t)$? At this condition, it is well known that the frequencies of the forward scattering and the backward scattering are the same.

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Select the backward-scattering from (A) ~ (E), when $(\vec{v}_1, \vec{v}_2) \to (\vec{v}'_1, \vec{v}'_2)$ is the forward-scattering, as shown in Fig.1.

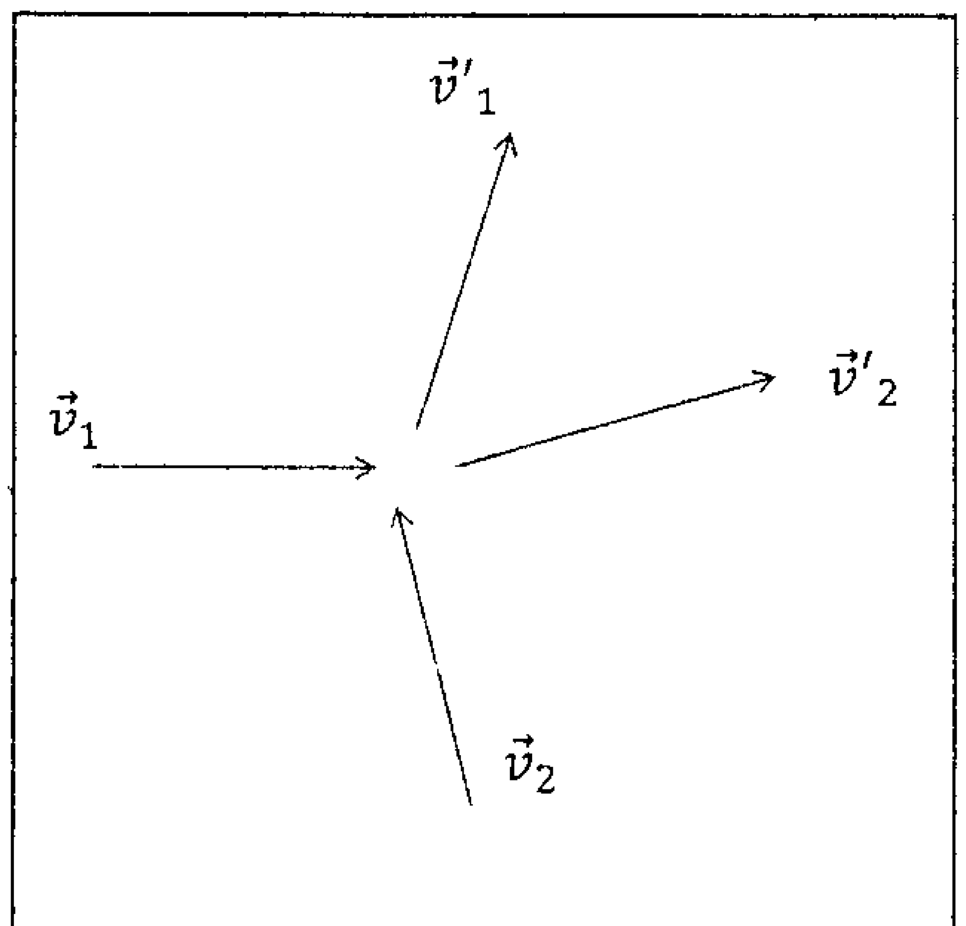
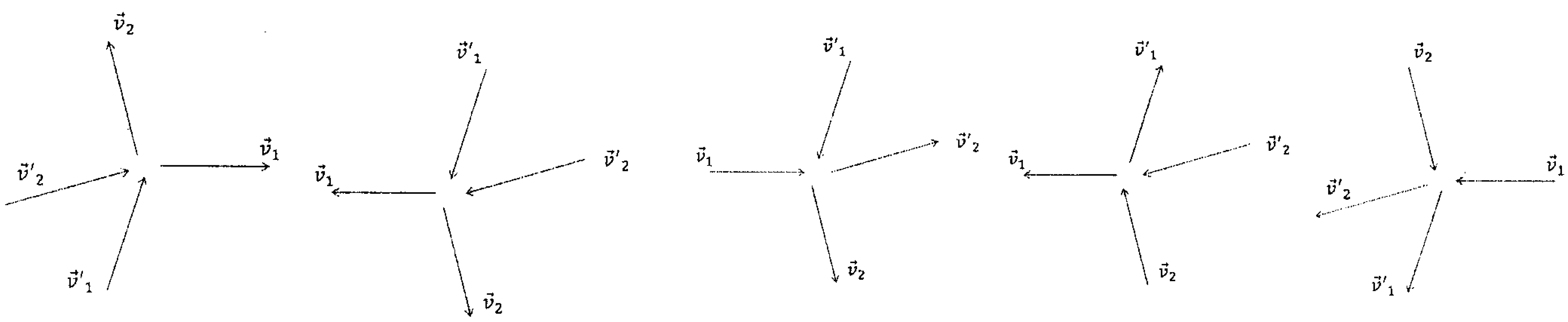



Fig. 1: Forward Scattering




(a) (b) (c) (d) (e)

(A) (B) (C) (D) (E)

4. Using the following equaiton:

$$\tau = \frac{1}{\int d^3\vec{v}_1 \int d\omega\sigma \cdot |\vec{v} - \vec{v}_1| f_1'(\vec{r}, \vec{v}_1, t)} = \frac{1}{\int d^3\vec{v}_1 \int d\omega\sigma \cdot |\vec{v} - \vec{v}_1| f_1(\vec{r}, \vec{v}_1, t)},$$

we can obtain the first order Boltzmann's equation within the mean-field approximation:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \cdot \frac{\partial f}{\partial \vec{v}} = \frac{f^0 - f}{\tau} = \frac{f' - f^0}{\tau},$$

where $f^0$ is Maxwell distribution. Assuming that the distribution function is homogenous and stationary under a constant electric field along x-axis ($E_x$), we can obtain the distribution function as follows

$$f = f^0 \cdot \left(1 + \frac{\tau q E_x v_x}{k_B T}\right)$$

Using this distribution, calculate the average value of $v_x$ and then you can derive the mobility ($\mu$). Select the correct equation for the mobility from (A)~(E).

(A) $\langle v_x \rangle = \frac{1}{2}\mu E_x, \quad \mu := \frac{2\tau q}{m}$

(B) $\langle v_x \rangle = 2\mu E_x, \quad \mu := \frac{\tau q}{2m}$

(C) $\langle v_x \rangle = \mu E_x, \quad \mu := \frac{\tau q}{m}$

(D) $\langle v_x \rangle = \mu E_x, \quad \mu := \frac{2\tau}{m}$

(E) $\langle v_x \rangle = \mu E_x, \quad \mu := \frac{\tau - q}{m}$

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5. A spaceship is measured to be 100 m long while it is at rest with respect to the observer A. If this spaceship now flies by the observer B with a speed of $2.4\times10^{8}$ m/s, or 0.8 C where C is the speed of light, what length will the observer B find for the spaceship?
(A) 100 m
(B) 80 m
(C) 60 m
(D) 40 m
(E) 20 m.

參考用

6. Please select which is the Planck's law for the spectral energy density u of the blackbody radiation in the following equations.
(A) $u(f,T)=\frac{8\pi f^2}{c^3}k_BT$
(B) $u(f,T)=\frac{8\pi f^2}{c^3}\left(\frac{hf}{e^{hf/k_BT}-1}\right)$
(C) $u(f,T)=\frac{8\pi f}{c^3}\left(\frac{hf}{e^{hf/k_BT}-1}\right)$
(D) $u(f,T)=\frac{8\pi f^2}{c^2}\left(\frac{1}{e^{hf/k_BT}-1}\right)$
(E) $u(f,T)=\frac{8\pi f^3}{c^3}e^{-hf/k_BT}$

,where f is the frequency, C is the speed of light, $k_B$ is the Boltzmann's constant, T is the absolute temperature, and h is the Planck's constant.

**Physical constants**

| | |
|---|---|
| *Electron rest mass* | $m_e=9.1095\times10^{-31}kg$ |
| *Proton rest mass* | $m_p=1.6726\times10^{-27}kg$ |
| *Neutron rest mass* | $m_n=1.6750\times10^{-27}kg$ |
| *Hydrogen atomic mass* | $m_H=1.6736\times10^{-27}kg$ |
| *lightv elocity* | $c=2.998\times10^{8}\ m/s$ |
| *Electron charge* | $e=1.602\times10^{-19}Coul$ |
| *Planck's constant* | $h=6.626\times10^{-34}J\cdot s$ |
| *Permittivity of free space* | $\varepsilon_0=8.854\times10^{-12}F/m$ |

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(二)複選題： 7~12 題 每題5分 答錯倒扣1分

7. For a particle trapped in a box with infinitely high barrier $U(x)\to\infty$ at the boundaries, x=0 and L. Derive the wave function $\Psi$ and calculate the expectation values of $\langle x\rangle$, $\langle x^3\rangle$ and $\langle x^3\rangle-\langle x\rangle^3=?$ (5%)

(A) $\Psi(x)=\frac{2\pi}{L}\cos\frac{n\pi x}{L}$, $\langle x\rangle=\frac{L}{2}$, $\langle x^3\rangle=\frac{L^3}{8}$, $\langle x^3\rangle-\langle x\rangle^3=0$

(B) $\Psi(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle x\rangle=\frac{L}{2}$, $\langle x^3\rangle=\frac{L^3}{4}$, $\langle x^3\rangle-\langle x\rangle^3=\frac{L^3}{8}$

(C) $\Psi(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle x\rangle=\frac{L}{2}$

(D) $\Psi(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle x\rangle=\frac{L}{2}$, $\langle x^3\rangle=\frac{L^3}{8}$, $\langle x^3\rangle-\langle x\rangle^3=0$

(E) $\langle x^3\rangle=\frac{L^3}{4}\left(1-\frac{3}{(n\pi)^2}\right)$, $\langle x^3\rangle-\langle x\rangle^3=\frac{L^3}{8}\left(1-\frac{3}{(n\pi)^2}\right)$

參考用

8. What is the angular momentum $L$ of an electron in the hydrogen atom and what is the correct mechanism or model used to calculate this angular momentum $L$ ? (5%)

(A) $L=1.055\times10^{-34}J\cdot s$ for an electron at ground state, n=1, according to quantum mechanics

(B) $L=0$ for an electron at ground state, n=1, according to quantum mechanics

(C) $L=1.055\times10^{-34}J\cdot s$ for an electron at ground state, n=1, according to Bohr model using de-Broglie wavelength theory

(D) $L=2\hbar$ for an electron at n=2, according to Bohr model using de-Broglie wavelength theory

(E) $L=0$ or $\sqrt{2}\hbar$ for an electron at n=2, according to quantum mechanics

9. A static Schrodinger equation of Particle-1 (position $\vec{r}_1$ and energy $\varepsilon_1$) and particle-2 (position $\vec{r}_2$ and energy $\varepsilon_2$) are described as follows:

$$\begin{array}{cc} Particle-1 & Particle-2 \\ -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}_1)=\varepsilon_1\psi(\vec{r}_1) & -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}_2)=\varepsilon_2\psi(\vec{r}_2). \end{array}$$

Here we assumed that the mass are the same, $m$, so-called "homogenuity". With no interaction between these two particles, two-body Schrodinger equation is written as follows:

$$\left(-\frac{\hbar^2}{2m}\nabla_1-\frac{\hbar^2}{2m}\nabla_2\right)\psi(\vec{r}_1,\vec{r}_2)=E\psi(\vec{r}_1,\vec{r}_2),\quad E=\varepsilon_1+\varepsilon_2.$$

Here assume that the wavefunction is written as:

$$\psi(\vec{r}_1,\vec{r}_2)=\sum_{k,l}a_{kl}\phi_k(\vec{r}_1)\phi_l(\vec{r}_2),\quad where\quad \psi(\vec{r})=\sum_k a_k\phi_k(\vec{r}).$$

This two-body wavefunction is then composed of the symmetric and the antisymmetric wavefunctions:

$$\psi_{kl}{}^S(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\{\phi_k(\vec{r}_1)\phi_l(\vec{r}_2)+\phi_l(\vec{r}_1)\phi_k(\vec{r}_2)\}$$

$$\psi_{kl}{}^A(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\{\phi_k(\vec{r}_1)\phi_l(\vec{r}_2)-\phi_l(\vec{r}_1)\phi_k(\vec{r}_2)\}$$

Expanding the two body wavefunction to N-body wavefunction, the symmetric wavefunction has the proporty of:

$$\psi^S(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_j,\cdots,\vec{r}_i,\cdots,\vec{r}_N)=\psi^S(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_i,\cdots,\vec{r}_j,\cdots,\vec{r}_N)$$

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The antisymmetric wavefunction has the propety of:

$$\psi^A(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_j,\cdots,\vec{r}_i,\cdots,\vec{r}_N)=-\psi^A(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_i,\cdots,\vec{r}_j,\cdots,\vec{r}_N)$$

By this way, the exchange of $\vec{r}_i$ and $\vec{r}_j$ causes minus sign in the antisymmetric wavefunction. Select the wavefunction that can satisfy Pauli's exclusion principle from (A) ~(E).

(A) $\psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)=\frac{1}{\sqrt{2}}\{\psi^A(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)+\psi^S(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)\}$

(B) $\psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)=\frac{1}{\sqrt{2}}\{\psi^A(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)-\psi^S(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)\}$

(C) $\psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)=\psi^A(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)\psi^S(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)$

(D) $\psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)=\psi^A(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)$

(E) $\psi(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)=\psi^S(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_N)$

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10. This is Boltzmann's H-function:

$$H:=\int d^3\vec{v}(\log f-1)f=\langle\log f-1\rangle.$$

Here $f$ is a distribution function, and it is assumed that the system is homogenous and has no external field. Select appropriate properties of H-function from (A) – (E).

(A) Progressively with time, H-function is monotonically decreased to the minimum value ($H_0$) that is calculated with Maxwell distribution.
(B) Progressively with time, H-function is decreased to the minimum value ($H_0$) that is calculated with Maxwell distribution and is then increased from a minimum value.
(C) Progressively with time, H-function is oscillatively increased and decrease around the value ($H_0$) that is calculated with Canonical distribution.
(D) Progressively with time, H-function is increased to a maximum value ($H_0$) that is calculated with Maxwell distribution and is then deccreased from a minimum value.
(E) Progressively with time, H-function is monotonically decreased to the minimum value ($H_0$) that is calculated with a stationary distribution.

11. Please indicate which are incorrect in the following terms. (A) The Compton-effect experiment proves that X-ray behaves like particles. (B)The Compton-effect experiment proves that X-ray behaves like waves. (C) The photoelectric effect proposed by Einstein explains that the maximum kinetic energy of photoelectrons is linearly dependent on the light frequency. (D) The photoelectric effect proposed by Einstein explains that there is time lag between the start of light illumination and the start of the photocurrent. (E) Planck's law for the blackbody radiation verifies that the light behaves like waves. (5%)

12. Please choose the correct ones in the following terms. (A)There are four visible spectral lines found in the emission of the discharged hydrogen atom. (B) The ionization energy of the hydrogen based on Bohr's model is 13.6 eV. (C) Rutherford's model of the atom proves that the number of the α particles entering the detector per unit time is proportional to the atomic number Z of the target atom. (D) Rutherford's model of the atom proves that the number of the α particles entering the detector per unit time is inversely proportional to the distance from the point where α particles strike the atomic foil to the zinc sulfide screen. (E) Millikan's oil-drop experiment proves that the charge of the electrons is quantized

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(三)　非選擇題：13~16 題 每題 10 分

13. A relativistic free particle of rest mass $m$ is moving with speed $v$.

(A) Write down its momentum and total energy. (2%)

(B) Find the relationship between its momentum and total energy. (2%)

(C) At what speed does the kinetic energy of the particle equal to its rest energy. (2%)

(D) Find the phase velocity of the de Broglie wave associated with the particle. (2%)

(E) If it has kinetic energy KE. Show that its de Broglie wavelength is given by $\lambda = \dfrac{hc}{\sqrt{\mathrm{KE}(\mathrm{KE}+2mc^2)}}$, where $h$ is the Planck constant, $c$ is the speed of light. (2%)

14. A hydrogen atom consists of one electron and one proton. Assume that the proton's mass is infinitely heavy and the electron's mass is $m$. No gravitational force is considered in this problem.

(A) Show classically that the energy of the system is $-\dfrac{e^2}{8\pi\varepsilon_0 r}$. (2%)

(B) Use the Bohr's model to find the quantized energy levels. (3%)

A hydrogenic atom is an ion, such as $\mathrm{He}^+$ or $\mathrm{Li}^{2+}$, whose nuclear charge is $+Ze$ and which contains a single electron. The nucleus has a finite mass of $M$ and the electron's mass is still $m$.

(C) Find the energy levels of the hydrogenic atom by modifying the result of (b). (3%)

(D) If the ground state energy of a hydrogen atom is $-13.6\,\mathrm{eV}$, find the first excited energy level of the $\mathrm{He}^+$ ion. The nuclear mass of $\mathrm{He}^+$ is $M_{\mathrm{He}^+}$. (2%)

15. At time $t = 0$, assume a linear simple harmonic oscillator normalized wave function as

$$\Psi(x,0) = \sqrt{\frac{1}{6}}\cdot\phi_0(x) + \sqrt{\frac{1}{3}}\cdot\phi_2(x) + N_3\cdot\phi_3(x)$$

(A) Determine the value of $N_3$ (5%)

(B) Determine the complete wave function at $t \neq 0$ (5%)

16. Considering the free electron gas system in a metal with $N$ electrons within volume $V$, at absolute temperature $T = 0$. Assuming the density of state function as $g(E) = \dfrac{8\sqrt{2}\pi m^2 V}{h^3}\sqrt{E}$

(A) Plot the Fermi-Dirac distribution function at T=0 (f(E) vs. T) (2%)

(B) Determine the Fermi energy as $E_F = \dfrac{h^2}{2m}(\dfrac{3N}{8\pi V})^{2/3}$ (4%)

(C) Determine the total energy as $E_{tot} = \dfrac{3}{5}N\cdot E_F$ (4%)