Magnetic inverse Compton scattering above polar caps

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Abstract. Following Kundt & Schaaf (1993), it is shown that under certain conditions, inverse Compton scattering is the dominant mechanism for braking escaping charges above polar caps rather than the usually invoked curvature radiation. These conditions are: (i) the polar-cap temperature is that indicated by X-rays from PSRs and (ii) either $\gamma < m_e c^2/kT = 10^{3.5} T_{0.5}^{-1}$ or $\gamma < 10^7 (\sigma/\sigma_T)^{0.35} \alpha_s^{0.5} T_{0.5}^{0.5}$, where $\gamma$ is the Lorentz factor of escaping charges, $T$ polar-cap temperature, $\alpha$ curvature radius of magnetic fieldlines, $\sigma$ cross section of magnetic inverse Compton scattering, and $\sigma_T$ Thomson cross section. The cyclotron resonance of magnetic inverse Compton scattering sets an upper bound on the energy of those escaping charges if the strength of the parallel electric field $E$ satisfies the condition $E \lesssim 10^{3.5} G B_{13} T_{0.5}$, with $B$ being magnetic field strength.

Keywords: radiation mechanisms: non thermal – pulsars: general – X-rays: stars

1. Introduction

Although it is well established that pulsars (PSRs) are fast rotating, magnetized neutron stars, since their discovery, they continue to challenge the knowledge and imagination of astrophysicists. The high brightness temperatures of radio emission from PSRs are not yet understood. The origins of radiation at other frequencies are also in controversies. Goldreich & Julian (1969) proved that there must be a magnetosphere surrounding a rotating, magnetized neutron star. After that, many different models have been proposed to account for the actual mechanisms of how PSRs manage to radiate (for reviews and general observational descriptions, see Michel 1982, 1991a; Backer 1988; Lyne & Graham-Smith 1990). Despite the various controversies in different pulsar models, charged particles escaping from polar caps are invoked in most of them. A polar cap is a region in which open fieldlines intersect a neutron star’s surface. Charged particles escape through the open-fieldline sectors due to centrifugal losses near the speed-of-light cylinder and are believed to result in electric fields parallel to magnetic fieldlines above the polar caps because of the deviation of the space charge density from the corotation charge density. It is these electric fields which drag charged particles out of neutron stars to replenish the centrifugal losses and to give rise to radiation in different frequency regimes. To understand further detail it is important to know how energetic these charged particles accelerated by the parallel electric fields can be.

On the other hand, there are many pulsars observed at X-rays which suggest the temperature at the polar caps is several $10^6$ K (e.g., Halpern & Holt 1992; Halpern & Ruderman 1993, for Geminga; Ogelman & Finley 1993, for PSR 1055–52; Becker & Trümper 1993, for PSR J0437–4715; Yancopoulos et al. 1994, for PSR 1929+10; Ogelman 1994). With this temperature, there will be a hot lepton corona above the polar caps of scale height (Kundt & Schaaf 1993)

$$H = kT/m_e g = 10^{3.5} T_{0.5},$$

which could be affected by the electric field above the polar cap corona. Unfortunately, the parallel electric field is not well derived so far. Its strength is uncertain and its temporal behavior is highly controversial. But to some extent it is agreed that there forms a voltage above a polar cap and the charges accelerated by the voltage produce $\gamma$-rays (via inverse Compton scattering, see Sect. 2), which in turn ignite a pair-production cascade. In Ruderman & Sutherland’s model (1975) a vacuum gap forms above the polar cap and this gap breaks down as its height grows so that the mean free path of pair production by a $\gamma$-ray photon (emitted through curvature radiation in Ruderman & Sutherland 1975) in the pulsar’s magnetic field is comparable with the height of the gap. In some other models the voltage above the polar cap is argued to be oscillatory, that is, it changes its sign periodically (Sturrock 1971; Kundt & Schaaf 1993). An oscillatory discharge avoids the problem of return currents.

So far, the charges escaping from polar caps are thought to gain energy due to the electric acceleration and lose energy through curvature radiation which is usually ignorable except for the case of a curvature-radiation-reaction limited version of polar cap and outer gap models in which the Lorentz factor of charges is large (e.g. Ruderman & Sutherland 1975; Cheng et al. 1986; Michel 1991b). Depending on the details of different models, the Lorentz factor of the primary escaping charges is
frequently taken a large value (~ $10^5$). But as pointed out by Kundt & Schaaf (1993) and indicated by X-rays from PSRs, if we interpret that as thermal emission from stellar surfaces, there should be a hot (dense) photon bath above polar caps. This photon bath will exert a braking force on escaping charges via scattering. Because of this new insight, it is worth investigating how significantly inverse Compton scattering can affect the Lorentz factor which the escaping charges can get. In Sect. 2, it is shown first that above polar caps, inverse Compton scattering is the dominant mechanism for braking escaping charges rather than curvature radiation. The effect of magnetic inverse Compton scattering is explored in Sect. 3. And then, in Sect. 4, an upper bound to the energy of those escaping charges is obtained.

2. Compton scattering or curvature radiation?

Charges accelerated by the parallel electric field above polar caps will lose energy when travelling through the hot photon bath above the polar cap corona via scattering those X-ray photons up to γ-rays. However, in the earlier literature, the mechanism for producing γ-rays is thought to be curvature radiation (e.g. Ruderman & Sutherland 1975). The energy-loss rate of an electron or positron due to curvature radiation is, when expressed in terms of the time derivative of the Lorentz factor,

$$\dot{\gamma}_{\text{curv}} = \frac{1}{mc^2} \frac{2}{3} \gamma^4 \frac{e^2 c^2}{a^2} = 10^{-8.3} \text{s}^{-1} \gamma_{2.5}^4 a_{-8}^{-2},$$

where $a$ is the local curvature radius of the magnetic fieldlines, and that due to inverse Compton scattering is

$$\dot{\gamma}_{\text{Compton}} \approx \frac{1}{mc^2} \frac{4}{3} \gamma^2 3kT n_e \sigma T,$$

$$\gamma_{\text{Compton}} = 10^{-8.5} \text{s}^{-1} \frac{\sigma}{\sigma_T} \gamma_{2.5}^2 T_{6.5}^4,$$

where $n_e :=$ black-body density = $(2.4/\pi^2)(kT/\hbar c)^3 = 10^{20.9} \text{cm}^{-3} T_{6.5}^3$, $\sigma$ is the (magnetic) Compton cross section, $\sigma_T$ is the Thomson cross section, and an isotropic distribution is assumed. Equivalently, the corresponding $e^{-1}$-folding distances $\ell := cE/E$ (e.g. Ruderman & Sutherland 1975) are

$$\ell_{\text{curv}} = 10^{21.3} \text{cm} \gamma_{2.5}^{-1} a_{-8}^2$$

and

$$\ell_{\text{Compton}} = 10^{3.5} \text{cm} \frac{\sigma}{\sigma_T}^{-1} \gamma_{2.5}^{-1} T_{6.5}^{-4}.$$  

It is obvious that even without considering the cyclotron resonance, the Compton $e^{-1}$-folding distance is much shorter than that of curvature radiation, that is,

$$\frac{\ell_{\text{Compton}}}{\ell_{\text{curv}}} = 10^{-17.8} \frac{\sigma}{\sigma_T}^{-1} \gamma_{2.5} a_{-8}^{-2} T_{6.5}^{-4}.$$  

On the other hand, if the Lorentz factor is so large that $\gamma > mc^2/kT = 10^{3.5} T_{6.5}^{-1}$, we take the extreme Klein-Nishina limit (Blumenthal & Gould 1970, Eq. (2.59)):

$$\dot{\gamma}_{\text{Compton}} = \frac{1}{mc^2} \frac{\sigma_T}{16} \left( \frac{4\pi kT}{mc^2} \right) \int \ln \left( \frac{4\pi kT}{mc^2} - 1.98 \right).$$

The quantity in the right parenthesis does not change much with different $\gamma$s and $T$'s. It is about 2.86 for $\gamma = 10^5$ and $T = 10^{6.5}$K. Taking that to be order of unity, then

$$\dot{\gamma}_{\text{Compton}} = 10^{9.8} \text{s}^{-1} \frac{\sigma}{\sigma_T} T_{6.5}^2.$$ (8)

The corresponding $e^{-1}$-folding distance is

$$\ell_{\text{Compton}} = 10^{5.7} \text{cm} \frac{\sigma}{\sigma_T}^{-1} \gamma_{2.5} a_{-8}^{-2} T_{6.5}^{-2},$$

which makes the ratio of $\ell_{\text{Compton}}$ to $\ell_{\text{curv}}$ (Eq.(9) to Eq.(4)) be

$$\frac{\ell_{\text{Compton}}}{\ell_{\text{curv}}} = 10^{-8.1} \frac{\sigma}{\sigma_T}^{-1} \gamma_{2.5} a_{-8}^{-2} T_{6.5}^{-2}.$$ (10)

We can conclude that the energy loss of those escaping charges due to curvature radiation is ignorable as long as (cf. Kundt & Schaaf 1993)

$$\gamma \ll 10^{7} \frac{\sigma}{\sigma_T} 0.25 \gamma_{5.0} 0.5 T_{6.5}^{0.5}.$$ (11)

One should keep in mind that the conclusion is obtained under the assumption of an isotropic photon distribution, and we have taken the quantity in the right parenthesis in Eq.(7) as order of unity.

When charges begin to be electrically accelerated, they have a small Lorentz factor, so at the beginning, inverse Compton scattering is the dominant mechanism for braking charges above polar caps rather than curvature radiation. The presence of a magnetic field makes the cross section $\sigma$ different from the Thomson cross section $\sigma_T$, especially when cyclotron resonance occurs. How important is the cyclotron resonance? Can the resonance braking compete with the acceleration by the parallel electric field and set an upper bound on the energy of the escaping charges? These are the subjects we shall deal with in the following sections.

The production of γ-rays via inverse Compton scattering near a neutron star has been considered as a mechanism for gamma-ray bursters (Ho & Epstein 1989; Dermer 1990; Smith & Epstein 1993). It was proposed to be the main mechanism for igniting pair-production cascades above the polar cap (Kundt & Schaaf 1993) and further used to account for the γ-ray emission from pulsars (Dermer & Sturmer 1994; Sturmer & Dermer 1994). The radiative braking of relativistic $e^\pm$ in strong magnetic fields was also investigated by Kardashev et al. (1984).

3. Magnetic inverse Compton scattering

In this section we are interested in the energy-loss rate of relativistic $e^\pm$ due to magnetic inverse Compton scattering in a photon bath. The energy-loss rate is, when expressed in terms of the time derivative of the Lorentz factor,

$$\dot{\gamma} = - \int d\epsilon_s \int d\zeta_s \frac{dN_{ph}(\epsilon_s, \zeta_s; \gamma)}{d\epsilon_s d\zeta_s} (\epsilon_s - \epsilon),$$ (12)

where $\epsilon := h\nu/mc^2$ is the dimensionless energy of the unscattered photon and $\epsilon_s$ is that of the scattered photon. $\zeta_s$ is the
cosine of the angle between the propagation of the scattered photon and the motion of the electron. The differential photon-production rate by an electron with Lorentz factor $\gamma$ is (Ho & Epstein 1989)

$$\frac{dN_{\gamma\gamma}(\epsilon, \zeta; \gamma)}{d\epsilon d\zeta} = c \int d\epsilon' \int d\zeta' n_{\epsilon\zeta}(\epsilon, \zeta, \epsilon', \zeta')(\frac{d\sigma'}{d\epsilon' d\zeta'})(\frac{d\epsilon'}{d\zeta'})(1 - \beta \zeta)$$

(13)

where $\zeta$ is the same as $\zeta_s$ but for unscattered photons, and all the symbols with a prime stand for the corresponding quantities in the comoving frame. $n_{\epsilon\zeta}(\epsilon, \zeta) d\epsilon d\zeta$ is the differential number density of photons with energies between $\epsilon$ and $\epsilon + d\epsilon$ directed in the cosine interval $\zeta$ to $\zeta + d\zeta$.

Eq. (12) can be further written as

$$\dot{\gamma} = -c \int d\epsilon \int_0^{\gamma(1 - \beta \zeta)^{-1}} d\epsilon' n_{\epsilon\zeta}(\epsilon, \zeta)(1 - \beta \zeta) \times \int_{\epsilon - \epsilon'}^{\epsilon'} d\epsilon' \int d\zeta' \left( \frac{d\sigma'}{d\epsilon' d\zeta'} \right) .$$

(14)

The upper limit of the integration over $\epsilon$ is set so that the Klein-Nishina correction can be ignored, i.e. $\epsilon' < 1$. The Klein-Nishina reduction gets important only when $\gamma > 10^3 T_{B,6.5}^{-1}$. The magnetic Compton cross section is well approximated by

$$\frac{d\sigma'}{d\epsilon' d\zeta'} = \frac{3\pi}{8} \epsilon \epsilon' \epsilon' - \epsilon' \times \left[ (1 - \zeta'^2)(1 - \epsilon'^2) + \frac{1}{4} (1 + \zeta'^2)(1 + \epsilon'^2)(g_1 + g_2) \right] ,$$

(15)

provided $\epsilon' \ll 1$ (Herold 1979). The terms $g_1$ and $g_2$ are

$$g_1(u) = \frac{u^2}{(u + 1)^2} , \quad g_2(u) = \frac{u^2}{(u - 1)^2 + u^2} ,$$

(16)

where $u := \epsilon'/\epsilon_B$, $\epsilon_B := h\omega_B/m_e c^2$, $\omega_B := eB/m_e c$, and $w := 2\epsilon_B/\alpha$, $\alpha$ is the fine-structure constant.

With the cross section Eq. (15) and noting that $\epsilon' = \gamma\epsilon(1 - \beta \zeta)$ and $\zeta' = (\zeta - \beta)/(1 - \beta \zeta)$, after integrating over $\epsilon'$ and $\zeta'$, we have (Appendix A)

$$\dot{\gamma} = \frac{3}{8} \omega B \int d\zeta \int d\epsilon' n_{\epsilon\zeta}(\epsilon, \zeta) \beta \epsilon' \times \left[ -\frac{4}{3} \zeta'(1 - \zeta'^2) - \frac{2}{3} \zeta'(1 + \zeta'^2) (g_1 + g_2) \right] ,$$

(17)

which is suitable for computation.

In Eq. (16), if $u$ is much larger than unity, we have $g_1 \rightarrow 1$ and $g_2 \rightarrow 1$. This is the case of either $\gamma \gg \omega_B/10^{-2} B_{12}/m_e c^2$ or the magnetic field weakening, and the cross section approaches that of the non-magnetized case. On the other hand, if $u \ll 1$, we have $g_1 \rightarrow 0$ and $g_2 \rightarrow 0$. This holds either for $\gamma \rightarrow 1$ or for a very strong magnetic field. The presence of magnetic fields will reduce the cross section for a charge with relatively low energy. The cyclotron resonance occurs when $u \approx 1$, that is,

$$\gamma \epsilon(1 - \beta \zeta) \approx \epsilon_B .$$

(18)

In such a case, $g_2 = 1/\omega^2 = 10^{6.2} B_{13}^{-2}$ because $\omega = 10^{-3.1} B_{13}$. But for a given photon distribution and a given strength of magnetic fields, the resonance is influential only when the resonance condition is met in the range where most photons are distributed. For a black-body photon bath, taking $\epsilon \approx 3kT/m_e c^2$ and $(1 - \beta \zeta) \approx 1$ in Eq. (18), the resonance peaks when the Lorentz factor of the electron is

$$\gamma_{\text{peak}} \approx 10^2 B_{13} T_{B,6.5}^{-1} .$$

(19)

The above discussion is shown in Fig. 1, in which an isotropic black-body photon distribution is taken, that is, in Eq. (17),

$$n_{\epsilon\zeta}(\epsilon, \zeta) = 4\pi \left( \frac{m_e c^3}{\hbar} \right)^3 \frac{\epsilon^2}{\epsilon_{\text{em}}^2 e^{\epsilon/\epsilon_B} - 1} ,$$

(20)

and $T = 10^6 K$, $B = 10^{13}$ G. For lower $\gamma$ the energy-loss rate is mainly contributed by the term without $g_1$ and $g_2$ in the integrand in Eq. (17). It is smaller than that of the non-magnetized case ($g_1 = g_2 = 1$). As $\gamma$ increases, resonance gets more and more important, and the $g_2$ term dominates. The same computation with $B = 10^{12}$ G is shown in Fig. 2.
To simulate the situation above a polar cap, the photon distribution is no longer isotropic and the attenuation of the photon density should be taken into account. Given the surface temperature and the strength of the magnetic fields, the energy-loss rate $\dot{\gamma}$ is a function of both $\gamma$ and the position above the polar cap. Assuming a circular polar cap and neglecting the curvature of the stellar surface and the magnetic field lines, the function $\dot{\gamma}(\rho_0, z_0; \gamma)$ can be easily obtained by replacing the part $\int d\zeta (\rho_0, z_0) \zeta d\zeta$ in Eq. (17) by $d^2A (\zeta(\rho_0, z_0)/4\pi)$, where $d^2A$ is the surface integral over the whole polar cap, $r$ is the corresponding distance from the stellar surface to the point denoted by $\rho_0, z_0$, and $n_\nu d\nu = (8\pi^2/c^3(e^h/kT - 1))d\nu$ is the black-body photon number density. Figure 3 and Fig. 4 show the energy-loss rate as a function of $\gamma$ at the position directly above the center of a polar cap and with a height of one half polar cap radius. The general behavior of $\dot{\gamma}$ in this computation is similar to that in Fig. 1 and Fig. 2. The only difference is that the lack of head-on photons makes the influence of resonance occur at a higher $\gamma$ by a factor of about 3. Therefore, the Lorentz factor for the peak resonance above a polar cap is raised from that in Eq. (19) (roughly) to

$$\gamma_{\text{peak}} \approx 10^{2.5} B_{13}^{1 - 6.5}. \quad (21)$$

From Fig. 1 – Fig. 4 one can see that the influence of the resonance is significant for some range of $\gamma$ around $\gamma_{\text{peak}}$, of order $\approx 0.5$ in logarithm.

4. An upper bound to the energy of escaping charges

How significant is the cyclovolt resonance? As mentioned in the last section, given a photon distribution, the peak resonance occurs when the resonance condition Eq. (18) is met for most photons. To explore its significance, we can define an average resonance cross section $\langle \sigma \rangle_{\text{res}}$ as (see Eq. (15))

$$\langle \sigma \rangle_{\text{res}} = \frac{\int_0^\infty d\epsilon \int d\zeta n_{\epsilon, \zeta} \int d\epsilon' \int d\zeta' \left( \frac{3}{8} \delta(\epsilon' - \epsilon')(1 + \zeta'^2g_2) \right)}{\int_0^\infty d\epsilon \int d\zeta n_{\epsilon, \zeta}} \quad (22)$$

where $\zeta'^2$ in Eq. (15) is taken 1 for relativistic charges and $g_2$ is defined in Eq. (16). For a black-body photon bath, noting that at peak resonance $\gamma kT (1 - \beta \zeta) \approx c_B m_B c^2$ holds and $(1 - \beta \zeta)$ is of order unity for most angles, we have (Appendix B)

$$\langle \sigma \rangle_{\text{res}} \approx \frac{\sigma_{\gamma} \sigma_T}{4\epsilon_0} = 10^{2.5} \sigma_T B_{13}^{1 - 6.5}. \quad (23)$$
Eq. (23) is valid only for
\[ B > kT m_e c / e \hbar = 10^{10.3} G T_{6.5} \]  
so that the peak resonance can take place. In Fig. 1 - Fig. 4 the relative magnitude of the energy-loss rate for magnetized and non-magnetized Compton scattering around \( \gamma_{\text{peak}} \) confirms the approximations made in deriving Eq. (23), which gives the information of how significant the cyclotron resonance is. Here we would like to point out one mistake in Kardashev et al. (1984): Their Eq. (17) should read \( f_{\text{rad}}^{(B)} \big|_{\text{max}} \approx 10^{3.7} f_{\text{rad}}^{(0)} B_{12}^{-1} \) (the factor \( \gamma^2 \) in their Eq. (6) is dismissed). Consequently, the correct form of their Eq. (39), the ‘\( e^{-1} \)-folding distance’ of resonant Compton scattering, which they call the ‘mean free path’ inadequately, should be \( L_B \approx 10^{2.2} \text{cm} T_{6.5}^{-1} \). For comparison, we should rewrite Eq. (3) for a photon distribution of isotropy in a hemisphere, that is, the angle between photon’s and particle’s motion ranges only from 0 to \( \pi/2 \), which is the case discussed in Kardashev et al. (1984) and in Dermer (1990). In such a case, we have (see, e.g. Melrose 1980, vol. I, p.133)
\[ \gamma_{\text{Compton}} \approx \frac{1}{m_e c^2} \frac{1}{3} \gamma^2 3kT \frac{n_e}{2} c \sigma = 10^{8.6} \text{s}^{-1} (\sigma/\sigma_T) \gamma_{2.5}^2 T_{6.5}^{4.4}, \]  
cf. Eq. (3). So the \( e^{-1} \)-folding distance, when the peak resonance occurs (i.e. Eqs. (21) and (23) apply), is
\[ \ell_{\text{Compton}} = 10^{1.9} \text{cm} T_{6.5}^{-1}. \]  
There is a discrepancy of a factor of 2 between Eq. (26) and Kardashev’s result. This discrepancy will get smaller if we use \( \sigma_T = 6.65 \times 10^{-25} \text{cm}^2 \) instead of \( 10^{-24} \text{cm}^2 \) employed in all the above discussion. On the other hand, Eq. (26) agrees with results obtained in Dermer (1990); see curves 2 and 3 in his Fig. 8 (but he also uses the inadequate terminology, ‘mean free path’, which is in general different from what is discussed here). Especially, Eq. (26) shows that at peak resonance, the \( e^{-1} \)-folding distance depends only on temperature’s cubic power inversely. This is perfectly shown in Dermer’s Fig. 8.

Whether the cyclotron resonance can set an upper bound on the energy of the charges escaping from polar caps depends on the strength of the parallel electric field which accelerates those charges. If the electric field is strong enough, those charges will be accelerated to a Lorentz factor beyond \( \gamma_{\text{peak}} \) before scattering takes place. On the other hand, if the electric field is weak, there will be an upper bound on the Lorentz factor, with which the energy gained due to electric acceleration and that lost due to Compton scattering are comparable with each other. A simulation is shown in Fig. 5 for the Lorentz factor of the escaping charge as a function of its distance from the polar cap. For lack of knowledge about electric fields above a polar cap, we assume constant fields. In addition, the height of the region in which electric acceleration acts is not well understood, so the result shown in Fig. 5 may not be realistic.

For the lower curves with weaker electric fields, \( \gamma \) grows at first up to a value such that \( \gamma = \gamma_E + \gamma_M \) is nearly zero, where the subscripts ‘E’ and ‘M’ denote the electric acceleration and magnetic Compton scattering respectively. Then \( \gamma \) increases slowly. In this computation (\( T = 10^{6.5} \text{K}, B = 10^{13} \text{G} \)) \( \gamma_{\text{peak}} \approx 10^{3.5} \), see Eq. (21) and Fig. 3. After \( \gamma \) grows beyond the range in which resonance is influential, \( \gamma_M \) decreases and \( \gamma \) increases rapidly to the value \( \gamma_E \). Due to the attenuation of the photon density as well as a smaller angular range of the incoming photons from the polar cap, \( \gamma_M \) takes smaller values at higher altitudes, so for a weaker electric field the take-off of the curve occurs at a larger height. For curves with stronger electric fields, the effect of magnetic inverse Compton scattering is relatively ignorable.

For resonant Compton scattering to play an important role in producing \( \gamma \)-rays above polar caps, it is required that near a polar cap \( \gamma_E \ll \gamma_M \) at resonance. The energy-loss rate at resonance is (Eqs. (21), (23), and (25))
\[ \gamma_M \approx 10^{11.1} \text{s}^{-1} B_{13} T_{6.5}^{2.5}, \]  
and for electric acceleration we have
\[ \gamma_E = e \beta E / m_e c \approx 10^7 \text{s}^{-1} E_0. \]  
So the strength of the parallel electric field should be
\[ E \lesssim 10^5 \text{G} B_{13} T_{6.5}^{2}. \]  
The corresponding upper bound on \( \gamma \) is
\[ \gamma \lesssim \gamma_{\text{peak}} \approx 10^{5.5} B_{13} T_{6.5}^{-1}, \]  
see Eq. (21). Because the resonance (\( \sigma/\sigma_T \gg 1 \)) effects a broad range around \( \gamma_{\text{peak}} \), the Lorentz factor of the accelerated charges
could be as large as $10^3$, depending on the actual spatial and temporal structure of the electric field. On the other hand, too weak an electric field is not desired because (i) the charges must be energetic enough to produce hard $\gamma$-rays for the succeeding pair-production, and (ii) for too low a $\gamma$, there is no significant resonance, and the mean free path for creating a $\gamma$ photon will be too long (see the discussion below). From Fig. 3 one can infer that the approximate lower bound is (by requiring $\gamma \gtrsim 10^{2.3}$)

$$E \gtrsim 10^3 G B_{13} T_{6.5}^2.$$  \hspace{1cm} (31)

In Goldreich & Julian (1969), the potential difference between the pole and the last open fieldline is $\frac{1}{2} \left( \frac{\partial B}{\partial x} \right)^2 \rho B$. Then one can roughly estimate the electric field to be $E_{12} \sim \frac{\partial B}{\partial x} B \sim 10^5 G \Omega_2^2 B_{13}$. Michel (1974) obtained an electric field of strength $< (4m_e c^2 \Omega_2 B / e)^{6/5} = 10^{5.7} G \Omega_2^{1/5} B_{13}^{1.5}$. Ruderman & Sutherland (1975) assumed a vacuum gap developed above the polar cap, and their electric field is $E_{12} = \frac{2 R \phi}{h}$, where $h$ is the height of the gap and $h \ll R$. If taking $h \gtrsim R_{pc}$, then $E_{12} \sim \left( \frac{\partial B}{\partial x} \right)^{1.5} B \sim 10^6 G \Omega_2 B_{13}$. In Arons & Scharlemann (1979), $E_{12} \sim \left( \frac{\partial B}{\partial x} \right)^{2.5} B \sim 10^{2.5} G \Omega_2 B_{13}$. In these models, magnetic inverse Compton scattering is not considered. These diverse values of electric field strengths reveal the poor understanding of the actual processes which make a neutron star 'visible'. On the other hand, they also show that the condition in Eq.(29) is not so exotic. A physical picture of the polar-cap region containing the consideration of magnetic inverse Compton scattering is required and now in preparation (Chang 1995). If the Lorentz factor of escaping charges is really bounded as we discuss here, that is, $\gamma \lesssim 10^3$, one immediate consequence is that the current-polar-cap models for $\gamma$-rays from PSRs have to be reexamined. In Harding & Daugherty (1993) and Daugherty & Harding (1994), based on earlier polar-cap models, they take an 'injection' energy of primary charges corresponding to a Lorentz factor of $\gamma \sim 10^7$. In Dermer & Sturmer (1994) and Sturmer & Dermer (1994), the Lorentz factor of primary charges is taken $\lesssim 10^5 \sim 10^6$.

A neutral excess of a factor $\xi \approx 10^4$ in the pulsar's $e^\pm$ winds is required for two reasons: (i) the number density of $e^\pm$ responsible for the size of pulsar wind nebulae (Kundt 1986; Kundt & Chang 1992) is $\xi$ times of $\rho_{eq} / e$, where $\rho_{eq}$ is the Goldreich-Julian corotation charge density and (ii) the radio emission needs high densities of the sources (e.g., Ruderman & Sutherland 1975; Kundt & Schaff 1993). These pairs should form within a distance not too far away from the star, so at first there must be enough $\gamma$-rays. The mean free path of Compton scattering is

$$l_{Compton} \approx \frac{1}{B_{13}} \frac{\sigma}{\sigma_T} \gtrsim 10^{-3} \text{cm} B_{13} T_{6.5}^{-3}$$  \hspace{1cm} (32)

where Eq.(33) holds for the cyclotron resonance. The length of the region in which the electric acceleration acts and the charge can produce enough $\gamma$-rays via resonant Compton scattering is then

$$l_{acc} = \xi l_{Compton} = 10^4.9 \text{cm} B_{13} T_{6.5}^{-3} \xi_4.$$  \hspace{1cm} (34)

This length is often of order of the polar cap radius

$$R_{pc} = R \frac{\sqrt{\gamma}}{c} = 10^{4.3} \text{cm} \Omega_1^{1/2}.$$  \hspace{1cm} (35)

The total voltage is, when Eq.(29) holds and therefore Eqs.(33) and (34) can be used,

$$e \phi_{acc} = e L l_{acc} \lesssim 10^{12.9} \text{eV} B_{13} T_{6.5}^{-1} \xi_4,$$  \hspace{1cm} (36)

which is smaller than the voltage across the polar cap (Goldreich & Julian 1969; Kundt & Schaff 1993)

$$e \phi_{pc} = 10^{14.2} \text{eV} B_{13} \Omega_1^2.$$  \hspace{1cm} (37)

Thus, we have shown that resonant Compton scattering can provide the necessary number of $\gamma$-rays for creating the huge neutral excess.

For msec pulsars, if the magnetic field at the polar caps is really so weak as inferred from the timing observation and the assumption of a pure dipole field (Taylor et al. 1993; Thorsett 1994), the cyclotron resonance will not be effective, or $\gamma_{peak}$ will be too small to make hard enough $\gamma$-rays and to be an upper bound for a wide range of electric field strengths (see Eqs.(21) and (29)). The magnetic field of msec PSRs need not be weak if one considers the presence of higher multipoles (Krolik 1991; Kundt & Schaff 1993; Chang 1994). The electric field parallel to magnetic fieldlines above the polar caps is still uncertain. If it does not meet the condition Eq.(29), we will need additional mechanisms, like triplet pair production (Kardashev et al. 1984; Mastichiadis et al. 1986), to get the large number density of the pair plasma, because the mean free path of Compton scattering $l_{Compton}$, Eq.(32), is too long.

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Appendix A: derivation of Eq.(17)

This appendix provides some intermediate steps omitted in the derivation of Eq.(17). The factor $(e_\sigma - e)$ in Eq.(14) can be written as $e_\nu / \gamma (1 - \beta_\nu e) - e_\nu / \gamma (1 - \beta_\nu e)$. Noting that $(\gamma (1 - \beta_\nu e))^{-1} = \gamma (1 + \beta_\nu e)$ (also, $(\gamma (1 - \beta_\nu e))^{-1} = \gamma (1 + \beta_\nu e)$), the integration over $e_\nu$ in Eq.(14) yields

$$\int \delta (e_\nu - \epsilon) de_\nu = \gamma e_\nu \beta \epsilon (\xi_\nu - \xi'_\nu),$$  \hspace{1cm} (A1)

where the delta function comes from the differential cross section, Eq.(15), and only related terms are shown here. The term containing $\xi'_\nu$ in Eq.(A1) gives zero contribution when integrating over $\xi'_\nu$ in Eq.(14). Because $\gamma (1 - \beta_\nu e) de_\nu = de'_\nu$, we get Eq.(17).
Appendix B: average resonance cross section of magnetic inverse Compton scattering

The numerator in the definition of \( \langle \sigma \rangle_{\text{res}} \), Eq.(22), can be further written as

\[
\int d\epsilon \int d\zeta n_{\epsilon, \zeta} \int d\epsilon' \int d\zeta' \left( \frac{3 \sigma_T}{8} \delta(\epsilon' - \epsilon)(1 + \zeta'^2) \frac{\Omega}{2} \right) \\
= 2\pi \left( \frac{m_e c}{h} \right)^3 \sigma_T \int d\zeta \left( \frac{\epsilon_B}{\gamma(1-\beta\zeta)} \right)^3 \left( \frac{u^2}{\gamma} - 1 \right)^2 \frac{u^2}{(u-1)^2 + w^2} ,
\]

where Eq.(20) is used for the photon distribution and the integrations over \( \epsilon' \) and \( \zeta' \) are done. At peak resonance, \( \gamma kT(1-\beta\zeta) \approx \epsilon_B m_e c^2 \) holds and \( (1-\beta\zeta) \) is of order unity for most angles. The terms containing \( u \) give a contribution of roughly \( 2/w \) after integrating over \( u \). We gain another factor of 2 from the integration over \( \zeta \). The photon number density is simply

\[
\int d\epsilon \int d\zeta n_{\epsilon, \zeta} = n_{\gamma} = \frac{2\pi}{\pi^2} \frac{4}{(kT/hc)^3}.
\]

So we get

\[
\langle \sigma \rangle_{\text{res}} \approx \sigma_T \left( \frac{4\pi c}{\gamma} \right)^3 \left( \frac{\epsilon_B}{\gamma} \right)^3 \frac{1}{1.7} \frac{4}{w} \approx \frac{\sigma_T}{4w} ,
\]

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