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ON THE ROLE OF PHOTOSPHERIC CONVECTION IN W URSAE MAJORIS STARS

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ABSTRACT

We correct the derivation of the source function in the atmospheres of contact binaries given in an earlier communication by Anderson and Shu. This correction affects the cases when convection is present in the photosphere. In our new treatment photospheric convection is more efficient for reducing limb darkening. This result does not, however, modify the numerical examples considered in our earlier paper.

Subject headings: convection — stars: W Ursae Majoris

I. INTRODUCTION

In an earlier paper (Anderson and Shu 1977, hereafter AS) we developed a theory for the light curves of contact binaries, with applications directed in particular to W Ursae Majoris stars. Since the publication of AS we have discovered an oversight in our treatment of the photospheres of these stars. In AS we implicitly assumed that the radiation energy density is given by aT^4 for an atmosphere in LTE. This assumption is justified at large optical depths, but setting the mean specific intensity equal to the source function at moderate optical depths is valid only if radiative equilibrium prevails. Thus, our treatment requires modification if convection carries part of the total flux in the *photospheric* layers. To facilitate comparisons with our earlier paper, the equations here are numbered identically, except for a prime, as their counterparts is AS.

II. PHOTOSPHERES OF CONTACT BINARIES

We adopt a modified version of Eddington's approximation to treat limb darkening and reflection in the atmospheres of contact binaries. For a plane-parallel gray atmosphere in LTE, the first two moments of the equation of transfer read

$$\frac{dF_{\rm rad}}{d\tau} = c(E_{\rm rad} - aT^4)\,, \tag{3a$'$}$$

$$\tfrac{1}{3}c\,\frac{dE_{\rm rad}}{d\tau} = F_{\rm rad}\,. \tag{3b$'$}$$

In equation (3b′) we have adopted the Eddington closure relation: $P_{\rm rad} = E_{\rm rad}/3$, where $E_{\rm rad}$, $F_{\rm rad}$, and $P_{\rm rad}$ are, respectively, the radiation energy density, flux, and pressure. As in AS, we assume that radiation at optical depth τ carries a fraction $R(\tau)$ of the total flux F:

$$F_{\rm rad} = FR(\tau)\,, \qquad F_{\rm conv} = F[1 - R(\tau)]\,, \tag{7$'$}$$

with $R(0) = 1$. If $R(\tau) = 1$ for all $\tau \geq 0$—i.e., if radiative equilibrium prevails—equation (3a′) implies $cE_{\rm rad} = caT^4$ (absorption rate = emission rate), and we then recover the analysis of AS. More generally, we can eliminate $E_{\rm rad}$ and $F_{\rm rad}$ from equations (3′) and (7′) and obtain, upon integration,

$$\tfrac{1}{3}caT^4 = F\left[K_0 + \int_0^\tau R(t)dt - \tfrac{1}{3}R'(\tau)\right]\,, \tag{8$'$}$$

where K_0 is an integration constant. Notice that $R(\tau)$ must not decrease from unity faster than $\exp(-3^{1/2}\tau)$ if the temperature is required to rise inward. The case $R(\tau) = \exp(-3^{1/2}\tau)$ corresponds to an isothermal atmosphere. An isothermal atmosphere exhibits, of course, no limb darkening; and it is probably the limiting form of an atmosphere in which convection can reasonably be expected to carry heat.

The emergent specific intensity at $\tau = 0$ is given by

$$I(\mu) = \int_0^\infty \frac{caT^4}{4\pi}\,e^{-\tau/\mu}\,\frac{d\tau}{\mu}\,,$$

where μ is the cosine of the angle between the viewing direction and the local zenith. Substituting equation (8′) into the above relation yields, after two integrations by parts,

$$I(\mu) = \frac{3F}{4\pi}\left\{K_0 + \mathscr{R}(\mu) + \frac{1}{3\mu}[1 - \mathscr{R}(\mu)/\mu]\right\}, \qquad \mathscr{R}(\mu) = \int_0^\infty R(\tau)e^{-\tau/\mu}d\tau . \tag{9′}$$

To determine the value of K_0, we require the flux F to be radiated into the ω_S steradians occupied by the local sky:

$$F = \int_{\omega_S} \mu I(\mu)d\omega = 3F\left(\frac{\omega_S}{4\pi}\right)[K_0\langle\mu\rangle_S + \langle\mu\mathscr{R} + \tfrac{1}{3}(1 - \mathscr{R}/\mu)\rangle_S] , \tag{10a′}$$

where the angular brackets are defined such that, for any function $f(\mu)$,

$$\langle f\rangle_S \equiv \frac{1}{\omega_S}\int_{\omega_S} f(\mu)d\omega . \tag{10b′}$$

Solving for K_0, we obtain

$$K_0 = (3\langle\mu\rangle_S S)^{-1}[1 - \langle 3\mu\mathscr{R} + (1 - \mathscr{R}/\mu)\rangle_S S] , \tag{11′}$$

where we have written $S = \omega_S/4\pi$ to denote the fraction of the celestial sphere occupied by the local sky.

We write the law of gravity brightening as $F = \sigma\bar{T}_e^4(g/\bar{g})^{4\beta}$, with $\beta = \frac{1}{4}$ for common radiative envelopes and $\beta = 0$ for common convective envelopes. Thus, in the modified Eddington-Barbier approximation we can now express the emergent bolometric and monochromatic specific intensities and the T-τ relation as

$$I(\mu) = \frac{3\sigma\bar{T}_e^4}{4\pi}(g/\bar{g})^{4\beta}\left\{K_0 + \mathscr{R}(\mu) + \frac{1}{3\mu}[1 - \mathscr{R}(\mu)/\mu]\right\}, \tag{14′}$$

$$I_\nu(\mu) = B_\nu[T(\tau)] \text{ at a value of } \tau \text{ where } \int_0^\tau R(t)dt - \tfrac{1}{3}R'(\tau) = \mathscr{R}(\mu) + \frac{1}{3\mu}[1 - \mathscr{R}(\mu)/\mu] , \tag{15′}$$

$$T(\tau) = \bar{T}_e(g/\bar{g})^\beta\left\{\tfrac{3}{4}\left[K_0 + \int_0^\tau R(t)dt - \tfrac{1}{3}R'(\tau)\right]\right\}^{1/4} . \tag{16′}$$

For radiative photospheres: $R(\tau) = 1$, $\mathscr{R}(\mu) = \mu$, equations (11′), (14′), (15′), and (16′), with K_0 denoted by Q_0, reduce to the corresponding formulae given by AS. For convective photospheres, the presence of the additional term, $[1 - \mathscr{R}(\mu)/\mu]/3\mu$, in equation (14′) implies that photospheric convection is more efficient for suppressing limb darkening than estimated by AS. In particular, zero limb darkening corresponds to $R(\tau) = \exp(-3^{1/2}\tau)$, $\mathscr{R}(\mu) = \mu/(3^{1/2}\mu + 1)$, and not to $R(\tau) = 0$, $\mathscr{R}(\mu) = 0$, as claimed by AS. *It is not necessary for the atmosphere to be fully convective to eliminate limb darkening.* This conclusion is based solely on the functional form of $R(\tau)$ and does not depend on a detailed formulation of convection theory.

Nevertheless, equation (13) of AS, which includes reflection in an atmosphere exhibiting zero limb darkening, is recovered by the present analysis because both treatments rely on an isothermal atmosphere to produce zero limb darkening. Thus, apart from the label $\mathscr{R}(\mu) = 0$, the numerical results presented in Figures 3 and 4 of AS are still valid, and none of our physical conclusions are changed by the considerations of the present paper. If anything, our identification is strengthened that convection in the photosphere leads to reduced limb darkening and, thus, may contribute significantly to the W-type behavior of W UMa stars of late spectral type.

III. DISCUSSION

Figure 1 compares the zero limb darkening case $R(\tau) = \exp(-3^{1/2}\tau)$ with the radiative flux fraction obtained in model atmosphere calculations by Auman (1969) and by Carbon and Gingerich (1969). These models employ a local mixing length formulation. The differences between dashed and dotted curves for similar values of ($\log_{10} T_e$, $\log_{10} g$) is a measure of the theoretical uncertainty. In particular, significantly different opacity contributions and mixing length ratios were used. Nevertheless, $R(\tau)$ in both calculations remains substantially above the value $\exp(-3^{1/2}\tau)$ for optical depths $\tau \lesssim 1$. Taken at face value, Figure 1 suggests that reduced limb darkening occurs only in stars of very late spectral type, e.g., M stars.

However, mixing length theory is weakest precisely when it is applied to phospheric convection. Toomre, Latour, and Zahn (1977) have demonstrated that an anelastic formulation of stellar convection theory leads to strong overshooting motions which are not present in local mixing length treatments. Böhm-Vitense (1970) and Böhm-Vitense and Canterna (1974) come to a similar conclusion concerning A and F stars using semiempirical arguments. We speculate that convective overshoot in the atmospheres of cool stars may carry an appreciable fraction of the total flux even at small optical depths. A further effect, which is suppressed explicitly by the anelastic

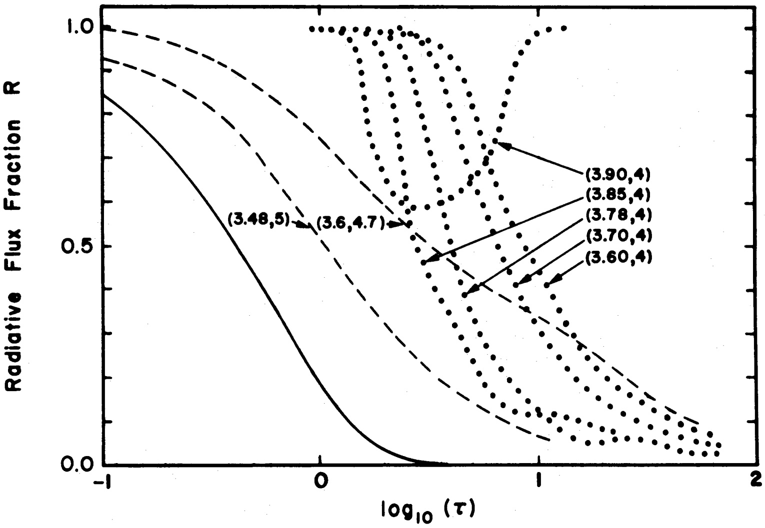

FIG. 1.—The radiative fraction $R(\tau)$ of the total flux for main-sequence stars with convection. *Solid line*, $R(\tau) = \exp(-3^{1/2}\tau)$ corresponding to an isothermal atmosphere. *Dashed lines*, models published by Auman (1969). *Dotted lines*, models published by Carbon and Gingerich (1969). The notation is: ($\log_{10} T_e$, $\log_{10} g$). The reference wavelength for the optical depth scale used by Auman is 1.17 μm; that by Carbon and Gingerich is 0.5 μm.

formulation, involves the transport of energy by acoustic waves generated by the convective motions (see Stein and Liebacher 1974). Active chromospheres would be a by-product of such nonradiative transport. (For a discussion of chromospheric activity in W UMa stars, see Hall 1976.)

Given the theoretical uncertainties, it is wisest to admit that for relatively cool stars we are confident of the limb darkening law for only the Sun. Perhaps sufficiently accurate observations of W UMa stars, coupled with judicious interpretation from the model extremes (full limb darkening and zero limb darkening), could help provide empirical data on the role of photospheric convection in stars of late spectral type.

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