

國 立 清 華 大 學 命 題 紙

98 學年度工業工程與工程管理學系 甲、乙、丙、丁組 碩士班入學考試

科目 統計學 科目代碼 1401,1501,1601,1701 共 2 頁第 / 頁 \*請在【答案卷卡】內作答

1. (5%) Let  $X$  and  $Y$  are independent random variables with variance  $\sigma_x^2 = 2$  and  $\sigma_y^2 = 1$ , find the variance of the random variable  $Z=2X-3Y+5$ .

2. (5%) Let  $Y$  be distributed with Pareto distribution with parameters  $X_0$  and  $\theta$ . The probability density function of such random variable is

$$f(y) = \begin{cases} \frac{\theta X_0^\theta}{\theta+1} & y > X_0 \text{ with } X_0, \theta > 0 \\ y & \text{otherwise} \end{cases}$$

Find the variance of  $Y$ .

3. (20%) Let  $x_1, x_2, \dots, x_n$  be a random sample from a normal distribution with mean  $\mu$  and variance  $\sigma^2$ .

1) Find the maximum likelihood estimates of  $\mu$  and  $\sigma^2$  given in this example.

2) Show that  $\bar{x} = \sum_{i=1}^n \frac{x_i}{n}$  is a sufficient statistic for the parameter  $\mu$  if  $\sigma=1$ .

3) Show that  $\bar{x} = \sum_{i=1}^n \frac{x_i}{n}$  and  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$  are jointly sufficient statistics for the parameters  $\mu$  and  $\sigma^2$ .

4. (10%) Let  $f(X;\theta) = \frac{e^{-.5(X-\theta)^2}}{\sqrt{2\pi}}$  be a probability density function. Assuming  $H_0:\theta=\theta_0$ , and  $H_1:\theta=\theta_1$  where  $\theta_0 > \theta_1$ , find the best critical region,  $C$ , for this test.

5. (10%) If  $(X, Y)$  has a bivariate normal distribution, find the conditional distribution of  $Y$  given  $X=x$ .

6. (20%) Consider a doctor pondering the following dilemma: "If I'm at least 80 percent certain that my patient has this disease, I will recommend surgery; whereas if I'm not quite as certain, then I recommend additional tests that are expensive and sometimes painful." Now, initially I was only 60 percent certain that Mr. Jones has the disease, so I ordered the Test A, which always gives a positive result when the patient has the disease and almost never gives a positive result when he is healthy. The test result was positive, and I was all set to recommend surgery. However, Mr. Jones informed me, for the first time, that he is a diabetic (糖尿病患者). This information complicates this decision because, although it doesn't change my original 60 percent estimate of his chances of having the disease, it does affect the interpretation of the results of the Test A since Test A, while never yielding a positive result when the patient is healthy, does unfortunately yield a positive result 30 percent of the time in the case of diabetic patients not suffering from the disease. Now what should the doctor do? more tests or immediate surgery?

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7. (20%) A manufacturer would like purchase new inspection equipment. In particular, there are 10 kinds of equipments can be selected. Therefore, he collects 350 products drawn from a factory for comparing the power of different inspection equipments and the test is determined as follows:

$H_0$  : the product is not defective

$H_1$  : the product is defective

In particular, among the 350 products, 50 products are known to be defective. The inspection results of using different equipments are recorded in the following table.

Equipment	Defective product		Qualified product	
	n	n/50	m	m/300
A	17	34%	12	4%
B	33	66%	45	15%
C	40	80%	90	30%
D	10	20%	3	1%
E	48	96%	147	49%
F	42	84%	120	40%
G	28	56%	15	5%
H	40	80%	60	20%
I	15	30%	6	2%
J	28	56%	18	6%

If you are the decision maker, which type of equipment would you purchase? Why? Please derive specific decision rules and also highlight remarks such as false positive and false negative rate.

8. (10%) Consider the following probability density function:

$$f(x) = c(1 + \theta x), \quad -1 \leq x \leq 1$$

- 1) Find the value of the constant  $c$ .
- 2) What is the moment estimator for  $\theta$ .
- 3) Show that  $\hat{\theta} = 3\bar{X}$