國　立　清　華　大　學　命　題　紙

八十八學年度 資訊工程學系 系（所）＿＿＿＿組碩士班研究生招生考試

科目 基礎計算機科學 科號 4101 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10%) The pigeonhole principle states that if a function f has n distinct inputs but less than n outputs, than there exist two inputs a and b such that $a \neq b$ and $f(a)=f(b)$. Give an efficient algorithm to find the values of a pair of a and b for which $f(a)=f(b)$. Assume that the inputs are 1, 2, ..., n, and $f(x)$ can be computed in $O(1)$ time for every x, $1 \le x \le n$. What's the time complexity of your algorithm?

2. (10%) A lower triangular matrix is a square matrix for which all the elements above the main diagonal are zero. Let A be a lower triangular $n \times n$ matrix such that row i has i nonzero numbers, $1 \le i \le n$. The total number of nonzero numbers in A is $n(n+1)/2$.
   (a) Give a data structure to store A such that the data structure is exactly of size $n(n+1)/2$ and the value of $A[i, j]$ can be obtained from it in $O(1)$ time for each pair of i and j, $1 \le i, j \le n$. (5%)
   (b) Describe the detailed steps for obtaining the value of $A[i, j]$ from the data structure, $1 \le i, j \le n$. (5%)

3. (5%) Let p be a pointer pointing to a node of a list. Assume that the list is a doubly linked circular list and has length>2. Write a sequence of Pascal (or C) statements to delete the node pointed by p from the list.

4. (25%) Which of the following statements are true? Justify your answer briefly?
   (a) Consider the problem of scheduling 13 examinations in 13 days so that two examinations given by the same instructor are not scheduled on consecutive days. It is always possible to schedule the examinations if no instructor gives more than 7 examinations. (5%)
   (b) A graph in which there has at most one path between every pair of vertices is a tree. (5%)
   (c) It is possible to rank players in a round-robin tennis tournament such that player a will be ranked higher than player b if a beats b, or a beats a player who beats b, or a beats a player who beats another player who beats b, and so on. (5%)
   (d) A sequence $\Delta = [d_1, d_2, ..., d_n]$ of integers, $n - 1 \ge d_1 \ge d_2 \ge ... \ge d_n \ge 0$, is called graphical if there exists an undirected graph with no self-loops having Δ as its degree sequence. The sequence [5,5,3,3,2,2,2] is graphical. (5%)
   (e) The language $L = \{a^k b^k \mid k > 0\}$ is a finite state language. (5%)

5. (10%)
   (a) Design an algorithm to sort three numbers. You should make it to have the least number of comparisons in the average case. (7%)
   (b) How many comparisons are required in the best, worst, and average cases in your algorithm, respectively? (3%)

6. (6%)
   (a) Which of the following sorting algorithms are in the worst case complexity $\Theta(n^2)$, where n is the number of input data? (3%)

   (b) Which of the following sorting algorithms are in the worst case complexity $O(n^2)$? (3%)

   (1) insertion sort (2) bubble sort (3) merge sort (4) quick sort
   (5) shell sort (6) heap sort (7) selection sort

7. (9%) Which of the following operations are more efficient (with respect to asymptotic complexity) when they are implemented for the array of sorted numbers than for the array of random numbers?
   (1) Find the maximum value.
   (2) Compute the arithmetic mean.
   (3) Find the median.
   (4) Find the middle value. (The middle value of a sequence of numbers is the median of distinct values of the numbers. For example, 4 is the middle value of sequence 1, 1, 1, 1, 1, 1, 1, 4, 5.)
   (5) Find the mode (i.e., the value that appears the most times).
   (6) Find the majority if exists (i.e., the value that appears at least $n/2$ times, where n is the number of input data).

8. (5%) Find all integer pairs (x, y) that satisfy $24x+40y=8$.

9. (5%) Let $f: Z^+ \to Z^+$, $f(n)$ is defined as the number of positive integers less than or equal to n that are relatively prime to n. Find $f(1763)$.

10. (5%) State the Fermat last theorem.

11. (5%) Let p be prime. Prove that $a^p \equiv a \bmod p$ for any positive integer a.

12. (5%) At least how many bits are required to convert each of 10 digits, 0, 1, ..., 9 and 26 capital letters A, B, C, ..., Z to a binary bit string without causing conflicts?